Proceedings of The Institute of Acoustics

THE COUPLING OF PLATES BY BARS

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INTRODUCTION

The sound insulation of two layer wall constructions is determined by the plate-airgap-plate system and the structural junctions between the layers. The former is the upper limit, the sound reduction index is characterized by a 18 dB/octave slope curve between the resonance and the limiting frequency. The average velocity-level difference has a 6 dB/octave curve in this range. The structural junctions reduce the improvement in sound reduction index caused by the second layer: the lower limit is a constant improvement, proportional to the increase of mass. In this case the velocity-level difference will be about constant. There is a lack of calculation methods of two layer constructions, with structural junctions at the perimeter especially in the cases of softening the rigid junctions. Among the great variety of practical configurations a calculation method of the pillar-connections is presented.

THE ELEMENTS OF THE MODEL

The calculation model consists of two thin, finite, simply supported plates, the first is excited by distributed pressure, the second is excited by line-moment, caused by the pillars.
The pillars are considered to be rectangular bars in torsion, excited by moment, both ends fastened.
There are important boundary conditions between the bars and the plates, characterizing also the rigidity of the connections.
The shematical view of the system is shown in fig.1.
The equations of motion of a sipmly supported plate are
eq(1) - eq(5) /1/

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$$D\left(\frac{\partial^2}{\partial^2}+\frac{\partial^2}{\partial y^2}\right)^2 Y-\rho_p h\omega^2 Y=P(xy) \qquad (1)$$

where $$D=\frac{E\,h^3}{12(1-\gamma^2)} \qquad (2)$$

E: modulus of elasticity
Y: normal displacement
ρ_p: density of the plate
h: thickn ess of the plate
ω: circular frequency
P: exciting pressure

$$Y=\sum_n\sum_m Y_{nm}\,\psi_{nm}(xy) \qquad (3)$$

Y_{nm} modal displacement

$$\psi_{nm}(xy)=\sin\frac{n\pi}{l}x\,\sin\frac{m\pi}{w}y$$

$$Y_{nm}=\frac{P_{nm}}{\rho_p h\,\sigma_{nm}} \qquad (4)$$

$$P_{nm}=\frac{4}{lw}\iint P(xy)\,\psi_{nm}(xy)\,dxdy$$

The angle-rotation χ_x, belonging to the x axis is:

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$$\chi x = \frac{\partial Y}{\partial x} = \sum\sum Y_{nm} \varnothing_{nm}(xy) \quad (5)$$

$$\sigma_{nm} = \omega_{nm}^2 - \omega^2 + j\delta_E \, \omega_{nm}^2$$

ω_{nm}: resonant curcular frequency of the n, m mode

The line-force exciting function can be expressed by eq(6):

$$P_{LF}(xy) = \delta(x_0) \cdot P(y) \cdot K \quad (6)$$

The line moment exciting function is the following:

$$P_{LM}(xy) = \lim_{\Delta \to \varnothing} \left\{ K \cdot P(y) \cdot [\delta(x_0 + \Delta) - \delta(x_0)] \right\} = \delta'(x_0) \cdot P(cy) \cdot M \quad (7)$$

$$K \cdot \Delta = M, \text{const}$$

When expressing the modal terms $n\pi/l \cdot \cos(n\pi/l \cdot x_0)$ will belong to $\delta'(x_0)$.

the equation of motion of the bar in torsion is eq(8)-eq(13):

$$\frac{\partial^2 \Omega(y)}{\partial y^2} + \frac{\Theta'}{T} \omega^2 \Omega(y) = + \frac{j\omega}{T} M(y) \quad (8)$$

Ω: angular velocity of the bar

Θ': moment of inertia,

$$\Theta' = \frac{\rho_b (ba^3 + ab^3)}{12} \quad (9)$$

: torsional modulus

ρ_b: density of the bar

T: torsional rigidity

$$T: \alpha \cdot G \cdot b^3 \cdot a \quad (10)$$

α: function depends on the a-b relation

M(y): exciting moment per unit length

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$$\Omega = \sum_{i} \frac{j\omega \, M_i}{\theta' \; \sigma_i} \; \sin \frac{i\pi}{w} y \tag{11}$$

M_i: the modal form of M

$$M_i = \frac{2}{w} \int_0^w M(y) \sin \frac{i\pi}{w} y \, dy \tag{12}$$

$$\sigma_i = -\omega_i^2 \left(1 + j\,\delta_G\right) + \omega^2 \tag{13}$$

The elastic inner layer will be characterized by its moment compliance CM related to unit length:

$$CM = \frac{\beta}{M'} \tag{14}$$

where

β : rotation of angle

M': moment per unit length

If the angle of rotation is little, ($tg\beta \approx \beta$) C_M will given by eq(15):

$$CM = \frac{2d}{h^3 E} \tag{15}$$

d: thickness of the elastic layer

h: width - " -

E: komplex modulus of elasticity

EQUATIONS OF MOTION, BOUNDARY CONDITIONS

For plate 1 eq(1) can be directly used with eq(16):

$$P(xy) = F(xy) - \delta'(x=0)\, P_{13}(y) - \delta'(x=l)\, P_{14}(y) \tag{16}$$

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For plate 2:

$$.P(xy) = \delta'(x-0)\,P_{32}(y) + \delta'(x-l)\,P_{42}(y) \tag{17}$$

For bar 3

$$M(y) = M_{13}(y) - M_{32}(y) \tag{18}$$

For bar 4

$$M(y) = M_{14}(y) - M_{42}(y) \tag{19}$$

There are two sets of boundary conditions: 4 equations to the angle of rotations and 4 to the ballance of moments.

It is apposed, that at x = 0 and x = 1 $\chi_y=0$, only χ_x are existing. In fig. 2 the angle relations of plate 1-bar 3 boundary are shown. Since the bending waves have the greatest importance in short distance vibration-propagation, all the other types of waveforms are neglected.

$$\chi_1(x=0,y) = \chi_3(y) + \chi_{13}(y) \qquad \chi_1(x=l,y) = \chi_4(y) + \chi_{14}(y) \quad (20/a,b)$$

$$\chi_3(y) = \chi_2(x=0,y) + \chi_{23}(y) \qquad \chi_4(y) = \chi_2(x=l.y) + \chi_{24}(y) \quad (20/c,d)$$

Based on the electrical-mechanical analogy it is clear, that this type of boundary condition correspondes to a paralell connection (moment-voltage, angular velocity-current), therefore the moments, acting onto all the elements will be equal. For eg.the moment, acting to the elastic layer, between 1 and 3 vill be M_{13}.
The equations of ballance will be the following:

$$M_{13}(y) = P_{13}(y) \qquad M_{23}(y) = P_{23}(y) \quad (21/a,b)$$

$$M_{14}(y) = P_{14}(y) \qquad M_{24}(y) = P_{24}(y) \quad (21/c,d)$$

SOLVING THE EQUATIONS

Substituting the equations of ballance and the equations of motion to the boundary conditions of angle-rotation leads to

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the following (here only the modified form of 20/a is given, all the others have the same structure)

$$\sum_{n}\sum_{m} \frac{F_{1nm} - \frac{2n\pi}{l^2} M_{13m} - \frac{2n\pi}{l^2}(-1)^n M_{14m}}{\rho_{P1} h_1 \sigma_{1nm}} \sin\frac{n\pi}{l} y \cdot \frac{n\pi}{l} =$$

$$= \sum_{m} \frac{M_{13m} - M_{23m}}{\theta_3' \, \sigma_{3m}} \sin\frac{m\pi}{w} y + CM_{13} \sum_{m} M_{13m} \sin\frac{m\pi}{w} y \qquad (22)$$

The equation can be multiplied by $\sin\frac{i\pi}{w}y$ and then integrated. Completing this operation to all of eg.20 a set of four equations is got, with four unknown variables M_{13m} M_{12m} M_{14m} M_{24m} The final form is the following:

$$\underline{A} \; \underline{\underline{B}} \cdot \underline{M} \qquad (23)$$

$$\underline{A} = \begin{bmatrix} \Lambda_{1m} \\ 0 \\ \Gamma_{1m} \\ \theta \end{bmatrix} \qquad = \underline{M} \begin{bmatrix} M_{13m} \\ M_{23m} \\ M_{14m} \\ M_{24m} \end{bmatrix}$$

$$\Lambda_{1m} = \sum_{n} \frac{F_{1nm}}{\rho_{P1} h_1 \sigma_{1nm}} \frac{n\pi}{l}$$

$$\Gamma_{1m} = \sum_{n} \frac{F_{1nm}}{\rho_{P1} h_1 \sigma_{1nm}} \frac{n\pi}{l}(-1)^n$$

$$\underline{\underline{B}} = \begin{bmatrix} \alpha_{1m} + K_{3m} + CM_{13} & -K_{3m} & +\beta_{1m} & \phi \\ K_{3m} & -\alpha_{2m} - K_{3m} - CM_{23} & \phi & -\beta_{2m} \\ +\beta_{1m} & \phi & \alpha_{1m} + K_{4m} + CM_{14} & -K_{4m} \\ \phi & \beta_{2m} & +K_{4m} & -\alpha_{2m} - K_{4m} - C \end{bmatrix}$$

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$$\alpha_{im} = \sum_n \frac{2}{l} \left(\frac{n\pi}{l^2}\right)^2 \frac{1}{\rho_{pi} h_i \sigma_{inm}} \qquad i = 1,2$$

$$\beta_{im} = \sum_n \frac{2}{l} \frac{n\pi}{l^2} \frac{1}{\rho_{pi} h_i \sigma_{inm}} (-1)^n \quad i = 1,2$$

$$K_{im} = \frac{1}{\theta_i' \sigma_{im}}$$

From the equations above the modal forms of M can be calculated, that is the motions of the plates can be determined.
For evaluating the coupling it is further necessary to determine the power, getting into plate 2. On the basis of [3]:

$$W_{2in} = Re \int_0^l \int_0^w P_2(xy)\, V_2(xy)\, dxdy$$

Expressing $W_{2\ in}$ in detailes it can be found, that there are parts of power, coming from bar 3 and 4 independently, and there are parts of cross-power.

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Fig. 1.
Shematical view of the plate-bar system

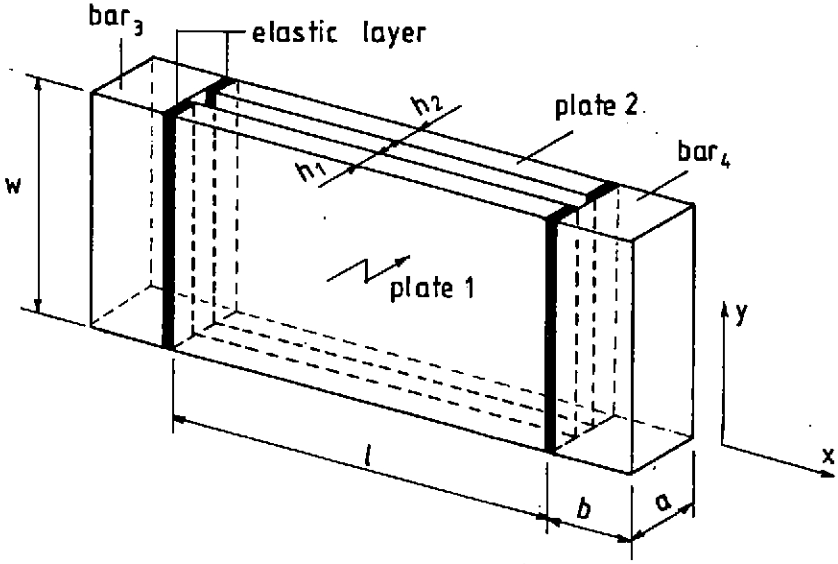
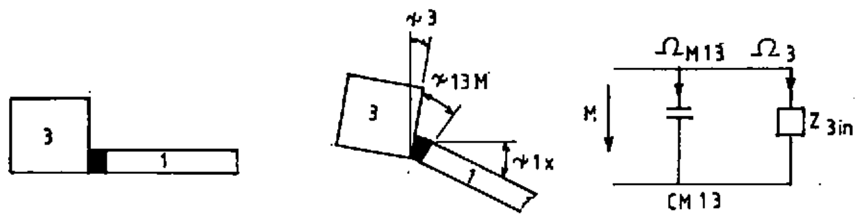


Fig. 2.
Angle relations

equilibrium position rotation electrical analogy



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A STUDY OF THE SOUND TRANSMISSION BETWEEN TWO FLATS, BEFORE AND AFTER THE INSTALLATION OF SOUND INSULATING MATERIAL

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Environmental Health Officers [EHO] often receive complaints relating to neighbourhood noise because under the Control of Pollution Act 1974 they are given powers to deal with noise nuisances. An example of the type of complaint received by an EHO is when noise is transmitted through a party wall from an adjacent dwelling, causing annoyance to the complainant. This type of complaint raises the question whether the walls and floors which separate adjacent dwellings should be provided with additional sound insulating material, in order to reduce the transmission of sound.

This paper reports on a study carried out of a conversion of a prewar end terrace property into two separate dwellings in 1987. The study investigated the effects on airborne and impact sound transmitted between the two dwellings, an upstairs and a downstairs flat, and the results when sound insulating material was installed in the floor which separated the first floor flat from the ground floor flat. [1].

The conversion of the property was brought to the attention of the Environmental Health Department at Cleethorpes Borough Council during the course of the procedure followed by officers of the Planning Department when a planning application had been submitted. At Cleethorpes Borough Council the Planning Department circulated details of planning applications to the EHO's in order to receive their comments relating to standards that would be required if planning approval was given.

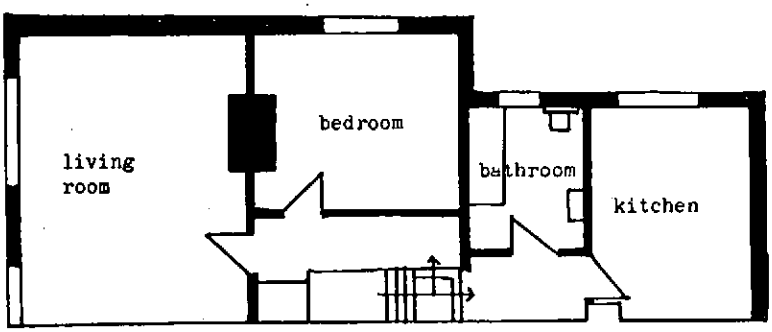
Under regulation 5 of the Building Regulations 1985 the conversion of a property into two flats is considered to be a material change of use, however under regulation 6 the requirements of E1/2/3 which relate to airborne and impact sound, do not need to be satisfied.

Thus there is no requirement under the Building Regulations 1985 for sound insulation to be installed in a property which is being converted into flats.

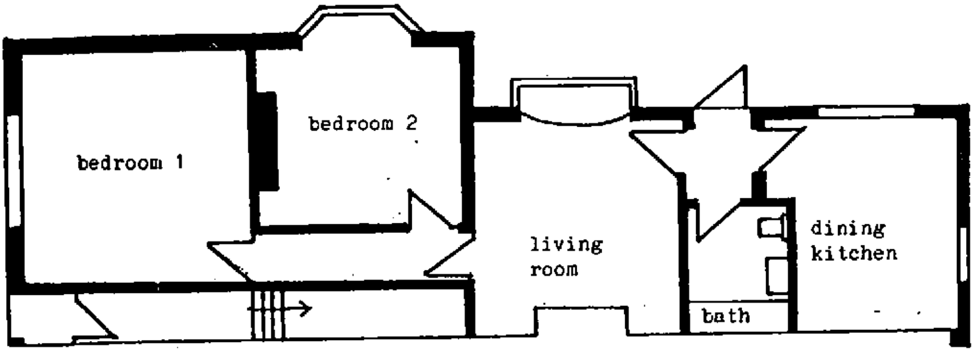
Therefore the Environmental Health Officers at Cleethorpes Borough Council adopted a policy in respect of planning applications which related to the conversion of a property into separate dwellings. The Environmental Health Officer made a request for the planning department to include a condition of the planning consent to require the installation of sound insulating material between separate dwellings.

At the property concerned, the plans of the proposed conversion indicated that the living room of the first floor flat would be above the bedroom of the ground floor flat (Figure 1). Therefore sound insulation was necessary between the two separate dwellings in order to prevent the transmission of noise into the downstairs bedroom. Furthermore the first floor living room was also next to the bedroom of the adjacent property, and this would result in the transmission of noise through the party wall.

The Environmental Health Officer made a recommendation to the planning department to include a condition on the planning consent requiring sound insulation in the floor of the first floor flat and in the party wall of the adjacent property.



PROPOSED FIRST FLOOR PLAN



PROPOSED GROUND FLOOR PLAN

Figure 1: Plan showing layout of property

The builder consequently submitted a scheme which indicated how the transmission of airborne and impact sound between the two dwellings would be reduced.

The material which was chosen by the builder was Rocksil Insulation Mat, which is produced by Pilkington, and consisted of rolls of unfaced, low density, non-combustible rock fibre.

The 50mm thick mat was to be installed in the ceiling of the ground floor flat by installing 100 mm x 38 mm ceiling joists at 400 mm centres below the existing ceiling, with a 13 mm plasterboard finish. The insulation mat is laid over the new joists between the new and the existing ceilings (Figure 2). In the party wall the mat was to be placed against the wall held in between 75 mm x 38 mm timber studs at 400 mm centres, with a 13 mm thick plasterboard and skimmed plaster board finish.

In order to calculate the effectiveness of the sound insulating material, the transmission of airborne and impact sound between the two flats was measured before and after the material had been installed.

The method used to measure the transmission of airborne and impact sound were in accordance with the procedures outlined in BS 2750 part 4 and part 6 respectively.

For the purpose of this study the measurements of the transmission of airborne and impact sound were restricted to the first floor front room (proposed living room) and ground floor front room (proposed bedroom).

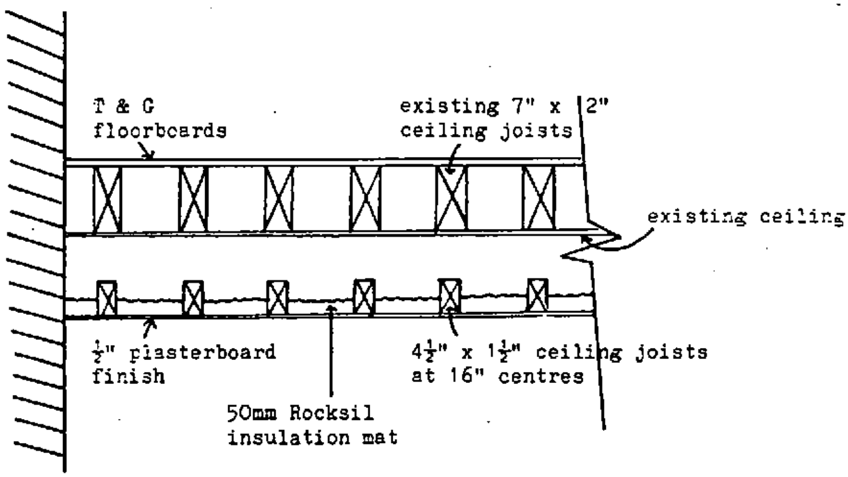


Figure 2: Cross-Section showing construction of new ceiling

METHOD USED TO MEASURE AIRBORNE SOUND INSULATION

In accordance with BS 2750 part 4, the sound which was generated in the source room was steady and had a continuous spectrum in the frequency range under consideration. [2]. When the measurements were made before the installation of the insulating material, the sound was generated by a Bruel and Kjaer isotropic sound source type 4205. However after the insulating material had been installed this sound source was found to be not powerful enough to allow measurements to be made in the receiving room because of the increased sound insulation, so a Bruel and Kjaer random noise generater type 1402 was used instead, attached to a loudspeaker. The loudspeaker was placed within the source room to give a sound field which was as diffuse as possible, and at such distances from the test specimen (floor) that the direct radiation upon it was not dominant. The sound pressure level (SPL) within the source room and the receiving room was measured in octave bands. The sound level meter which was used was a Bruel and Kjaer type 2209, which had a B & K external filter type 1613 attached to it. A bandpass filter set type

1612 was attached to the noise generator, then the octave band setting of the sound source and the sound level meter could be increased in sequence. The octave bands under consideration was taken to be centre frequency 63 Hz up to 4000 Hz.

Within the source room the measurements were made at 6 different positions for all the centre frequencies from 63 Hz to 4 kHz inclusive. This enabled an average SPL to be obtained at each octave band centre frequency. In the receiving room the measurements were taken at 6 different positions for the centre frequencies 63 Hz; 125 Hz; 250 Hz and 500 Hz, and at 3 different positions for centre frequencies, 1 kHz; 2 KHz and 4 KHz. These measurements were taken using the first floor front room as the source room and the ground floor front room as the receiving room. Then the experiment was repeated but interchanged using the first floor front room as the receiving room and the ground floor front room as the source room.

This method was used before and after the installation of Rocksil Insulation Mat.

Using the average SPL that were calculated for each centre frequency, the sound pressure level difference was determined. The level difference is the difference in the space and time average SPL's produced in the two rooms by a sound source in one of the rooms. Each time the measurements were carried out, a detailed description of all the surfaces within the two rooms was noted, with regard to the area and the material of construction (Figure 3.1 and 3.2). This enabled absorption coefficients to be determined for each surface within each room. These measurements were necessary in order to calculate the Sound Reduction Index (SRI) for each room. BS 2750 part 4 states that the apparent SRI may be evaluated using

$$R' = L_1 - L_2 + 10 \log \frac{S}{A} \text{ dB}$$

where S = Area of test material
A = Equivalent absorption area in the receiving room
L_1 = Average SPL in source room
L_2 = Average SPL in receiving room

METHOD USED TO MEASURE IMPACT SOUND INSULATION

In accordance with BS 2750 part 6 the impact sound was generated by a tapping machine which was placed on the floor under test. For the purposes of this study the tapping machine was placed in one position at the centre of the floor instead of the 4 positions suggested in the British Standard. The B & K tapping machine used had 5 hammers placed in a row, the time between successive impacts being 100 $\pm$ 5 ms.

The impact sound levels in the receiving room (ground floor front room) were measured using the B & K type 2209 sound level meter, which had a B & K external filter type 1613 attached to it.

The measurements of the SPL's were taken at 6 different positions in the receiving room for the centre frequencies 63 Hz; 125 Hz; 250 Hz and 500 Hz and at 3 of the positions for the centre frequencies 1 KHz; 2 KHz and 4 KHz. This enabled an average SPL in a specific frequency band to be calculated.

	Area	Material
FLOOR	4 m 770 = W 3 m 920 = L	Wooden boards insulation mat below
WALLS	1 = 3 m 920 x 2 m 680 2 = 4 m 770 x 2 m 660 minus windows 3 = 3 m 940 x 2 m 670 4 = 4 m 770 x 2 m 670 minus door	Plaster Plaster on brickwork - front Wallpaper Plaster
CEILING	4 m 770= W 3 m 920 = L	Artex on ceiling joists
WINDOW	1 = 1 m 160 = W 1 m 550 = H 2 = 770 mm = W 1 m 550 = H	Glass in wooden frames
DOORS	760 mm x 1 m 990	Wooden, 35 mm thick

Figure 3.1: Area and materials of surfaces of upstairs room after treatment.

Using this calculation the normalised impact sound pressure level, L_n, could be determined for each centre frequency by,

$$L_n = L_i + 10 \log \frac{A}{A_o} \text{ dB}$$

where L_i = Average SPL in a specific frequency band in the receiving room
A = Equivalent absorption area of receiving room
A_o = The reference equivalent absorption area = $10m^2$

This method was used before and after the installation of Rocksil Insulation Mat.

THE RESULTS FOR AIRBORNE SOUND

The difference between the sound pressure levels was measured in the two rooms at each octave band centre frequency, before the installation of insulation. This was then compared with the differences in the sound pressure levels after the installation of Rocksil Insulation Mat. From the difference in the two it became evident that the transmission of airborne sound between the rooms had been reduced by the presence of insulation material.

	Area	Material
FLOOR	3 m 910 = W 3 m 930 = L	Concrete
WALLS	1 = 2 m 400 x 3 m 910 minus window 2 = 2 m 400 x 3 m 930 3 = 2 m 400 x 3 m 910 minus door 4 = 2 m 400 x 3 m 930	Plaster* Plaster Plaster Plaster
CEILING	3 m 910 x 3 m 930	Plaster on insulation mat, joists etc.
WINDOW	2 m 230 = W 1 m 80 = L	Glass in wooden frames
DOORS	750 mm x 2 m 30	Wooden

Figure 3.2: Area and materials of surfaces of downstairs room after treatment.

From the measured SPL's and the calculated absorption coefficients, the SRI's for the 2 rooms were calculated for each octave band centre frequency.

From the results of the measurements when the sound source was in the downstairs room, it could be seen that before the installation of insulation, the SRI's ranged from 13.43 dB at 125 Hz, gradually increasing to 30.93 dB at 4 KHz. After Rocksil Insulation Mat had been installed the SRI's were increased to 33.74 dB at 125 Hz, gradually increasing to 65.31 dB at 4 KHz (Figure 4).

The results of the measurements taken when the sound source was in the first floor front room followed a similar trend. This indicates that:

a) The SRI was generally lower at the lower octave band centre frequency, gradually increasing with the centre frequency. Thus the reduction in the transmission of airborne sound is greatest at the high frequencies as expected. This factor is important because high frequency noise can be the cause of annoyance to people.

b) The installation of the Rocksil Insulation Mat plasterboard treatment had effectively increased the SRI for each octave band centre frequency, thus the transmission of airborne sound was reduced by the presence of the treatment.

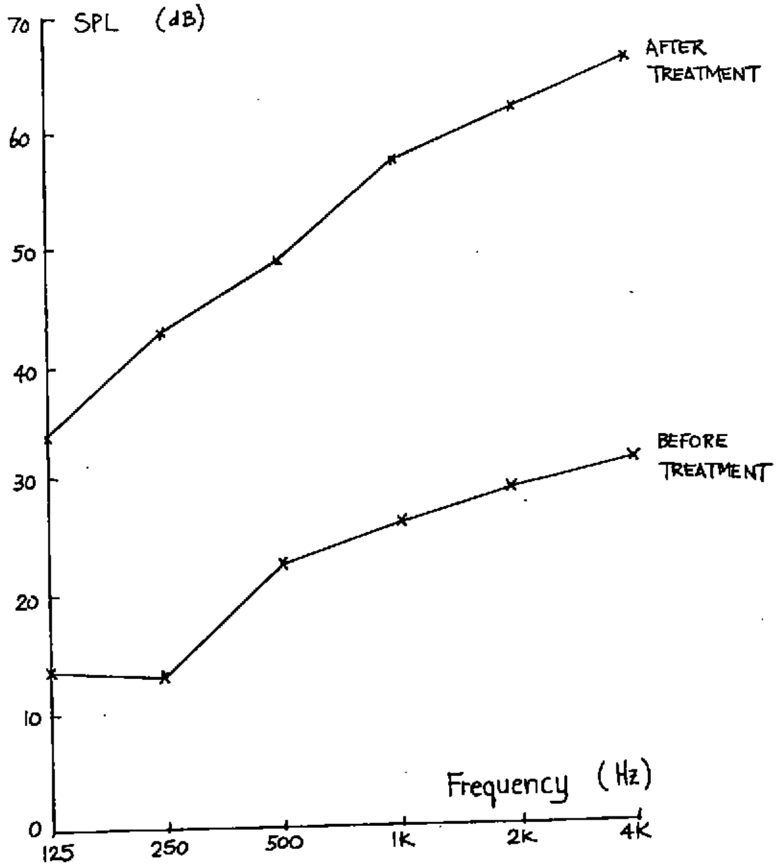


Figure 4: Comparison of SRI before and after treatment

THE RESULTS FOR IMPACT SOUND

Analysis of the normalised impact sound pressure levels, which were calculated before and after the installation of insulation, revealed that there was a general trend that the impact SPL decreased at the higher frequencies.

After the installation of the Rocksil Insulation Mat the normalised impact SPL at each octave band centre frequency was lower than it was before the insulation was installed (Figure 5).

Therefore it was found that the installation of Rocksil Insulation Mat and plasterboard treatment had effectively reduced the transmission of impact sound between the two rooms.

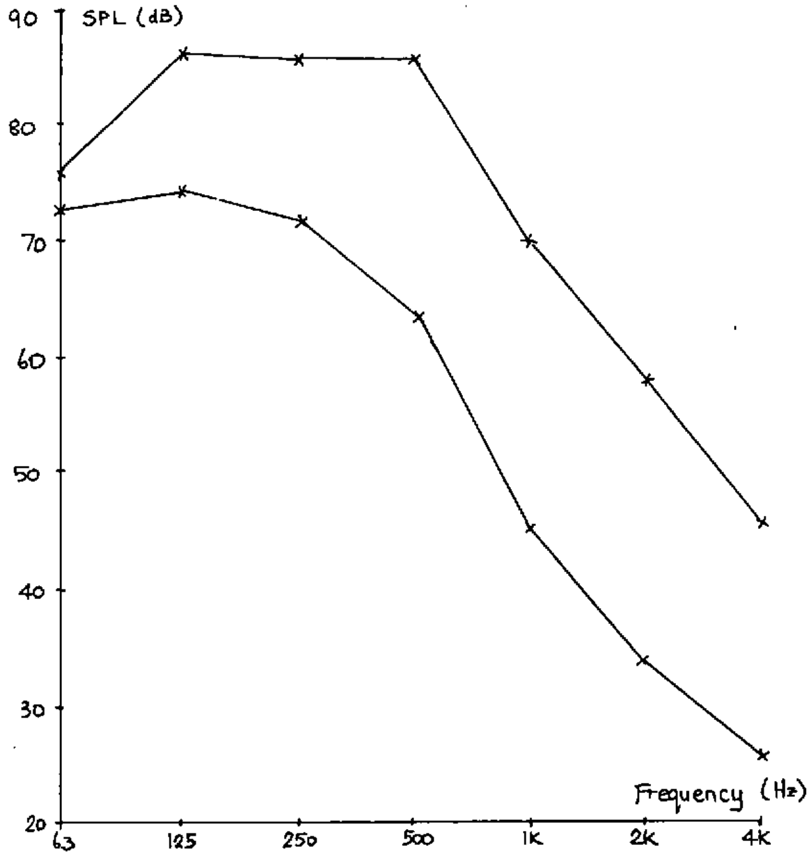


Figure 5: Comparison of impact sound measurements before and after treatment

CONCLUSION

On analysis of the results of the measurements and calculations for the transmission of the sound between the two separate dwellings before and after the treatment, it was concluded that the transmission of both airborne sound and impact sound was effectively reduced to acceptable levels in the two dwellings.

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