24th INTERNATIONAL CONGRESS ON SOUND AND VIBRATION
23–27 July 2017, London

NONLINEAR VORTEX INDUCED VIBRATIONS OF INCLINED MARINE RISERS

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In this paper, we study the nonlinear dynamics of statically deflected offshore structures undergoing vortex induced vibrations. We consider inclined risers that have catenary-like profiles where the riser structure is modelled as an Euler-Bernoulli beam with mid-plane stretching and under self-weight. The statically deflected structure is subjected to vortex induced vibration forces and its equation of motion is coupled with Van Der Pol oscillator. Then, we use mode shapes, extracted from the linear eigenvalue problem, in Galerkin method. This reduces the coupled system to a set of nonlinear ordinary differential equations. We use long time integration with Runge-Kutta method to integrate the coupled system and obtain the response while varying the inclination angle, internal velocity, and external velocity. The numerical results reveal the effects of the nonlinearity on the response of the structure, modal interaction, and resonance interactions.

Keywords: Vortex Induced Vibrations, Inclined Risers, Galerkin.

1. Introduction

Marine risers serve as the main link to transport oil and products in oil drilling and production facilities. Because of their importance, the design aspects of the riser structures have to be carefully studied. This is due to the fact that riser structures are subjected to different types of internal and external loadings as part of fluid-structure interactions and this increases the complexity of the design problem. The forces on the risers structure can be exerted by environmental factors, such as sea currents, geometrical factors, and factors induced by internal fluid flow. These forces can combine to generate static and dynamic stresses, such as vortex-induced vibration, which hinders the structural life of the riser due to fatigue damage [1]. From a structural point of view, the riser model is different from traditional structural beams because risers are under variable self-weight axial force. In addition, when the riser becomes at an inclination the self-weight force induces a distributed load that causes a static initial deflection of the structure [2]. In the presence of a nonlinear geometrical feature, the static deflection of the riser structure influences its dynamics producing complex dynamics such as vortex induced vibrations VIV.

The phenomenon of vortex induced vibrations with a wake oscillator of a vertical marine riser is a well addressed topic in research. A complete review and listing of the relevant works are found in

[3-5]. As examples, we mention the early work of Nordgren [6] and Hover et al. [7] who paved the way to study the nonlinear dynamics of statically deflected structures. Several researches have considered experimental works on steel catenary risers (SCR) to study the effects of the nonlinear geometry on the VIV dynamics [8-11]. These results featured modal interaction and some aspects of travelling waves. The numerical analysis of VIV of deflected structures [12-16] showed the modal transition behavior as well as some signs of chaotic dynamics in the riser structure.

The vortex-induced vibrations is a two dimensional phenomena. In the absence of a formulation describing the inline vortex vibrations, which is based on the oblique shedding characteristics of the current flow, studies are restricted to the context of cross flow vibrations. In this work we perform a numerical analysis on the cross flow vortex-induced vibrations of inclined structure where the effects of nonlinear mid-plane stretching and internal fluid are accounted for. In addition, the fluid forces on the riser structure are quantified using Morison formulation [17] and Van Der Pol oscillator to study the nonlinear dynamics of vortex-induced vibrations.

2. Mathematical Formulation

We consider studying the nonlinear interaction in inclined marine risers when the cross flow lift induces a motion in the direction of the static deflection of the riser, Fig. 1.

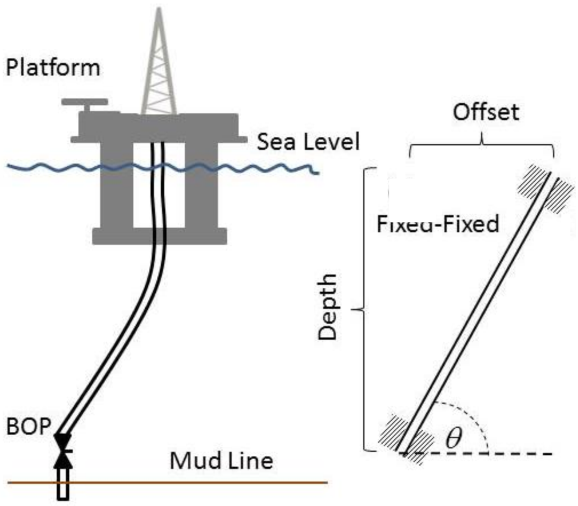

Figure 1: Schematic of an inclined riser used in oil production extending from the platform to the blow out preventer for fixed-fixed configuration of the riser.

The riser extends from the floating unit to the blow out preventer (BOP) near the seabed. Based on the depth, offset, and length of the pipe, the riser pipe will statically deflect as shown in Fig. 1. Subsequently, the static deflection along with the lift force induced by external current flow will contribute to the dynamics of the riser system. The external current flow in this case is assumed to be orthogonal to the plane of curvature of the pipe. The riser is modeled as an Euler-Bernoulli beam with variable axial tension including the nonlinear mid-plane stretching effects. Hence, the riser equation with the effects of internal flow becomes [18, 19]

$$
\begin{aligned}
& m\frac{\partial^2 \hat{y}}{\partial t^2} + EI\frac{\partial^4 \hat{y}}{\partial \hat{x}^4} + 2m_f U_i \frac{\partial^2 \hat{y}}{\partial \hat{x}\partial t} + \left(m_f Ui^2 - \left(T_e - WeSin(\theta)(L-\hat{x})\right) - \frac{EA_p}{2L}\int_0^L \left[\left(\frac{\partial \hat{y}}{\partial \hat{x}}\right)^2\right] d\hat{x} \right)\left(\frac{\partial^2 \hat{y}}{\partial \hat{x}^2}\right) \\
& -WeSin(\theta)\left(\frac{\partial \hat{y}}{\partial \hat{x}}\right) + c_s \frac{\partial \hat{y}}{\partial t} = F_d - WeCos(\theta)
\end{aligned} \tag{1}
$$

where $\hat{y}$ is the response of the structure, $\hat{x}$ is the length along the structure, t is the time, m is the total mass, $m = m_f + m_s$, where m_f is mass of internal fluid, m_p is mass of riser pipe, EI is flexural rigidity, U_i is internal fluid flow, T_e is the applied tension, θ is the inclination angle, L is the length of the pipe, A_p is the cross section area of the pipe, D is the diameter, We is the apparent weight of the riser, F_d is the external dynamic force. The tension of the pipe T_e is assumed to vary linearly with the weight of the pipe. This work is considered a subsequent analysis to the static solution which is solved using boundary layer perturbation using the method of matched asymptotic expansion [20] and the linear dynamics [21] which is studied using Galerkin method. Then, the nonlinear equation is written as

$$m\frac{\partial^2 \hat{y}_d}{\partial \hat{t}^2} + EI\frac{\partial^4 \hat{y}_d}{\partial \hat{x}^4} + 2m_f U_i \frac{\partial^2 \hat{y}_d}{\partial \hat{x}\partial \hat{t}} + \left(m_f Ui^2 - T_e + WeSin(\theta)(L-\hat{x})\right)\left(\frac{\partial^2 \hat{y}_d}{\partial \hat{x}^2}\right) - WeSin(\theta)\left(\frac{\partial \hat{y}_d}{\partial \hat{x}}\right) + c_s\frac{\partial \hat{y}_d}{\partial \hat{t}}$$
$$-\frac{EA_p}{2L}\left(\int_0^L\left[\left(\frac{d\hat{y}_s}{d\hat{x}}\right)^2\right]d\hat{x} + \int_0^L\left[\left(\frac{\partial \hat{y}_d}{\partial \hat{x}}\right)^2\right]d\hat{x} + \int_0^L\left[2\frac{d\hat{y}_s}{d\hat{x}}\frac{\partial \hat{y}_d}{\partial \hat{x}}\right]d\hat{x}\right)\frac{\partial^2 \hat{y}_d}{\partial \hat{x}^2} - \left(\frac{EA_p}{2L}\left(\int_0^L\left[2\frac{d\hat{y}_s}{d\hat{x}}\frac{\partial \hat{y}_d}{\partial \hat{x}}\right]d\hat{x} + \int_0^L\left[\left(\frac{\partial \hat{y}_d}{\partial \hat{x}}\right)^2\right]d\hat{x}\right)\right)\frac{d^2\hat{y}_s}{d\hat{x}^2} = F_d \tag{2}$$

The riser is subjected to the drag and lift forces, where the first is governed by the linearized form of Morison equation [17] and the later is governed by Van Der Pol oscillator. The external current flow is assumed to be steady and uniform. These forces are written as

$$F_d = \underbrace{F_{inertia} + F_{drag}}_{Morison\ Equation} + \underbrace{F_{lift}}_{Van\ Der\ Pol\ oscillator} \tag{4}$$

The expressions of each term in Eq. (4) are given by

$$F_{inertia} = -C_A\rho_e A_e\frac{\partial^2 \hat{y}_d}{\partial \hat{t}^2}\ ,\ F_{drag} = -\frac{1}{2}C_D\rho_e DU_e\frac{\partial \hat{y}_d}{\partial \hat{t}}\ ,\ F_{lift} = \frac{1}{4}C_{L_0}\rho_e DU_e{}^2 q(\hat{x},\hat{t}) \tag{4a}$$

where C_A is the added mass coefficient, ρ_e is the density of external fluid, A_e is the external area of the riser pipe, C_D is the drag coefficient, C_{L_0} is the lift coefficient, U_e is the external current flow. The inertia mass of the riser becomes $m = m_f + m_s + C_A\rho_e A_e$. The description of the lift force using the Van Der Pol oscillator is written as [22]

$$\frac{\partial^2 q}{\partial \hat{t}^2} + \lambda\omega_s\left(q^2 - 1\right)\frac{\partial q}{\partial \hat{t}} + \omega_s{}^2 q = \frac{P}{D}\frac{\partial^2 \hat{y}_d}{\partial \hat{t}^2} \tag{5}$$

where P and λ are correction parameters obtained based on experimental results, ω_s is the vortex shedding frequency, which is governed by the external current flow. For further analysis, we introduce the following dimensionless variables

$$y = \frac{\hat{y}}{D} \qquad x = \frac{\hat{x}}{L} \qquad t = \sqrt{\frac{EI}{mL^4}}\hat{t} \tag{6}$$

where $\hat{y}$ denotes the dimensional variable and the dimensionless one is denoted by y. Substitute Eq. (6) into (2) we obtain

$$\frac{\partial^2 y_d}{\partial t^2} + \frac{\partial^4 y_d}{\partial x^4} + 2\sqrt{\beta}\bar{v}\frac{\partial^2 y_d}{\partial x\partial t} + \left(\bar{v}^2 - \bar{T} + \bar{\sigma}(1-x) - \bar{\eta}\left(\int_0^1\left[\left(\frac{dy_s}{dx}\right)^2\right]dx + \int_0^1\left[\left(\frac{\partial y_d}{\partial x}\right)^2\right]d\hat{x} + \int_0^1\left[2\frac{dy_s}{dx}\frac{\partial y_d}{\partial x}\right]d\hat{x}\right)\right)\left(\frac{\partial^2 y_d}{\partial x^2}\right)$$
$$-\bar{\eta}\left(\int_0^1\left[2\frac{dy_s}{dx}\frac{\partial y_d}{\partial x}\right]dx + \int_0^1\left[\left(\frac{\partial y_d}{\partial x}\right)^2\right]d\hat{x}\right)\left(\frac{d^2 y_s}{dx^2}\right) - \bar{\sigma}\left(\frac{\partial y_d}{\partial x}\right) + c\frac{\partial y_d}{\partial t} = \alpha_F q \tag{7a}$$

and Van Der Pol oscillator is given by

$$\frac{\partial^2 q}{\partial t^2} + \lambda \Omega_s \left(q^2 - 1\right)\frac{\partial q}{\partial t} + {\Omega_s}^2 q = P\frac{\partial^2 y}{\partial t^2} \tag{7b}$$

where

$$\beta = \frac{m_f}{m} \quad \bar{v} = \frac{\sqrt{\frac{m_f}{EI}}}{L}U_i \quad \bar{\sigma} = \frac{WeSin(\theta)L^3}{EI} \quad \bar{\eta} = \frac{A_p D^2}{2I} \quad c = \frac{C_D \rho_0 D U_e L^2}{2\sqrt{mEI}} \alpha_F = \frac{C_{L_0} \rho_e {U_e}^2 L^4}{4EI} \quad \bar{T} = \frac{T_e L^2}{EI} \quad \Omega_s = \omega_s \sqrt{\frac{m}{EI}} L^2 \tag{8}$$

We note the presence of quadratic and cubic nonlinearities, which arise due to the nonlinear stretching of the riser structure. Next, we use Galerkin expansion to solve the nonlinear equation and assume the solution to be of the form

$$y_d(x,t) = \sum_{i=1}^{n} \phi_i(x) u_i(t) \tag{9}$$

where $\phi_i(x)$ are the mode shapes extracted from the linear dynamics. Similarly, because of the synchronization between the fluid flow motion around the riser pipe and the motion of the riser pipe, the same Galerkin expansion of the riser can be employed in Van Der Pol Eq. (7b), which is expressed as

$$q(x,t) = \sum_{i=1}^{n} \phi_i(x) w_i(t) \tag{10}$$

Solution (9) is substituted back into Eq. (7a), which yields

$$\begin{aligned}&\sum_{i=1}^{n}\phi_i(x)\ddot{u}_i(t) + \sum_{i=1}^{n}\phi_i''''(x)u_i(t) + 2\sqrt{\beta}\bar{v}\sum_{i=1}^{n}\phi_i'(x)\dot{u}_i(t) - \bar{\sigma}\sum_{i=1}^{n}\phi_i'(x)u_i(t) + c\sum_{i=1}^{n}\phi_i(x)\dot{u}_i(t) - \bar{\eta}\int_0^1\left[\left(\sum_{i=1}^{n}\phi_i'(x)u_i(t)\right)^2\right]d\hat{x}\left(\frac{d^2 y_s}{dx^2}\right)\\&\left(\bar{T}_s - \bar{\sigma}x - \bar{\eta}\left(\int_0^1\left[\left(\sum_{i=1}^{n}\phi_i'(x)u_i(t)\right)^2\right]d\hat{x} + \int_0^1\left[2\frac{dy_s}{dx}\sum_{i=1}^{n}\phi_i'(x)u_i(t)\right]d\hat{x}\right)\right)\left(\sum_{i=1}^{n}\phi_i''(x)u_i(t)\right) - \bar{\eta}\int_0^1\left[2\frac{dy_s}{dx}\sum_{i=1}^{n}\phi_i'(x)u_i(t)\right]dx\left(\frac{d^2 y_s}{dx^2}\right) = \alpha_F\sum_{i=1}^{n}\phi_i(x)w_i(t)\end{aligned} \tag{11}$$

Next, we apply the orthogonality condition by multiplying Eq. (11) by $\phi_j(x)$ and integrating over the domain. This reduces Eq. (11) to

$$\begin{aligned}&\ddot{u}_i(t) + 2\sqrt{\beta}\bar{v}\sum_{i=1}^{n}\int_0^1\phi_j(x)\phi_i'(x)\dot{u}_i(t)dx + c\sum_{i=1}^{n}\int_0^1\phi_j(x)\phi_i(x)\dot{u}_i(t)dx + \int_0^1\phi_j(x)\left(\sum_{i=1}^{n}\phi_i''''(x)u_i(t)\right)dx - \int_0^1\phi_j(x)\left(\bar{\sigma}\sum_{i=1}^{n}\phi_i'(x)u_i(t)\right)dx\\&-\bar{\eta}\int_0^1\phi_j(x)\left(\int_0^L\left(\sum_{i=1}^{n}\phi_i'(x)u_i(t)\right)^2 d\hat{x} + \int_0^L 2\frac{dy_s}{dx}\sum_{i=1}^{n}\phi_i'(x)u_i(t)d\hat{x}\right)\left(\sum_{i=1}^{n}\phi_i''(x)u_i(t)\right)dx + \int_0^1\phi_j(x)\left(\bar{T}_s - \bar{\sigma}x\right)\left(\sum_{i=1}^{n}\phi_i''(x)u_i(t)\right)dx\\&-\bar{\eta}\int_0^1\phi_j(x)\left(\int_0^1 2\frac{dy_s}{dx}\sum_{i=1}^{n}\phi_i'(x)u_i(t)dx + \int_0^1\left(\sum_{i=1}^{n}\phi_i'(x)u_i(t)\right)^2 d\hat{x}\right)\left(\frac{d^2 y_s}{dx^2}\right)dx = \alpha_F w_i(t)\end{aligned} \tag{12}$$

Also, substituting solution (10) into Eq. (7b) and applying the orthogonality condition, we obtain

$$\ddot{w}_i(t) + \int_0^1\phi_j(x)\lambda\Omega_s\left(\left(\sum_{i=1}^{n}\phi_i(x)w_i(t)\right)^2 - 1\right)\sum_{i=1}^{n}\phi_i(x)\dot{w}_i(t)dx + {\Omega_s}^2 w_i(t) = P\ddot{u}_i(t) \tag{13}$$

where $\bar{T}_s = \bar{v}^2 - \bar{T} + \bar{\sigma} - \Gamma_s$ and $\Gamma_s = \bar{\eta}\int_0^1 \left[\left(\frac{dy_s}{dx}\right)^2\right] dx$.The partial differential equation of the structure is reduced to a set of ordinary differential equations that are coupled with the reduced Van Der Pol oscillator equations to form coupled systems.

3. Numerical Results

We use the data presented in Table 1. The parameters obtained for the forced Van Der Pol oscillator are $\lambda = 0.3$ and $P = 12$. The nonlinear dynamic responses are studied at configurations θ=15^o and θ=75^o. In addition, we define Strouhal relation to govern the frequency synchronization between the structure and the wake, which is given by $\Omega_s = V_r S \Omega_1$ and $V_r = \frac{2\pi U}{\Omega_1 D}$. For the purpose of convergence, ten modes are used in Galerkin method while updating the temporal initial conditions from the previous external velocity for two applied tension cases: (a) $2\bar{\sigma}$ and (b) $5\bar{\sigma}$.

Table 1. Properties of the riser used in the analysis.

Property	Value	Property	Value
Outside Diameter, D	0.26 *m*	Density of Riser, ρ_p	7850 *Kg/m*^{3}
Inside Diameter, D_i	0.2 *m*	Density of Sea Water, ρ_e	1025 *Kg/m*^{3}
Modulus of Elasticity, E	207 *GPa*	Density of Internal Fluid, ρ_i	998 *Kg/m*^{3}
Depth	150 *m*		

3.1 Frequency Analysis

The eigenvalue problem of Eq. (11) is first analysed to understand how the frequency varies with the applied tension and the inclined angle configuration while setting the internal velocity to zero ($\bar{v} = 0$). The frequency results are plotted in Fig. 2.

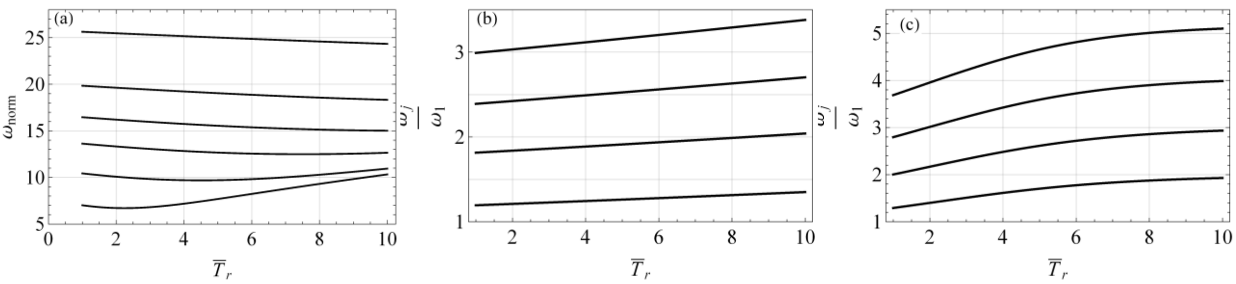

Figure 2: Natural frequency results $\bar{T}_r = \bar{T}/\bar{\sigma}$, $\omega_{norm} = \omega/\sqrt{\bar{\sigma}}$. (a) First natural frequency results for configurations: 85^o, 75^o, 60^o, 45^o, 30^o, 15^o from bottom to top respectively. (b) Lowest four frequency ratios at 15^o. (c) Lowest four frequency ratios at 30^o.

It is noted from Fig. 2(a) the influence of the static deflection on the structure, which causes a decrease in the frequency values as the applied tension increases. This behavior decreases with increasing configuration angle (from bottom of the plot 2a to top). In addition, we note also in Fig. 2(b) and Fig. 2(c) the commensurability of the frequency ratios, which can result into activation of modal interactions in the response of the structure. Because of the influence of the static deflection of the structure, there will be a possibility of higher order resonance activation that causes the nonlinear response to become complex. This behavior is attributed to the competition between the mid-plane stretching and the static deflection, which induces a softening behavior similar to what has been reported in arches [23] and cables [24]. From a qualitative aspect, similar results were produced by [25] confirming the results obtained in this work. Furthermore, we note that at the asymptotic level for configuration 75^o the frequency ratio tends to integers at higher applied tension values. In addition, we note that the spacing between the frequency values in configuration (b) is small compared to the spacing in configuration (c). Because the frequencies are closely spaced, the activation of more than one resonance is possible.

3.2 Nonlinear Oscillations

Next, the response of the structure is examined by capturing the steady state response of the structure while varying the reduced velocity. In order to capture the effect of the symmetric and non-symmetric modes, the response is monitored at $x = 0.3$. For convergence, ten modes are used in Galerkin procedure.

3.2.1 Effect of Inclination Angle

Here, we neglect the effect of the internal velocity be setting $\bar{v} = 0$ and we plot the response of configurations θ=15^o and θ=75^o with forward and backward sweeps of the external reduced velocity. The results are shown in Fig. 3 for applied tension values of $\bar{T} = 2\bar{\sigma}$ and $\bar{T} = 5\bar{\sigma}$.

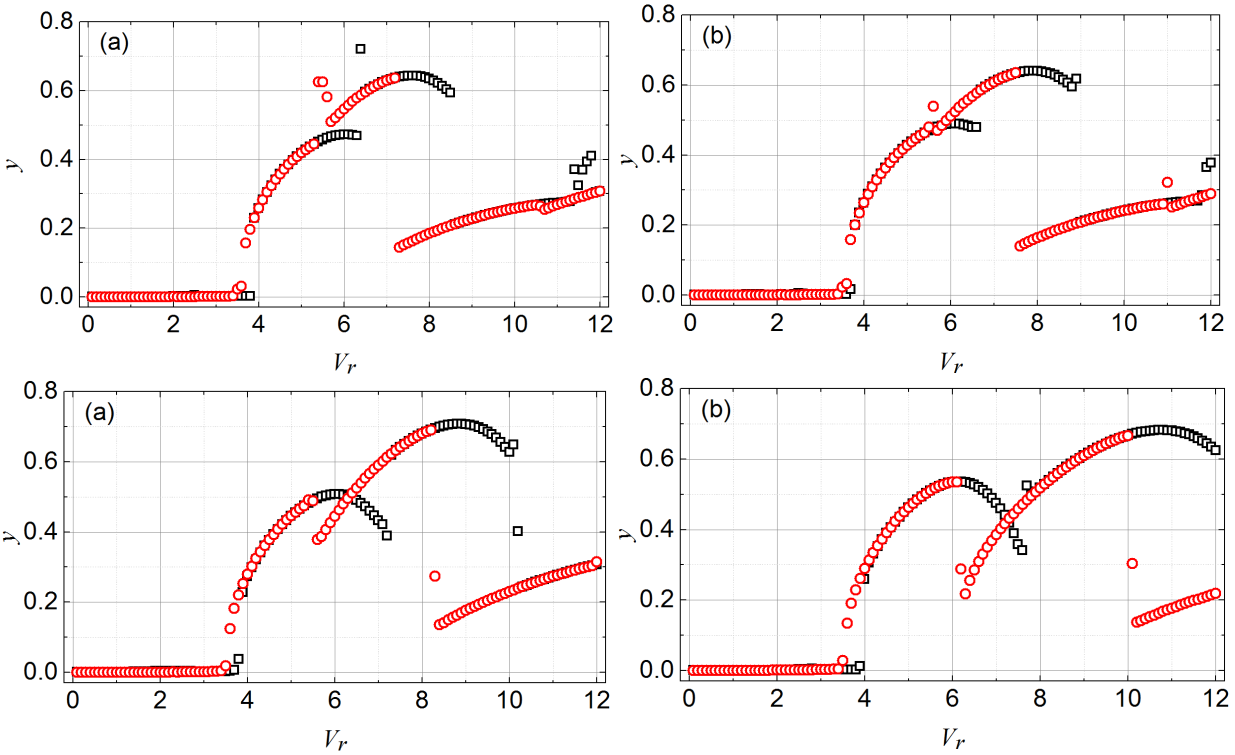

Figure 3: Nonlinear response of the structure y versus the external reduced velocity Vr. ('□'); forward sweep, ('○'); backward sweep. (a) $\bar{T} = 2\bar{\sigma}$, θ=15^o(b) $\bar{T} = 5\bar{\sigma}$, θ=15^o(c) $\bar{T} = 2\bar{\sigma}$, θ=75^o (d) $\bar{T} = 5\bar{\sigma}$, θ=75^o.

As observed from Fig. 3, the response of the structure is dominated by the hardening behaviour. We note the activation of the first, second, and third resonances while sweeping around the first natural frequency. Note that the fundamental natural frequency corresponds to a Strouhal number of 0.2

and an external reduced velocity of 5. Instead, due to the close spacing between the frequencies, we note that the second and third natural frequencies are being activated which results into a response with a wider band activated at several frequencies. We note also that increasing the applied tension in cases (b) and (d) causes the response of the second and third natural frequency to shift away from the first natural frequency. From Fig. 3 we note also the modal transition responses, which lay outside the response of the structure. This is because of the two competing resonances, which cause the response to change from a periodic one to have a beating-like characteristic.

3.2.2 Effect of Internal Velocity

In this section, we study the influence of the internal velocity on the response characteristics at $\bar{T}=2\bar{\sigma}$ and θ=15^{o} and we plot the response of the structure at internal velocities of $\bar{v}^2=\bar{\sigma}/3$ and $\bar{v}^2=\bar{\sigma}/2$. The results are plotted in Fig. 4.

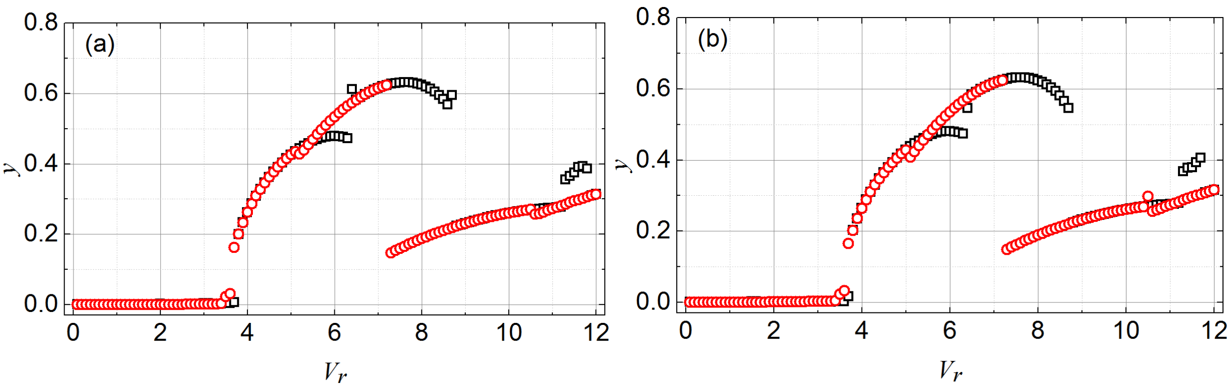

Figure 4: Nonlinear response of the structure y versus the external reduced velocity Vr. ('□'); forward sweep, ('○'); backward sweep at $\bar{T}=2\bar{\sigma}$, θ=15^{o}. (a) $\bar{v}^2=\bar{\sigma}/3$ (b) $\bar{v}^2=\bar{\sigma}/2$.

As noted from Fig. 4, introducing the internal velocity reduces the effect of the modal transition in comparison to the results plotted in Fig. 3 (a). This is due to the fact that the internal velocity has two effects. First, it shifts the axial force in the form of a compression force. Second, it introduces a gyroscopic term that couples the response of the modes. This result suggests that the internal velocity reduces the interaction in the modal transition regions and shifts the response towards a period one.

4. Conclusion

In this work, we presented the global nonlinear VIV dynamics response of the structure considering the effects of the static deflection, internal velocity, and mid-plane stretching. The presence of nonlinearity in the structure leads to complex dynamics involving interaction from several modes. These resonances can introduce non-periodic behaviours in the modal transition regions, which require further investigation.

5. ACKNOWLEDGMENTS

This research made use of the resources of the IT Research Computing at King Abdullah University of Science & Technology (KAUST) in Thuwal, Saudi Arabia. In addition, the first author acknowledges the support of Saudi Aramco.

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