APPLICATION OF ACOUSTIC RECIPROCITY PRINCIPLES

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THE APPLICATION OF RECIPROCITY PRINCIPLES TO ACOUSTIC AND VIBRO-ACOUSTIC PROBLEMS

1. Introduction

The most widely known principle of acoustic reciprocity was introduced in 1860 by Helmholtz [1]. It may be used to show that the positions of an omi-directional receiver and an omni-directional source operating in a linear, passive, time-invariant fluid system, bounded by rigid, or locally reacting impedance surfaces, may be exchanged, with no change in the received pressure. John Tyndall queried its validity when he found that the acoustic effect of a plane screen placed between a whistle source and a sensitive flame was not invariant with respect to movement of the screen to a reciprocal position: Lord Rayleigh, however, pointed out that the whistle was not an omni-directional source. The latter also published a general theorem of reciprocity for multi-degree-of-freedom vibrating systems in 1873, which includes Helmholtz's principle as a special case [2]. Later, Helmholtz was able to remove some of the constraints required by Rayleigh [3]. Both admitted dissipation functions which are positive definite quadratic forms of the generalised velocities.

The restriction to *locally reactive* impedance surfaces was removed by Lyamshev in 1959 [4,5]. In a most significant contribution to the advancement of theoretical and experimental studies of interaction between elastic plate and shell structures and conti-guous ideal fluids (herein termed, 'vibro-acoustics'), he developed reciprocal relationships between the acoustic fields generated by the action of vibrational forces applied to structures, and the normal velocity fields produced on the surface of those structures when insonified by acoustic sources in the fluid (the diffraction problem). These relationships may frequently be used to ease the experimental difficulties associated with applying controlled vibrational force inputs to operational structures for the purposes of investigating the resulting radiated sound [6].

In this paper, examples will be given of the application of various forms of reciprocity relationship to the solution of practical problems in various areas of engineering acoustics and noise control.

2. Application of Reciprocity to Duct Mode Directivity Measurement

A manufacturer of ventilation duct attenuators wished to obtain data on the directivity characteristics of sound radiated from the exits of rectangular ventilation ducts which contain terminal splitter silencers. Since no information appeared to be available in the literature, an experimental study was conducted at ISVR, Southampton University, as an MSc project [7]. Directivity is, of course, dependent upon the modal composition of the field in the duct: this is normally not known *a priori*. Therefore it was decided to evaluate the directivity associated with the incidence of individual duct modes on a terminal splitter silencer.

It would be virtually impossible, within the time and cost limitations of an MSc project, to devise an in-duct source to excite each mode selectively. Since it may be shown that sound absorbent materials behave reciprocally within their range of *linear* behaviour [6], it was possible to exploit the 'classical' principle of reciprocity between omni-directional sources and receivers. The ducts containing splitter silencers were mounted in an anechoic chamber, and a loudspeaker was mounted at a distance of about 6 metres from the duct exit face. The duct could be rotated about a pivot located on the exit plane, in order to vary the angle between the duct axis and the loudspeaker axis. The reciprocity principle requires that the source field has spherically symmetric field characteristics only over the area of the duct exit.

The reciprocity principle ensures that the transfer function between the volume velocity of a point source in the duct and the sound pressure at the loudspeaker position is identical to that which is obtained when source and reciever are exchanged. This transfer function is independent of the termination (opposite end of the duct to the exit), provided that the termination does not scatter energy from one mode into another (i.e., it must have uniform impedance). Below the lowest cut-off frequency of a duct, this transfer function is independent of position of the receiver on a duct cross-section, and indicates plane wave directivity. At higher frequencies, transfer functions were evaluated on a five-by-five point grid on a cross-sectional plane within the duct. By means of modal (cosine) decomposition of the transfer functions, the modal directivity of the modes up to (2,2) were determined. Measurements were made in planes both parallel and perpendicular to the splitter plane. Examples of the results are shown in Figures 1(a) and (b).

3. Prediction of the Radiation Directivity of a Submerged Body

One of the major problems in experimentally evaluating the far field directivity of vibrating structure submerged in water is that it is extremly costly and time-consuming to perform tests in the sea or in deep lakes, where reflections from bounding surfaces are sufficiently weak. Hydro-sounders (projectors) may be tested in anechoic tanks, or by means of pulsed inputs in reflective tanks. These techniques are not available to the engineer concerned with the reduction of noise radiated by large sumberged structures, partly because their decay times are normally too long for echoes to be excluded from the acoustic response.

The reciprocity relationship between source and receiver in fluids may be specialised to source positions on the surfaces of solid bodies. The complex amplitude of harmonic sound pressure generated in free space by a harmonic point monopole of complex volume velocity amplitude $\tilde{Q}$ is

$$\tilde{p} = (i\rho_0\omega\tilde{Q}/4\pi r)\exp(-ikr)\,. \qquad (1)$$

where r is the distance between source and observation point. The term $\exp(-ikr)/4\pi r$ is termed the 'free space' Green function. The relationship in equation (1) is modified by the presence of any scattering, or reflecting, surface within the fluid. When the point source operates immediately adjacent to a rigid body, the Green function is altered from that in equation (1): in the special case of an infinite, plane, rigid surface, the Green function is doubled.

The vibrating surface of any body of arbitrary geometry may be represented by an array of monopole sources distributed over the surface. The Green function appropriate to any one source position, and any one receiver position, may, by reciprocity, be determined by placing a point source at that receiver point, and measuring the resulting sound pressure at the location of the source on the *rigid* body: this is known as the blocked pressure (Figure 2). A particular advantage of this reciprocal measurement technique is that the Green functions for a large submerged body may be evaluated on a much smaller rigid model in an anechoic chamber, in *air*, over a frequency range scaled so as to maintain the non-dimensional ratio of wavelength to body dimension the same in prototype and scale model. In order to predict the far field generated in water by a body, it is necssary to measure the vibrational normal velocity field of the body at the corresponding points on the surface, and then combine them with the appropriate Green functions measured on the model (Figure 3).

The selection of the distribution of points at which to sample the surface velocity field of a complex structure is not a straight-forward matter, involving as it does a sampling process performed on a non-uniform, non-homogeneous field. Spatial aliassing of high wavenumber components into low wavenumbers (which radiate much more effectively) presents a serious source of error. However, if the process is performed properly, predictions are found to be satisfactory, as shown by a prediction for a submerged structure, made by ISVR in conjunction with another research organisation (Figure 4).

Of course, such reciprocal measurements may be made on any structure, full-size or model, in order to evaluate the 'influence' of surface vibration at any position on the pressure field at any selected receiver position. For example, a monopole source could be placed at the position of the driver's ear inside a car, and the sound pressure produced on the (rigid) body shell would indicate the contribution to the pressure *phasor* at the ear. [Note carefully that the contributions from all surface points must be added with due account of relative phase of the surface vibration and of Green function at the receiver point.] In this case also, the Green functions could be measured on a small-scale, rigid model of the car interior.

4. Sound Radiation from Point-excited Structures

As indicated above, Lyamshev extended the reciprocity principle to fluids containing elastic shell and plate structures, uniform and otherwise. The most useful result of this extended principle is that the transfer function btween the vibrational point force applied to a linear elastic structure and the resulting sound pressure at a point in a conti-guous fluid, is equal to the transfer function between the volume velocity of a point monopole located at the receiver point and the resulting vibrational velocity of the structure, at the point of application of, and in the same direction as, the force (Figure 5).

Consequently, if one needs to estimate the sound pressure produced in a fluid by the application of a mechanical force to a structure, it is not necessary either to measure the force, or to simulate it by a vibration generator. It is only necesary to place a calibrated, omni-directional source at the field point of interest, and measure the resulting acceleration (velocity) of the structure in the direction of the applied force: the origin of the force need not be removed for this measurement. For example, the source at the driver's ear, mentioned above, could be used to vibrate the car, and the transfer function to acceleration at engine mounting point may then be measure: the contribution of the engine mounting force to the sound pressure at the driver's ear, is thus quantified.

This technique has been extensively applied in the fields of ship radiated noise and building installations [8]. An example the direct and reciprocal results of an application to the problem of cabin noise generated in an aerospace structure excited by mechanical forces is shown in Figure 6. In this case the modulus of the transfer function was averaged over octave bands, which is not entirely rigorous, but often reasonably accurate.

5. Application to Cabin Noise Studies for Propeller-Driven Aircraft

Prediction of the noise generated in the cabin of a propeller-driven aircraft by the propeller noise involves theoretical evaluation of the sound field acting on the surface of the fuselage, together with calucations of the response of the fuselage structure, the influence of sound insulation, and the reponse of the air in the cabin, which is influenced by the presence of the trim and the furnishings. Although current computational programs for the calculation of cabin fields are powerful and often quite accurate, it is still common practice among aircraft manufacturers to test structures and insulation systems by applying to them simulated propeller acoustic fields of various degrees of sophistication. A fundamental problem is that the convection effects of the forward passage of an aircraft cannot properly be simulated in a static rig; flight and wind tunnel tests are prohibitively expensive and time consuming.

In the face of this problem, the ISVR is currently developing an experimental technique, based upon the Lyamshev reciprocity principle, for 'calibrating' model fuselages as 'transducers' of external pressure fluctations into sound in the contained air. If successful, it is hoped to extend it to full-scale fuselages. It is assumed that the fluctuating pressure distribution on the external surface of a fuselage may be sufficiently precisely estimated using one of the modern computations schemes developed, for example, by NASA in the USA. In principle, the transfer functions between these pressures and the sound pressure at an internal point could be determined experimentally by applying to the fuselage a controlled excitation force, or pressure. Point forces are inappropriate for two reasons: (i) they generate strong high-wavenumber components in the structural shell which play no significant part in the transmission process; (ii) it is not at all clear how the excitation points on the fuselage surface should be chosen - on frames? on panels? on stringers? at what density? It is very difficult to apply a uniform pressure over a limited area of the surface with a device which has to be moved around over the surface, and which may not influence the vibrational reponse of the structure, either by mechanical or acoustic loading.

In view of these uncertainties and potential difficulties, the appropriate transfer functions are generated by a reciprocal measurement process. A calibrated 'point' monopole source is placed at a receiver point of interest in the cabin space, and the resulting normal velocity of the shell is determined, *not at a point*, but as an integral over a finite area (a volume velocity): this is achieved by the use of a specially developed vibration volume velocity probe (Figure 7). Transfer functions to all contiguous areas subject to significant fluctuating pressure are then determined. The resulting set of transfer functions represents a calibration of the structure for the particular receiver point, since they may be combined with *any external pressure field* to estimate the resulting cabin sound pressure. In this way the relative merits and limitations of the various 'simulated' propeller fields, and also the effects of forward flight, may be investigated. A particular advantage of the volume velocity probe is that it acts as a wavenumber filter, and discriminates against high wavenumber components of the vibration field.

The test rig used to validate this new technique is illustrated in Figure 8, and the results of comparisons between sound pressures measured in the box and predicted from a measured external pressure field, are presented in Figures 9 and 10.

A fully automated, computer-controlled test rig for 1:4 model fuselages is about to be commissioned.

6. Relative Modal Radiation Efficiencies of a Vibrating Structure

P.W. Smith extended vibro-acoustic reciprocity principles to 'modal reciprocity' in the early 1960's [9,10]. It is sometimes of interest to ascertain which modes of a complex structure are the most efficient radiators of sound *power*. This may be done by insonifying the structure with a reverberant sound field, and measuring the relative modal responses. If the modal directivity is also of interest, the insonifying field must take the form of a plane wave, incident from the appropriate direction in an anechoic chamber.

7. Conclusions

Based upon my personal experience, I have described a few examples of the application of reciprocity principles to experimental problems in acoustics and vibro-acoustics. I am certain that there are many more such problems to which these principles may be applied with advantage. Next time you design an experiment to invetigate transfer functions, you might like to consider with it might not be performed more conveniently and even more accurately, by employing a reciprocal measurement technique.

Acknowledgement
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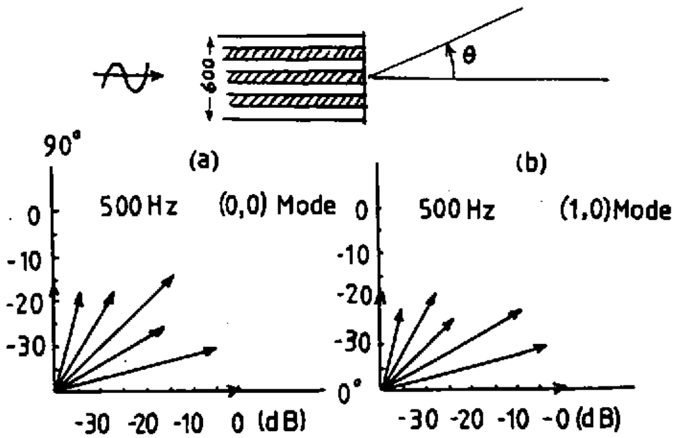

Fig.1 Modal directivity of a silenced duct

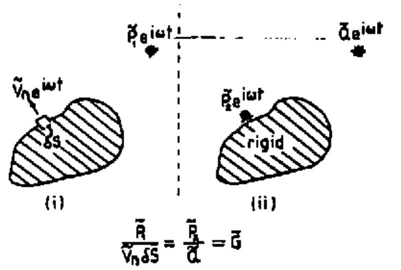

Fig.2 Reciprocal measurement of Green function

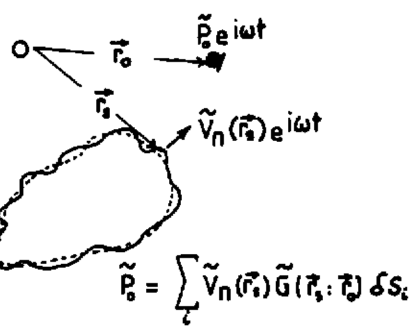

Fig.3 Application of Green functions to estimation of radiation

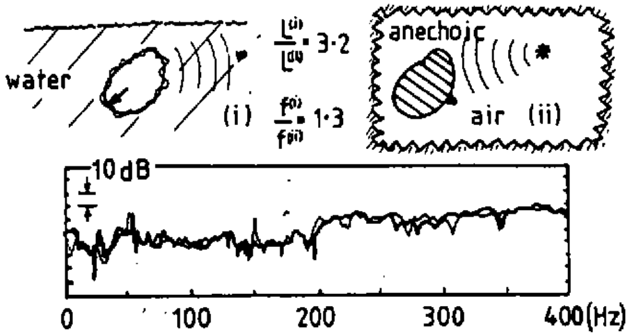

Fig 4 Sound pressure in water from measurement
—— and from vibration and G functions ——

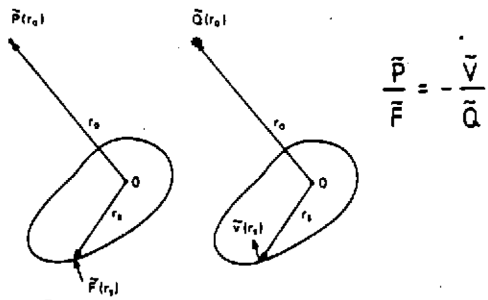

Fig. 5 Reciprocal cases of radiation and response.

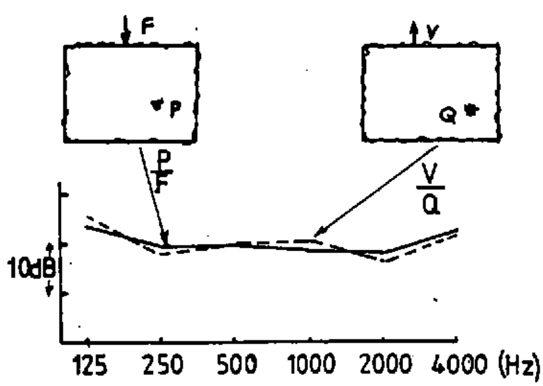

Fig 6 Application of 'Lyamshev' reciprocity

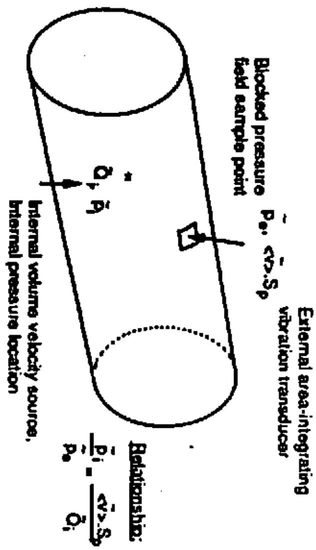

Figure 7. Modification of Lyamshev's relationship

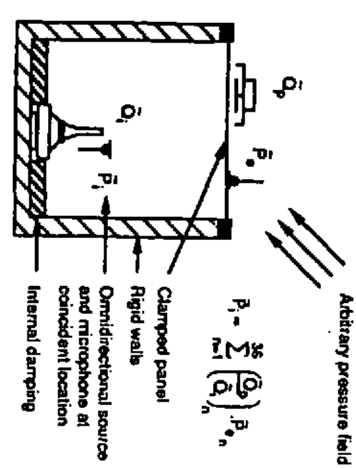

Figure 8. The preliminary test rig.

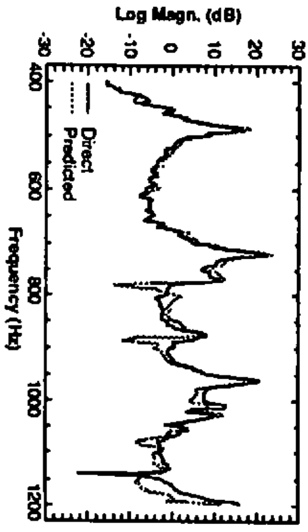

Figure 9. Comparison of direct and predicted pressure magnitude inside the reverberant box, with a damped panel.

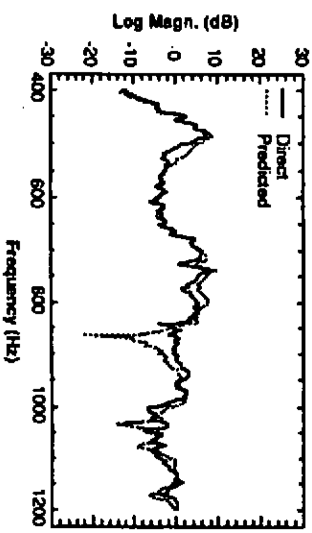

Figure 10. Comparison of direct and predicted pressure magnitude inside the acoustically damped box, with a damped panel.