

APPLICATION OF PLANAR MICROPHONE ARRAYS AND COMPRESSIVE SENSING FOR LOCALIZATION OF COMPRESSED AIR LEAKAGES

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Compressed air is a kind of expensive sources in modern industrial factories. Leakage is the largest waste of energy in a compressed air system. When air escapes through a small orifice of a compressed gas system, a turbulent noise spectrum is produced. A algorithm combined Orthogonal Matching Pursuit (OMP) algorithm and singular value decomposition is proposed for localization of compressed gas leakages. The method can show the locations of compressed gas leakages on super-resolution source maps in the low signal-to-noise (SNR) environment. The experiments are conducted in a mechanical laboratory, with the noise of an air compressor and environment noises served as the background noises. The SNR is very low in the laboratory. The leakage orifices are designed in the arbitrary positions in the three-dimensional space. The results obtained with the proposed method is compared with those with conventional beamformer (CBF) and the Tikhonov regularization (TIKR) method. The performances of the CBF method and TIKR method degrade due to low SNR in the laboratory. At the same time, the results show that the CS algorithm is computationally more effective and can present a super-resolution map. This work proves the feasibility of phased microphone array and a CS algorithm applied to the localization of compressed gas leakages.

Keywords: Compressed air, microphone array, compressive sensing

1. Introduction

Nowadays, compressed air is the most universal energy medium used by a number of industries. Despite all its advantages, compressed air is also an expensive energy resource. Energy costs contribute to 75% of the total costs for compressed air production [1]. A significant share of compressed air is lost on various accounts. Leaks are the most visible and most significant contributors to compressed air losses. Besides economic effect, the reduction of compressed air losses is significant for the environment. In order to reduce leak loss, it is necessary to locate the leakage source. Various methods have been developed and applied to leak detection. Huseynov et.al [2] proposed an ultrasonic gas leak localization system based on a distributed network of sensors. The system deploys highly sensitive miniature Micro-Electro-Mechanical Systems (MEMS) microphones and uses a suite of energy-decay (ED) and time-delay of arrival (TDOA) algorithms for localizing a source of a gas leak. Steckel [3] applied a random, sparse array of microphones in conjunction with an algorithm inspired by compressed sensing (CS) to finely localize air leaks. Veronesi et al. [4] applied Truncated Singular Value Decomposition (TSVD) into the sound source location. Hansen [5] proposed a

new method–Tikhonov Regularization (TIKR), which can also be employed to deal with source location. But least squares methods may underestimate the true power of sources severely [6]. Compress sensing (CS) [7, 8], a recently developed revolutionary theory, has also been employed in acoustic imaging [6, 9, 10]. We propose a method in this work, which combine Orthogonal Matching Pursuit (OMP) algorithm of CS and singular value decomposition (SVD). The proposed method named OMP-SVD method has high efficiency. Besides, it can locate the leakage source exactly even though the SNR is low.

2. Observation model

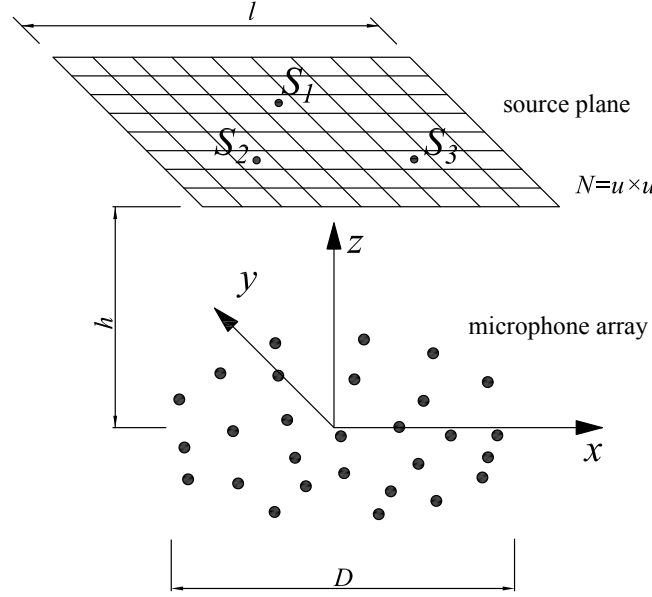


Figure 1: Acoustic signal propagation model

As is shown in Figure 1, it illustrates the model of acoustic signal propagating from the source plane which is h away from the planar microphone array. The planar array is an optimized random array with M sensors, which locate at known positions $\bar{\mathbf{P}} = [\bar{\mathbf{p}}_1, \dots, \bar{\mathbf{p}}_M]^T$, where $[\bullet]^T$ denotes the transpose operator. The source plane is discretized into $N = u \times u$ equidistant grids at known discrete positions $\mathbf{P} = [\mathbf{p}_1, \dots, \mathbf{p}_N]$. The measurements of sound pressure at M microphones of the array in time domain are divided into B blocks. The frequency-domain data of microphone array $\mathbf{Y}(f) = [y_1, \dots, y_M]^T$ can be obtained by averaging the data of all blocks:

$$\mathbf{Y}(f) = \frac{1}{B} \sum_{b=1}^B \mathbf{Y}_b(f). \quad (1)$$

where $\mathbf{Y}_b(f) = [y_{1b}, \dots, y_{Mb}]^T$ equals to the measurements data of microphones at b th block. The unknown vector \mathbf{X} comprises the source strength at all N grid nodes.

The pressure field at the m th microphone is given by:

$$y_m = \sum_{n=1}^N \frac{x_n \cdot e^{-jkr_{mn}}}{4\pi r_{mn}}, \quad (2)$$

where $r_{mn} = \|\bar{\mathbf{p}}_m - \mathbf{p}_n\|$ denotes the distance between the m th microphone and the n th grid node, $k = \omega/c$ is the wave number with c being the sound speed, $\omega = 2\pi f$ is the angular frequency with f being the desired frequency, and x_n is the source strength of the n th grid node.

The model can be compactly expressed in matrix form:

$$\mathbf{Y} = \mathbf{A}\mathbf{X}, \quad \mathbf{Y} = \begin{bmatrix} y_1 \\ \vdots \\ y_M \end{bmatrix}, \mathbf{A} = \frac{1}{4\pi} \begin{bmatrix} \frac{e^{-jkr_{11}}}{r_{11}} & \dots & \frac{e^{-jkr_{1N}}}{r_{1N}} \\ \vdots & \ddots & \vdots \\ \frac{e^{-jkr_{M1}}}{r_{M1}} & \dots & \frac{e^{-jkr_{MN}}}{r_{MN}} \end{bmatrix}, \mathbf{X} = \begin{bmatrix} x_1 \\ \vdots \\ x_N \end{bmatrix}, \quad (3)$$

where \mathbf{A} is an $M \times N$ matrix and defined as the measurement matrix. The i th element of the vector \mathbf{Y} consists of the average pressure fields of all blocks of the i th microphone.

In order to approach real cases, we take the background noise and errors into account. Thus, the model closer to reality can be depicted as:

$$\mathbf{Y} = \mathbf{A}\mathbf{X} + \mathbf{e}, \quad (4)$$

where \mathbf{e} denotes background noise and error.

3. OMP-SVD method for acoustic imaging

With the help of microphone array, we can get a $M \times B$ data matrix \mathbf{y} under desired frequency f from the B blocks of data:

$$\mathbf{y}(f) = [\mathbf{Y}_1 \dots \mathbf{Y}_B] \quad (5)$$

Similarly, the source strength \mathbf{X} is divided as a $N \times B$ matrix \mathbf{x} which consists of sources strength under desired frequency f . We use the SVD of the data matrix \mathbf{y} to reduce the sensitivity to noise. The results decomposes the data matrix into signal subspace and noise subspace, and to keep signal subspace. Mathematically, this translates into the following representation:

$$\mathbf{y} = \mathbf{U}\mathbf{\Lambda}\mathbf{V}^T. \quad (6)$$

Defined the reduced $M \times K$ dimensional matrix \mathbf{y}_{SV} , which involves most of the signal power as $\mathbf{y}_{SV} = \mathbf{U}\mathbf{\Lambda}\mathbf{D}_K = \mathbf{y}\mathbf{V}\mathbf{D}_K$, where $\mathbf{D}_K = [\mathbf{I}_K \mathbf{0}]'$. Here \mathbf{I}_K is a $K \times K$ identity matrix, $\mathbf{0}$ is a $K \times (B - K)$ matrix of zeros, and K is the estimated number of sources resulting from SVD. In addition, we transform the $N \times B$ signal matrix \mathbf{x} as $\mathbf{x}_{SV} = \mathbf{x}\mathbf{V}\mathbf{D}_K$, and let $\mathbf{e}_{SV} = \mathbf{e}\mathbf{V}\mathbf{D}_K$, to obtain the system

$$\mathbf{y}_{SV} = \mathbf{A}\mathbf{x}_{SV} + \mathbf{e}_{SV}. \quad (7)$$

In order to apply the CS algorithm to our work, we consider a new system which is equivalent to Eq. (7):

$$\bar{\mathbf{y}} = \bar{\mathbf{A}}\bar{\mathbf{x}} + \bar{\mathbf{e}}, \quad (8)$$

where $\bar{\mathbf{y}}$ is obtained by stacking all columns of \mathbf{y}_{SV} into a column vector, and similarly for $\bar{\mathbf{x}}$ and $\bar{\mathbf{e}}$. Let $\bar{\mathbf{A}}$ be defined as:

$$\bar{\mathbf{A}} = \begin{pmatrix} \mathbf{A} & & \\ & \ddots & \\ & & \mathbf{A} \end{pmatrix}, \quad (9)$$

where $\bar{\mathbf{A}}$ is block diagonal with K replicas of \mathbf{A} .

The OMP algorithm, a kind of greedy algorithm, can seek the optimal solution through iterations, which can be used to solve the equation (8). After each iteration of the OMP algorithm, we obtain a nonzero vector approximating the signal $\bar{\mathbf{x}}$. The detailed description and the steps of the OMP

algorithm for acoustic image can be found in our previous work [6, 10]. According to Eq.(8), we can find an approximate solution $\tilde{\mathbf{x}}$ with the help of OMP algorithm and transform $\tilde{\mathbf{x}}$ as the matrix $\tilde{\mathbf{x}}_{SV}$. Then we can get the strengths and locations of sources by averaging the source strengths of all columns of signal subspace $\tilde{\mathbf{x}}_{SV}$. The strength vector \mathbf{X}^* of uncorrelated sources can be obtained by

$$\mathbf{X}^* = \frac{1}{K} \sum_{k=1}^K \text{diag}[\mathbf{R}_{\mathbf{x}_{SV}(k)}], \quad (10)$$

where $\mathbf{R}_{\mathbf{x}_{SV}(k)} = \mathbb{E}[\tilde{\mathbf{x}}_{SV}(k)\tilde{\mathbf{x}}_{SV}(k)^H]$ [11]. Here $\tilde{\mathbf{x}}_{SV}(k)$ is the k th column of the matrix $\tilde{\mathbf{x}}_{SV}$, and \mathbb{E} is the mathematical expectation.

4. Experimental Results and Analysis

4.1 Experiment configuration of the gas leakage

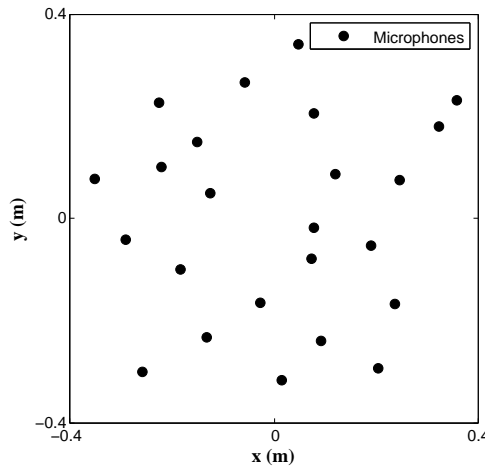


Figure 2: A 24 channels microphone array

In this experiment, we measured the data by using the planar microphone array shown in Fig 2.

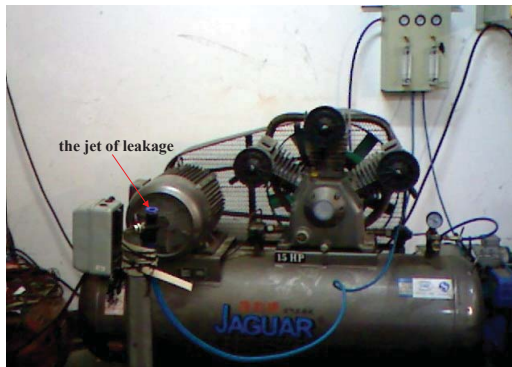


Figure 3: Source configuration of gas leakage

Figure 3 shows the experimental configuration, which includes an air compressor, a plastic pipe, bracing piece and a jet. Compressed gas was leaked from the jet in the experiment. The acoustic source at the jet generated the leakage noise. At the same time, The air compressor and other machines generated loud background noise in the environment. The observation zone of interest was a 0.78 m

$\times 0.58$ m plane and was discretized into 101×76 grids. The distance between the source plane and the microphone array was 1.1 m.

In the experiment, a 10 s recording was stored for each measurement. Thus we obtained data with many sampling points for each channel. Then the time domain sampling data were divided into 50%-overlapping blocks, where each block contained 1024 sampling points. We performed FFT for each data block after applying Hanning window to each block. Therefore, we obtained the measurement data of each block in frequency domain for each microphone.

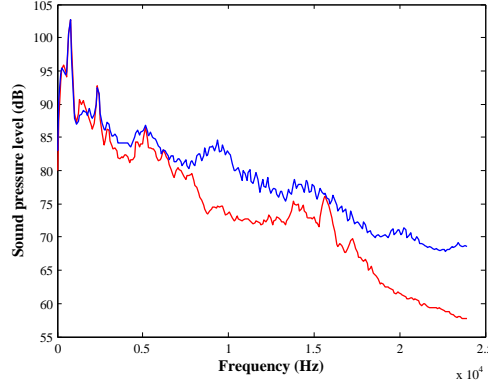


Figure 4: The spectrum of the gas leakage experiment

Because it is very difficult to obtain the background noise in the experiment, the measured sound is regarded as the background noise when there is no leakage. The experimental noise spectrum and the background noise spectrum are shown in Fig. 4. Because the power of experiment is almost equal to that of the background noise at some frequencies, the SNRs is very low at these frequencies positions. We roughly obtain the SNRs shown in Fig. 5 with the following equation:

$$\text{SNR}(f) = 10\log_{10}(\mathbf{P}_{\text{measure}}(f) - \mathbf{P}_{\text{noise}}(f)/\mathbf{P}_{\text{noise}}(f)), \quad (11)$$

where $\mathbf{P}_{\text{measure}}(f)$ and $\mathbf{P}_{\text{noise}}(f)$ are the powers of the gas leakage experiment and background noise at the frequency f , respectively. Here the obtained SNRs exist difference from the theoretical SNRs, which are merely the rough estimate for the theoretical SNRs.

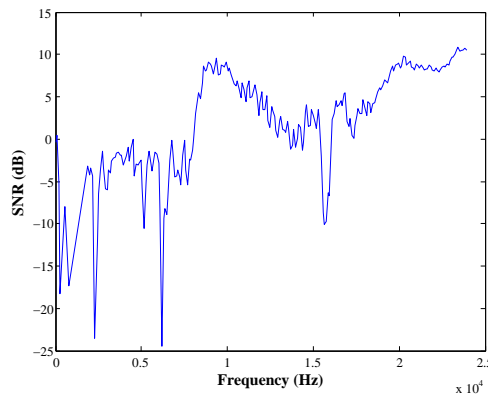


Figure 5: The SNR of the gas leakage experiment

In order to verify the performance of our proposed method in low SNR environments, the lowest SNR was chosen to obtain the source maps from the measurement data, where the frequency and SNR are 6187 Hz and -24 dB respectively. More, we obtained the source maps as SNRs equal to 0.2 dB and 9.5 dB, where the frequencies are 12937 Hz and 9375 Hz respectively. The source maps from the proposed OMP-SVD method were compared with those of the CBF method and the TIKR method.

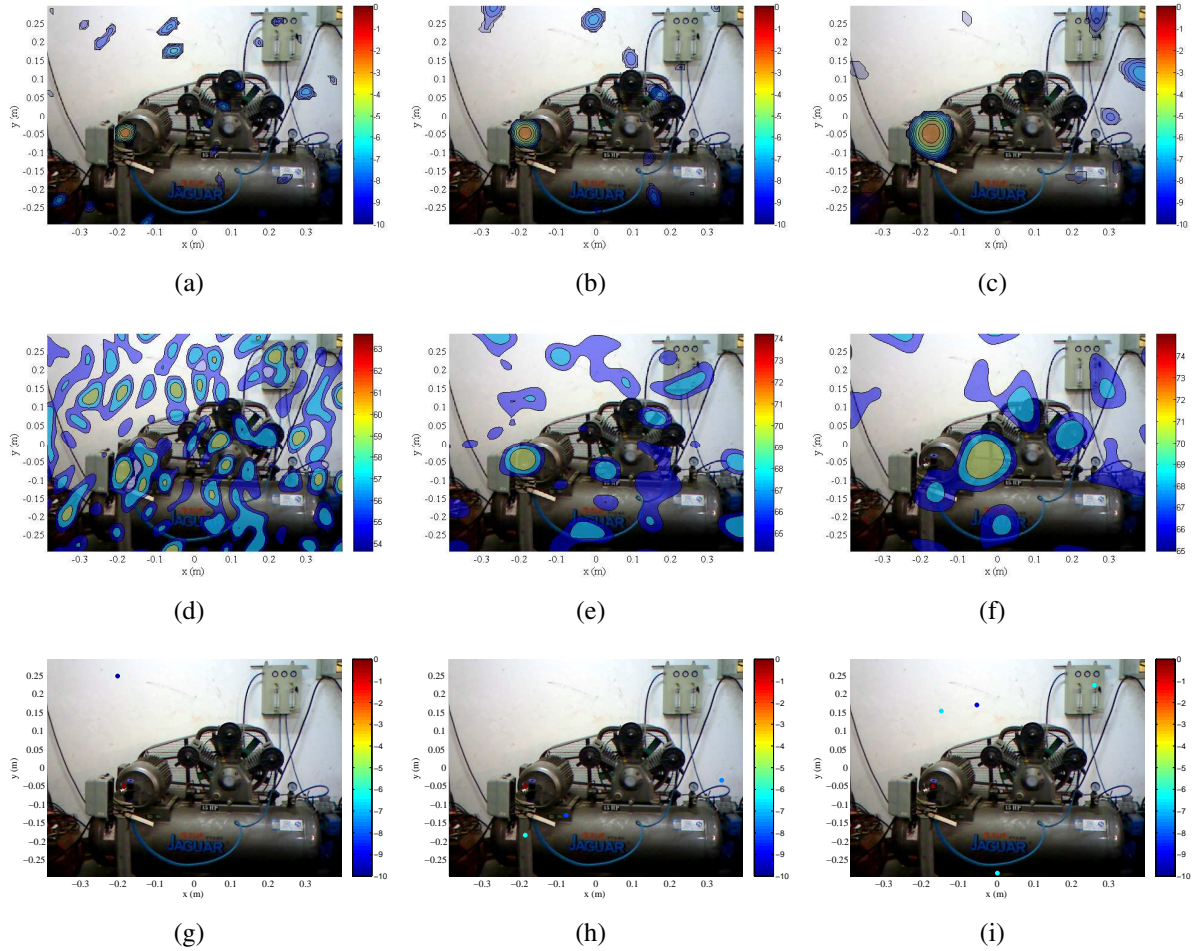


Figure 6: The source maps for the gas leakage experiment obtained by the CBF method as the SNRs equal to (a) 9.5 dB, (b) 0.2 dB, and (c) -24 dB. The source maps obtained by the TIKR method as the SNRs equal to (d) 9.5 dB, (e) 0.2 dB, and (f) -24 dB. The source maps obtained by the OMP-SVD method as the SNRs equal to (g) 9.5 dB, (h) 0.2 dB, and (i) -24 dB.

Figure 6 shows the source maps obtained by the CBF method, the TIKR method and the OMP-SVD method as the SNRs equal to 9.5 dB, 0.2 dB and -24 dB, where the frequencies were 12937 Hz, 9375 Hz and 6187 Hz respectively. For the OMP-SVD method, satisfying results were obtained and the main source could be located accurately for the lowest SNR -24 dB, which are shown in Fig. 6(i). Compared with the TIKR method, the proposed method is more robust, as it works for the low SNR issues. We conclude that the proposed method could be applied to the low SNR environments for acoustic imaging.

5. Conclusion

In this paper, we studied the performance of the OMP-SVD method in acoustic imaging, and compared the maps with those of CBF method and TIKR method.

For the experiment of gas leakage, we have found that the main source lied on the position of jet, which could be obtained by OMP-SVD method. However, the TIKR method could not be applied to the aeroacoustic imaging because of the unsatisfying results. The correction of experiment source maps obtained by the proposed method has been verified through comparing with the maps of CBF method.

Acknowledgments

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