TARGET MOTION ANALYSIS FROM DISTRIBUTED SENSORS AND SIMULTANEOUS SENSORS PATTERN CALIBRATION.

F. MARTINERIE, F. YWANNE

THOMSON SINTRA A.S.M.
1 av. Aristide Briand, 94117 Arcueil Cedex, FRANCE

1. INTRODUCTION

We address the problem of target motion analysis (TMA) using distributed sensors when the positions of these sensors are not completely known. We have recently presented a new method which simultaneously achieves TMA and the sensors pattern calibration from doppler measurements [1]. However the results obtained were corrupted by a slight residual bias. In the present paper, we explain the theoretical basis of our method and extend it to various types of measurement (ranges, dopplers, and azimuths). Moreover, we introduce an improvement based on an iteration of our algorithm and discuss the results obtained.

In the following, we firstly define our problem formulation. We then recall our algorithm and justify it from two different points of view. Next we formulate and discuss the residual bias, and introduce the improvement of our method. Simulation results are presented and discussed, and we finally state at which conditions the residual bias can be cancelled.

2. PROBLEM FORMULATION

2.1 Geometric description

The system consists of a moving target and a pattern of N distributed sensors lying in the same horizontal plane. The target motion is supposed to be rectilinear and unaccelerated. Consequently, it is completely characterized by the target vector : $X_T(t)=(x(t), y(t), v_x, v_y)^T$.
The N sensors locations are defined by the vector :

$$X_s = (x_{s1}, y_{s1}, \ldots, x_{sN}, y_{sN})^T.$$

The measurements coming from any sensor i can be of various types :

- ranges : $d_i(t) = ((x(t)-x_{si})^2+(y(t)-y_{si})^2)^{1/2}$
- dopplers : $f_i(t) = f_0(1- ((x(t)-x_{si})v_x+(y(t)-y_{si})v_y)/cd_i(t))$
- azimuths : $\beta_i(t) = \tan^{-1}(x(t)-x_{si}/y(t)-y_{si})$

2.2 Identifiability

Our purpose is to estimate simultaneously both the target kinematics parameters and the sensors locations. From ranges or doppler measurements, a geometrical analysis shows that the target parameters X_T and the distances between sensors can be easily derived. But additional information are required in order to localize them entirely. Nevertheless, in many applications, the sensors positions are not completely unknown : "a priori" positions (x_{api}, y_{api}) are available for each sensor i. The error attached on these a priori positions is symboled by a covariance matrix Σ_{ap}.
In the following section, we shall see how to use these information to make the probleme identifiable.

3. MAXIMUM LIKELIHOOD ESTIMATOR

3.1 "Random" formulation

Given $X_T(t)$ and X_S, the state vector we want to estimate can be defined :

$$X(t) = (X_T(t) \mid X_S)$$
$$= (x(t), y(t), v_x, v_y, x_{s1}, y_{s1}, \ldots, x_{sN}, y_{sN})$$

The second part of this state vector has a special status, since it is considered random, with known statistics (mean position $X_{ap} = (x_{ap1}, y_{ap1}, \ldots, x_{apN}, y_{apN})$ and associated variance Σ_{ap}).
Determinist parameters being a particular case of random ones, we can then consider that the whole state vector X(t) is random, with known a priori probability distribution.
In this case, H.L. Van Trees demonstrates in [2] that if an efficient estimator exists, it is necessarily the maximum a posteriori (MAP) estimator.
Given a set of measurements Θ, the solution of our problem is given by :

$$X(t) \text{ maximizing } p(X(t)|\Theta),$$

where $p(X(t)|\Theta)$ is the probability of X(t) knowing Θ.
This maximization is equivalent to the maximization of :

$$p(\Theta|X(t)).p(X(t)),$$

so that the optimal estimator is finally given by :

$$X(t) \text{ maximizing } p(X(t), \Theta),$$

which is the maximum joint likelihood estimator (MLE).

Under Gaussian assumptions, one can write :

$p(\theta,X(t)) = p(\theta|X(t)).p(X(t))$, with :

$$p(\theta|X(t)) = K.\exp -1/2(\theta-\theta(X(t)))^T \Sigma_\theta^{-1}(\theta-\theta(X(t)))$$
$$p(X(t)) = \exp -1/2(X_s-X_{ap})^T \Sigma_{ap}^{-1}(X_s-X_{ap})$$

(as $X(t) = (X_T(t) \mid X_s)$ and as X_s is determinist, the only contribution to $p(X(t))$ comes from the a priori statistic on X_s)

The maximization of this joint likelihood is then equivalent to the minimization of the following least mean square criterion :

$$J(X(t)) = - \text{Log } (P(\theta,X(t))$$
$$= (\theta-\theta(X(t)))^T\Sigma_\theta^{-1}(\theta-\theta(X(t))) + (X_s-X_{ap})^T \Sigma_{ap}^{-1}(X_s-X_{ap})$$

3.2 Determinist formulation

This criterion J can also be interpreted as the least mean square criterion attached to the following determinist problem :

If the a priori positions X_{ap} are interpreted as observations of the sensors positions, we can define a global vector of observations M :

$$M = (\theta,\ X_{ap})\ ,$$

with an associated variance matrix $\Sigma = \left(\begin{array}{c|c} \Sigma_\theta & 0 \\ \hline 0 & \Sigma_{ap} \end{array} \right)$.

The estimation of X(t), given M and Σ is then achieved by minimizing the following LMS criterion :

$$J = (M-M(X))^T \Sigma^{-1}(M-M(X))$$
$$= (\theta-\theta(X(t)))^T \Sigma_\theta^{-1}(\theta-\theta(X(t))) + (X_s-X_{ap})^T \Sigma_{ap}^{-1}(X_s-X_{ap})$$

This criterion is the same as the one derived in the "random" case.

So finally, there are two ways of obtaining the optimal estimator :

- The first consists in considering the sensors positions as random parameters and in then defining the corresponding MAP estimator.
- The second consists in considering the a priori data on sensors positions as an additionnal part of the observation vector, and in defining the corresponding LMS estimator.

Under Gaussian assumptions, this LMS estimator is known to be efficient, and robust.

3.3 Cramer-Rao Lower Bounds (C.R.L.B.)

The estimation variance of any unbiased estimator cannot be better than the Cramer-Rao Lower Bounds, defined as the inverse of the Fisher's information matrix (F.I.M.). Consequently, a comparison between estimation results and C.R.L.B. will allow us to state about the quality and the performances of our estimator. In the determinist case, the C.R.L.B. are given by :

$$C.R.L.B. = (F.I.M.)^{-1} = E\{(dL/dX)(dL/dX)^T)\}^{-1} \text{ , where :}$$
$$L = Log(p(M|X)), \text{ and } M = (\Theta, X_{ap})$$

In the "random case", it is shown in [2] that the bounds can be calculated according to the same formula, but with a different definition of L :

$$L = Log(p(\Theta,X))$$

The developpement of these expressions leads to the same result, since $p(M|X) = p(\Theta,X)$ (see § 3.1 and § 3.2), so the two formulations of our problem are in fact equivalent.

4. ALGORITHMS

4.1 Algorithm definition

Our problem now consists in finding the value of X(t) which minimizes the L.M.S criterion J. This minimization is achieved with the help of a Gauss-Newton algorithm. This algorithm is known to be robust and efficient (i.e. the results obtained are close to the C.R.L.B.). It is also unbiased in the case where the sensors positions are exactly known (see [1] and fig 1).

4.2 Residual bias

However, in the case where the sensors positions are random, an estimation bias appears, even when the sensors positions are jointly estimated with the target characteristics (cf fig 2). This bias can be formally calculated from a first order developpement of J around the minimal value of X :

$$\text{bias} = (F.I.M.)^{-1} . \left(\begin{array}{c|c} 0 & 0 \\ \hline 0 & \Sigma_{ap} \end{array} \right) . \left(\begin{array}{c} 0 \\ \hline E(X_s - X_{ap}) \end{array} \right)$$

It clearly appears in this expression that this bias is caused by the fact that we only have a single a priori position for each sensor, so that $E(X_s-X_{ap}) = X_s-X_{ap} \neq 0$.
This bias would be cancelled if we had a statistically representative set of observed sensors positions.

4.3 Algorithm improvement

We observed in our first simulations that the estimated positions of the sensors were located between the true positions and the a priori ones (see fig 2). This fact led us to introduce an iteration in our algorithm, with the estimated positions of sensors at step i used as the new a priori positions at step i+1. (see diagram 1)

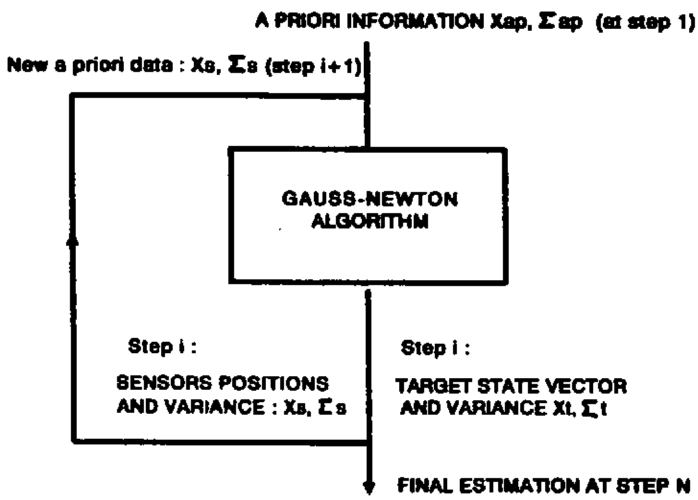


diagram 1 : algorithm improvement

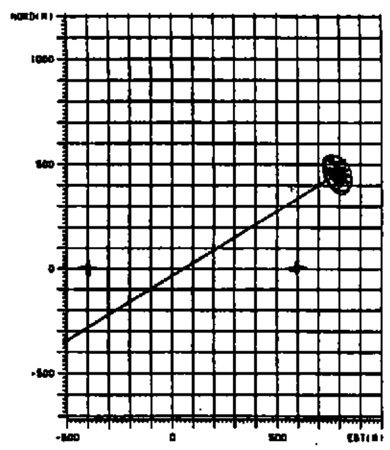
In the next section, simulations concerning this improvement are presented and results are discussed.

5. SIMULATIONS AND RESULTS

Monte-Carlo simulations have been computed to test our algorithms. The simulations presented below have been obtained with a single draw for sensor locations and either a hundred or ten draws for each measurement. The target position estimates are symbolized by "0", the a priori sensors positions by "+" and the true sensors positions by "*". For each set of estimates, two position uncertainty ellipses are represented : the one in thin line is deduced from the CRLB and that in thick line is calculated from the estimates. In the following, two different scenarii are considered : in the first one, only the distance between sensors is imperfectly known whereas in the second one, both coordinates x and y are unknown.

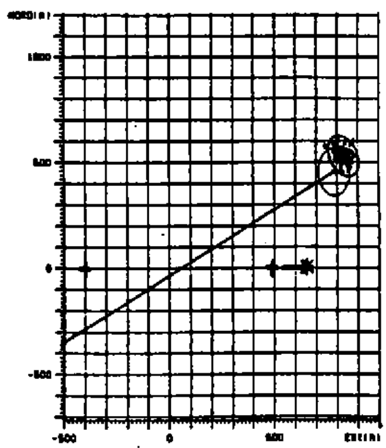
- Scenario 1 (pictures 1 to 3) : the pattern is constituted by a pair of sensors measuring doppler. On picture 1, the two sensors locations are exactly known and our algorithm proves efficient. In the two following examples, the location of one of the two sensors is imperfectly known : its a priori y-coordinate is correct but there is an error on the a priori x-coordinate and consequently on the assumed distance between the sensors.
The picture 2 represents the results achieved by our basic algorithm. One can observe that all the estimates are biased, but also that the estimated sensor position is located between the a priori position and the true one. This sensor localization improvement is due to the doppler measurements which contain information relative to the true distance between the two sensors. But these information are not sufficient because of the error introduced by the a priori x-coordinate.
The picture 3 illustrates the results obtained after ten iterations of our basic algorithm. We can see that in this case, our new algorithm converges on the true position of the sensor and the target parameters are then correctly estimated. This result follows from the fact that the a priori x-coordinate introduced from one iteration to another is even better.

- Scenario 2 (pictures 4 to 7): the pattern is constituted by three sensors denoted S_1 to S_3, each of them providing range measurements. On pictures 5 and 7, S_1 measures also azimuths. The results presented with the scenario 2 have all been obtained after ten iterations of our basic algorithm.
On pictures 4 and 5, the three sensors positions are to be estimated. The estimation results presented on picture 4 are biased. We shall notice that the estimated target and sensors positions can be derived from the true ones by a translation and a rotation. This is due to the fact that only the distances between sensors are identifiable from range measurements. The picture 5 shows that the introduction of azimuth measurements only eliminates the circular ambiguity of the problem : the rotation disappears but the estimated positions can still be derived from the true ones by a translation. Actually, the estimates obtained are correct in the set of axes relative to the sensors but are still biased in our own set of axes for lack of reliable absolute information. If no such information are available, the residual bias can not be cancelled.
On pictures 6 and 7, the position of sensor S_1 is now perfectly known. In the two cases, the coordinates x_{s3} and y_{s2} are correctly estimated, which is not wondering since these parameters can be determined from the true position of S_1 and the estimated distance between the sensors. However, when S_1 measures only ranges (picture 6), the remaining circular ambiguity causes a residual bias which completely disappears as soon as azimuths measurements are available (picture 7).



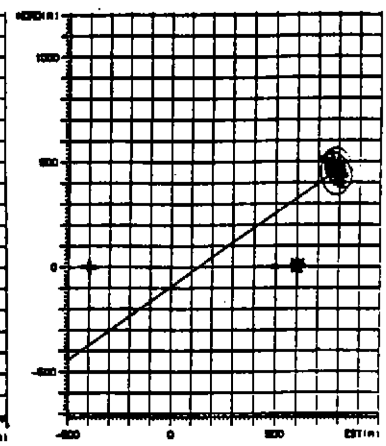
picture 1

TMA with known sensors locations



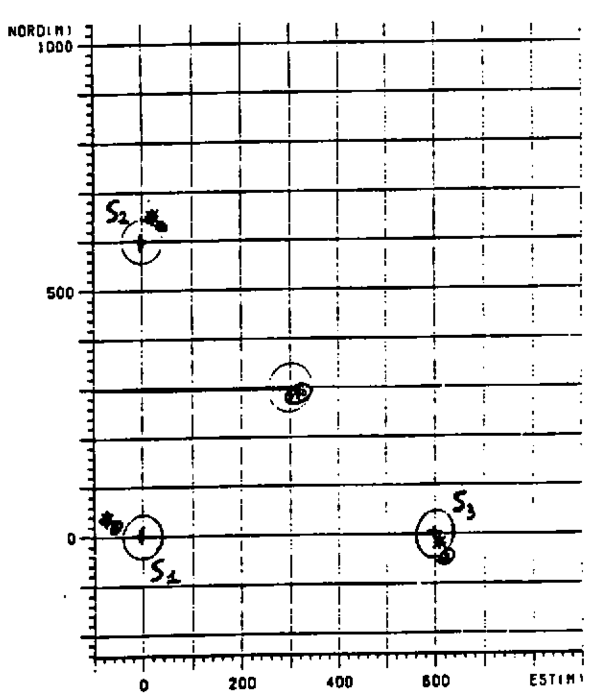
picture 2

Residual bias



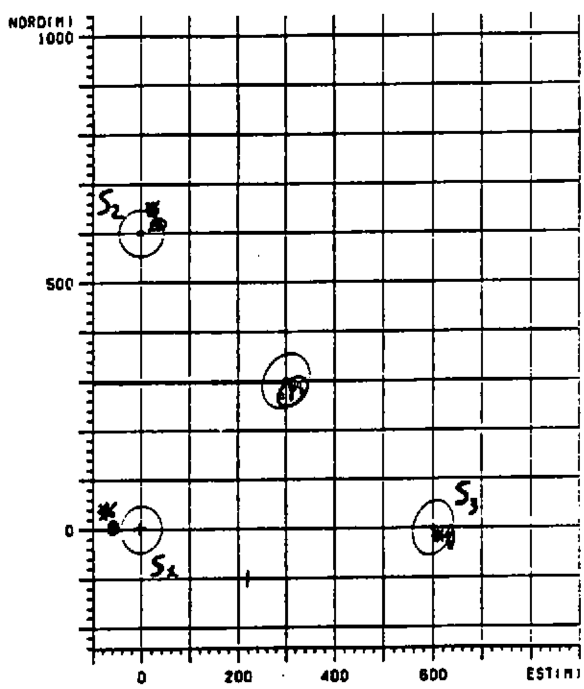
picture 3

Results from algorithm improvement



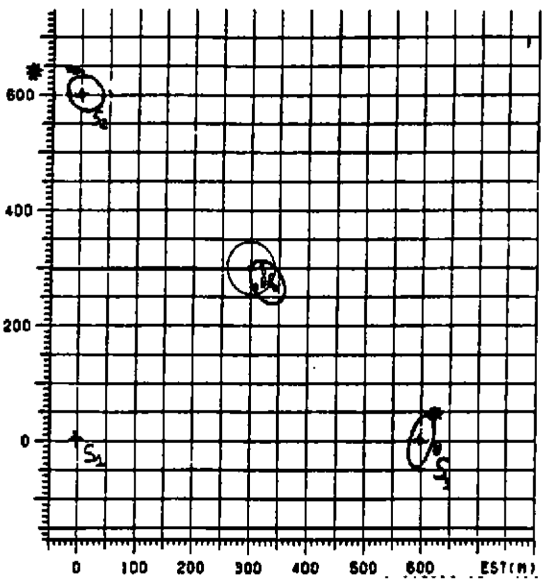
picture 4

Unknown locations for every sensor



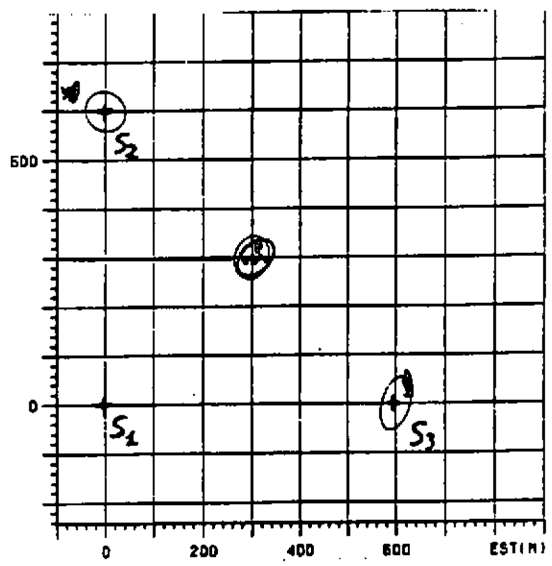
picture 5

Additional azimuths measurements



picture 6

Known location for sensor 1



picture 7

Known location for sensor 1 and azimuths

6. CONCLUSIONS

In this paper, we have examined the problem of TMA using imperfectly located sensors making various types of measurements. We have shown that an iteration of the algorithm presented in [1] led to unbiaised results providing that the true position of at least one sensor is known and azimuths measurements are available. Moreover, it has been explained that, because of identifiability reasons, the residual bias can not be cancelled unless these additionnal information are given.

7.ACKNOWLEDGEMENTS

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8.BIBLIOGRAPHY

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