Proceedings of the Institute of Acoustics

ANALYSIS OF SPHERICAL WAVE PROPAGATION OVER ABSORBING GROUND

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1. INTRODUCTION

The subject of this contribution is a problem which in this year celebrates its 80th birthday and which still is alive, if not vivacious – an impression which one gets if one counts the number of papers written in the last years obout this topic. It came into the world by SOMMERFELD's paper, [1], in 1909. Sommerfeld was interested in an analytical explanation of the observation of unexpected long ranges of radio waves propagating over sea. So he treated the propagation of spherical electromagnetic waves over an absorbing iinfinite plane. His explanation of the long range propagation was the existence of surface waves which are guided along the surface.

The intention ol the present contribution is to make some remarks to the pecularities of the literature in this field and to give some new solutions which may help to finish some controversial discussions. A more detailed discussion will be found in a lorthcoming book, [2].

A citation from SOMMERFELD's paper may serve as guideline:

"There exist two opposite opinions which – at least in a general sense – can be characterized by the conflict between "volume waves" and "surface waves"....This interesting wave type up to now was mainly hypothetical. There was no proof that it is developed from the waves which are radiated from the source. It is a main task of the present investigation to supply this proof and to decide the question: volume wave or surface wave."

And then he makes a restriction:

"It must be stated from the very beginning, that the answer will not be the same under all conditions and in all cases, because quite generally our simplified notations and conceptions mostly can describe only some borderline cases adequately, and cannot represent the general complexity of the phenomena."

This may be summarised by: "The existence of surface waves can be shown in principle, their importance in a concrete situation is still open". This, however, is different from the statement in one of the papers of HABAULT/FILIPPI, that the surface wave is nothing else but a mathematical artefact which only appears if one applies a special method of approximation to the exact solution and which (in their own solution) does not appear in the numerical results neither.

A first pecularity of the literature is, that generally RUDNICK, [3], is said to have been the first, in 1947, to transfer the electromagnetic solutions to sound fields. This is not true. The first was SCHUSTER, [4], who, in 1939, not only has treated the porous half-space, but also an absorber layer of finite thickness in front of a rigid wall, for which HABAULT/FILIPPI, [5], claim priority. It must be said, however, that SCHUSTER just has applied the electro-acoustic field analogies to SOMMERFELD's solutions, so he has not been aware that, in general, the electromagnetic solutions cannot be adopted to sound absorbers immediately (see below).

Another pecularity of the literature lies in the fact, that approximate results are mostly compared against each other. The numerical integration of the exact solution is said to be too complicated to be performed. It is hard, on this basis, to judge on the relative merits of different approximations. So it is no wonder that HABAULT/FILIPPI, [6], on this basis declare the results of THOMASSON, [7], to be wrong because in THOMASSON's approximation can appear sound pressure levels above an absorbing plane which are much higher than above a rigid plane, a result which – at the first glance – is curious, indeed, and which was not found by HABAULT/FILIPPI in their approximation.

A further pecularity of the literature must be mentioned in this context. Sometimes higher-order approximations are described. If, however, one applies them for numerical computations, one realizes that the results mostly are not much better than from the first-order approximations – or even worse. So the higher-order approximations have to be ruled out as a judge between different first-order approximations.

It is on this background that methods for direct numerical solutions of the exact equations are estimated to be of primary importance. Such solutions will be described shortly in what follows. Next the discussion about the surface waves should be decided. It can be shown that the motor for this long-lived discussion are a number of misunderstandings.

2. EXACT SOLUTIONS

The time factor be $\exp(-i\omega t)$, as usual in the literature. The geometry is depicted in **Fig.1**. A point source is placed in S on the z-axis in a height h above the absorbing plane $z=0$, the receiver is at $P(r,z)$ in a radial distance r from the z-axis and in a height z. The plane $z=0$ separates the upper medium with the characteristic wave number and wave impedance k_1, Z_1 from the lower medium which is either bulk reacting, then with the characteristic wave number and wave impedance k_2, Z_2, or which is locally reacting, then with the normalized (with Z_1) surface impedance Z. The sound pressure in the upper medium be $p_1(r,z)$, or (if we like to indicate the source height h explicitely) $p_1(r,z;h)$. We normalize with an appropriate pressure P_0 so that the free spherical wave of the point source reads $p_s(r,z)/P_0 = \exp(ik_1R_1)/(k_1R_1)$.

By the way, this is one more pecularity of the literature, that the cofactors of the expressions for the sound pressure show a large variety. Our proposal is to unify them by the normalization to a sound pressure P_0 which easily can be determined from the facts which are known about the sound source, either the pressure in 1 meter distance, or its sound power, or its volume flow.

In the case of a *bulk reacting half-space* $z<0$, using $k=k_2/k_1$ and $Z=Z_2/Z_1$, and in the special situation of a source height $h=0$ (i.e. $R_1=R_2=R$) (SOMMERFELD's situation), the exact solution is:

$$\frac{p_1(r,z;0)}{P_0} = 2(1+kZ)\int_0^\infty \frac{y\,J_0(yk_1r)\,e^{-k_1z\sqrt{y^2-1}}}{\sqrt{y^2-k^2}+kZ\sqrt{y^2-1}}\,dy =: 2(1+kZ)\cdot I \qquad (1)$$

The generalization to non-zero source heights, $h\neq0$, according to BREKHOVSKIKH, [8], then is:

$$\frac{p_1(r,z;h)}{P_0} = \frac{e^{ik_1R_1}}{k_1R_1} - \frac{e^{ik_1R_2}}{k_1R_2} + \frac{p_1(r,z+h;0)}{P_0} \tag{2}$$

This equation says that the sound field for an arbitrary source height is the direct wave minus a spherical wave from the mirror source at S' (see Fig.1) plus the sound field of a source with zero height, but now taken at a receiver height z+h . So, in principle, one would only need to know the sound field for h=0 .

Eq.(1) is based on a decomposition of spherical waves into zylindrical ones, [1]. By a decomposition into plane waves which then are reflected by the plane-wave reflection factor r(Θ) , one gets:

$$\frac{p_1}{P_0} = \frac{e^{ik_1R_1}}{k_1R_1} + \frac{p_r}{P_0}$$

$$\frac{p_r}{P_0} = i\int_0^{\pi/2} J_0(k_1 r\sin\Theta')e^{ik_1H\cos\Theta'}\cdot r(\Theta')\cdot\sin\Theta'\,d\Theta' + \tag{3}$$

$$+\int_0^{\infty} J_0(k_1 r\cosh\Theta'')e^{-k_1H\sinh\Theta''}\cdot r(\tfrac{\pi}{2}-i\Theta'')\cdot\cosh\Theta''\,d\Theta'' =: i\,I_1 + I_2$$

with H=h+z and with the reflection factors (4)

$$r(\Theta') = \frac{kZ\cos\Theta' - \sqrt{k^2-1+\cos^2\Theta'}}{kZ\cos\Theta' + \sqrt{k^2-1+\cos^2\Theta'}};\; r(\tfrac{\pi}{2}-i\Theta'') = \frac{kZ\tanh\Theta'' - \sqrt{1-(k/\cosh\Theta'')^2}}{kZ\tanh\Theta'' + \sqrt{1-(k/\cosh\Theta'')^2}}$$

Eq.(3) is suited for *locally reacting absorbers*, also, because the absorber is represented by the reflection factor only, which now is:

$$r(\Theta') = \frac{Z\cos\Theta' - 1}{Z\cos\Theta' + 1};\; r(\tfrac{\pi}{2}-i\Theta'') = \frac{Z\sinh\Theta'' + i}{Z\sinh\Theta'' - i} \tag{5}$$

3. EXACT INTEGRATION

The immediate numerical integration of the exact analytical solutions is difficult, indeed. The integrands show irregular oscillations due to the Bessel function $J_0(u)$ and due to the exponential function. The other factors can attain large magnitudes. So the steps of the integration variable in numerical integration schemes must be small and the integrand must be computed for a very large number of values of the integration variable (some 10 thousands). Then the result becomes questionable because of numerical rounding errors.

A relatively unsophisticated method starts from a replacement of the "end" of the integration to infinity by a decomposition of the integral in eq.(1) into

$$I = \int_0^{\infty}\ldots = \int_0^1\ldots + \int_1^2\ldots + \int_2^{y_{up}}\ldots + \int_{y_{up}}^{\infty}\ldots = I_a + I_b + I_c + I_d \tag{6}$$

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where the first two terms, I_a and I_b, take into account the change of the exponent from imaginary to real and the neighbourhood of poles of the reflection factor to the path of integration. So the integration steps must be small in these terms. They usually can be wider in the third term, I_c. The fourth term, I_d, is an approximation to the "end".

If y_{up} is chosen so that $y_{up}^2 >> 1$ and $y_{up}^2 >> |k^2|$, the last integral can be approximated by

$$I_d \approx \frac{1}{1+kZ} \int_{y_{up}}^{\infty} J_0(y\, k_1 r) e^{-y k_1 z}\, dy = \frac{1}{1+kZ} \cdot I_e \tag{7}$$

wherein the new integral I_e can be expressed by $I_e = f_{0,0}(\infty) - f_{0,0}(y_{up}) = -f_{0,0}(y_{up})$ where for the integrals of the type

$$f_{m,n}(x) = \int^{x} e^{-qy}\, y^m J_n(ty)\, dy \tag{8}$$

LUKE, [9], has given recurrence relations. They lead to:

$$\begin{aligned}
I_e &= -e^{k_1 z\, y_{up}} \cdot F_0(y_{up}) \\
F_m(x) &= A_m(x) + B_m F_{m-1}(x) - C_m F_{m-2}(x)\ ;\ m = 0, -1, -2, \ldots \\
A_m(x) &= \frac{x^m}{(k_1 R)^2}\left[k_1 r\, J_1(x\, k_1 r) + \left(\frac{m-1}{x} - k_1 z\right) J_0(x\, k_1 r)\right] \\
B_m &= (2m-1)\frac{k_1 z}{(k_1 R)^2}\ ;\ C_m = \frac{(m-1)^2}{(k_1 R)^2}
\end{aligned} \tag{9}$$

The convergence in the recurrence for F_0 is produced by the factors $A_m(y_{up})$ which are proportional to y_{up}^m with $m<0$. The A_m, B_m, C_m are proportional to $1/(k_1R)^2$, so the remainder integral will be proportional to $1/(k_1R)^6$ after the first recursion, already. The Bessel functions J_0, J_1 can easily be generated by a polynomial appproximation. With a suitable integration scheme (best suited is a Romberg scheme with an automatic halfing of the step width of integration until the result is stable within a preset relative error), the computation can be programmed on a personal computer. However, because at least about 10 steps must be placed into a "period" of the Bessel functions, several thousands of steps must be used for large $k_1 r$ in a distance plot, and the computation of one distance curve may take several hours.

The numerical examples shown below all assume a mineral fibre absorber (this is often applied for modelling absorbing ground !). It is known, see e.g. [10], that the characteristic wave number and the characteristic wave impedance of mineral fibre absorbers can be easily computed as functions of a single non-dimensional variable $E = Z_1/(\Xi\lambda_1)$, where Ξ is the flow resistivity of the absorber material (the medium 1 is air with the wave length λ_1). The diagrams show distance curves over r/λ_1 of the relative sound pressure level

$$L = 10\,\lg\left|\frac{p_1(r,z)}{p_h(r,z)}\right|^2 = 10\,\lg\left|\frac{p_1(r,z)/P_0}{p_h(r,z)/P_0}\right|^2 \quad \text{dB} \tag{10}$$

of the sound pressure ratio $p_1(r,z)/p_h(r,z)$ where $p_h(r,z)$ is the sound pressure

$$\frac{p_h(r,z)}{P_0} = \frac{e^{ik_1R_1}}{k_1R_1} + \frac{e^{ik_2R_2}}{k_2R_2} \tag{11}$$

above a rigid plane. So the diagrams do not contain the "geometrical attenuation" due to the spherical spread of the wave, and the reflection interference from a rigid surface for finite source heights is eliminated to some degree, also.

Fig.2 is an example of a distance curve computed by the method just described for a fixed source height h/λ_1 (in the diagrams is $\lambda_0=\lambda_1$) and for several receiver heights z/λ_1 over a bulk reacting absorber plane. The integration limit was placed at $y_{up}=8$ and the preset relative error for the integration scheme was 5% . A few thousands of integration steps were necessary; the generation of one curve took about 3 hours, even with the wide span of the preset relative error. The curves show the typical decrease with the square of the distance at their far ends. Because the surface has a spring-type reactance, the sound pressure must first decrease when the receiver height increases from zero value to small finite values (from general considerations of the boundary condition).The curves pass this test with good result, also. The maxima at low and medium distances where the relative sound pressure levels are positive, that is where the sound pressure in front of the absorber is higher than in front of the rigid plane, are at such places, where for the rigid plane deep interference minima between the direct and the reflected waves would be found. The depth of these standing wave minima is reduced by the absorber. This explains the maxima.

4. ACCELERATION OF CONVERGENCE.

The above integration method is not yet satisfactory. The problems arise from the slow decrease of the oscillations of the integrand. Another numerical integration method therefore starts from the integrals I_1 , I_2 in eq.(3), which, however, are not suited for numerical integration as they stand due to the strong variation of the "periods" of the Bessel functions. They first are modified by a transformation $y=\cos\Theta'$ in I_1 and $y=\sinh\Theta''$ in I_2 which leads (for a *bulk reacting half-space*) to:

$$\begin{aligned} I_1 &= \int_0^1 J_0(k_1 r\sqrt{1-y^2})\frac{kZ\,y - \sqrt{k^2-1+y^2}}{kZ\,y + \sqrt{k^2-1+y^2}}\, e^{+ik_1H\,y}\,dy \\ I_2 &= \int_0^\infty J_0(k_1 r\sqrt{1+y^2})\frac{kZ\,y - \sqrt{1+y^2-k^2}}{kZ\,y + \sqrt{1+y^2-k^2}}\, e^{-ik_1H\,y}\,dy \end{aligned} \tag{12}$$

If k_1H is small (or even zero), then the integrand in I_2 becomes small for $y\to\infty$ only by the decrese of the Bessel function $J_0(x)$, i.e. as $1/\sqrt{x}$, which is very slow. The same holds for I in eq.(1) (where we would have to replace z by H according to eq.(2)). We therefore first apply an acceleration of convergence.

For this we write I in eq.(1) as $I(a(y),b(y),c(y))$ and I_2 in eq.(12) as $I_2(A(y),B(y),C(y))$ with the factors and their asymptotic approximations for large y .

$$a(y) = J_0(y\,k_1 r) \rightarrow a_\infty(y) = \sqrt{\frac{2}{\pi k_1 r y}}\cos(y\,k_1 r - \pi/4)$$
$$b(y) = \frac{y}{\sqrt{y^2 - k^2} + kZ\sqrt{y^2-1}} \rightarrow b_\infty = \frac{1}{1+kZ} \qquad (13)$$
$$c(y) = e^{-k_1 H\sqrt{y^2-1}} \rightarrow c_\infty = e^{-k_1 H y}$$

and:

$$A(y) = J_0(k_1 r\sqrt{1+y^2}) \rightarrow A_\infty = J_0(y\,k_1 r)$$
$$B(y) = \frac{kZy - \sqrt{1+y^2-k^2}}{kZy + \sqrt{1+y^2-k^2}} \rightarrow B_\infty = \frac{kZ-1}{kZ+1} \qquad (14)$$
$$C(y) = C_\infty(y) = e^{-k_1 H y}$$

Then one evidently has:

$$I(a,b,c) = I(a,b_\infty,c_\infty) + \int_0^\infty a\cdot(bc - b_\infty c_\infty)\,dy \qquad (15)$$

and with

$$I(a,b_\infty,c_\infty) = \frac{1}{1+kZ}\frac{1}{k_1 R_2} \qquad (16)$$

one gets for I in eq.(1):

$$I(a,b,c) = \frac{1}{1+kZ}\frac{1}{k_1 R_2} + \int_0^\infty J_0(yk_1 r)\left[\frac{y\,e^{-k_1 H\sqrt{y^2-1}}}{\sqrt{y^2-k^2} + kZ\sqrt{y^2-1}} - \frac{e^{-k_1 H y}}{1+kZ}\right]dy \qquad (17)$$

Here the integrand decreases as $(1/y^2)\cdot\exp(-k_1 Hy)$, which means that the upper integration limit must not be extended so far.

Similarly we write for I_2 in eq.(12):

$$I_2(A,B,C) = I_2(A_\infty,B_\infty,C) + \int_0^\infty (AB - A_\infty B_\infty)C\,dy \qquad (18)$$

and get: (19)

$$I_2 = \frac{kZ-1}{kZ+1}\frac{1}{k_1 R_2} + \int_0^\infty \left[J_0(k_1 r\sqrt{1+y^2})\frac{kZy - \sqrt{1+y^2-k^2}}{kZy + \sqrt{1+y^2-k^2}} - J_0(k_1 r y)\frac{kZ-1}{kZ+1}\right]e^{-k_1 H y}\,dy$$

Although this looks more complicated than eq.(15), it has the advantage, that the Bessel function is included in the acceleration of convergence with only slow variations of the integrand for large y which allows for a reduction of the step number in the integration scheme. The computing time is reduced by this acceleration of convergence by a factor of about 1/4 to 1/5 and the precision altogether is improved.

Fig.3 is a repetition of Fig.2, but now with the integral in the shape of eq.(17). The relative error of the integration was preset to 2% .**Fig.4** shows distance curves for zero source height above a *bulk reacting absorber plane* (which cannot be computed by the method of eqs.(6) to (9)).

In the case of a *locally reacting absorber plane* z=0 , we start from eq.(2), now with

$$\frac{p_1(r,z+h;0)}{P_0}=2Z\int_0^\infty J_0(yk_1r)\frac{y}{Z\sqrt{y^2-1}-i}\,e^{-k_1H\sqrt{y^2-1}}\,dy \tag{20}$$

which, by the way, follows from eq.(1) in the limit $k\to\infty$ and Z representing the normalized surface impedance. With the integral written as in eq.(15) with the factors and their asymptotics:

$$\begin{aligned} &a=a_\infty=J_0(yk_1r)\;;\; b=\frac{y}{Z\sqrt{y^2-1}-i}\to b_\infty=\frac{1}{Z}\\ &c=e^{-k_1H\sqrt{y^2-1}}\to c_\infty=e^{-k_1Hy}\end{aligned} \tag{21}$$

one gets:

$$\begin{aligned}\frac{p_1(r,H;0)}{P_0}=&\frac{2}{k_1R_2}+2\int_0^1 J_0(yk_1r)\Big[i\cdot\frac{y\,e^{ik_1H\sqrt{1-y^2}}}{\sqrt{1-y^2}+1/Z}-e^{-k_1Hy}\Big]dy\,+\\ &+2\int_1^\infty J_0(yk_1r)\Big[\frac{y\,e^{-k_1H\sqrt{y^2-1}}}{\sqrt{y^2-1}-i/Z}-e^{-k_1Hy}\Big]dy\end{aligned} \tag{22}$$

This can be integrated numerically quite easily by a Romberg scheme with an increase of the upper integration limit of the second integral by steps until the variation of the result due to such an increase remains under a certain limit of the relative change.

The numerical example in **Fig.5** is for a locally reacting half-space of mineral fibres with the same parameters as in Fig.4, i.e. for a zero source height. Both diagrams are nearly coincident. Also for a source height $h/\lambda_1=1$, as in **Fig.6** , the difference to the corresponding Fig.3 for a bulk recting absorber is negligible.

This is a good place to come back to the remark that higher-order approximations in the literature often are not better than the first order approximations. Such higher-order approximations often start from identical transformations in which – as in eqs.(15) or (18) – the same term is added and subtracted and then the whole expression is rearranged. In eqs. (15), (18) we have separated the integral of the first term on the right-hand side. It is important to find an analytical solution for that integral. Problems of precision may arise if for this integral, too, approximations are used. It is important that the terms which were added and subtracted compensate each other at least down to the 4th or 6th decimal. If both parts of the identical transformation are treated differently and if the precisions of the different approximations are not good enough, the compensation will be disturbed and the final result will be spoiled. The paper by ATTENBOROUGH/HAYEK/LAWTHER, [11], is a good example for this.

5. PASS INTEGRATION

Most of the discussions about the surface waves come from the search for approximate solutions for the exact analytical integrals, and mostly the method of pass integration (integration along steepest descent) is the approximating method which is applied.

A starting point can be eq.(3) in which the integral for the reflected field p_r can be written:

$$\frac{p_r}{P_0} = i \int_0^{\pi/2 - i\infty} J_0(k_1 r \sin\vartheta)\, e^{ik_1 H \cos\vartheta}\, r(\vartheta) \sin\vartheta \, d\vartheta \tag{23}$$

whereof the path of integration is shown in **Fig.7** . This solution is applicable for the bulk reacting as well as for the locally reacting absorber plane depending on the reflection factor $r(\vartheta)$ which is selected. In order to make this integral suited for pass integration, the Bessel function $J_0(u)$ is replaced by the Hankel function $H_0^{(1)}(u)$ with the help of the relation:

$$J_0(u) = \frac{1}{2}\left[H_0^{(1)}(u) - H_0^{(1)}(u\, e^{i\pi})\right] \tag{24}$$

with the result:

$$\frac{p_r}{P_0} = \frac{i}{2} \int_{C_2} H_0^{(1)}(k_1 r \sin\vartheta)\, e^{ik_1 H \cos\vartheta}\, r(\vartheta) \sin\vartheta \, d\vartheta \tag{25}$$

The path of integration C_2 is shown in **Fig.7** .

For reasons which will become clear soon, we multiply and divide the integrand by $\exp(ik_1 r\cdot\sin\vartheta)$ and obtain as the integral to be solved:

$$\frac{p_r}{P_0} = \frac{i}{2} \int_{C_2} e^{ik_1 R_2 \cos(\vartheta - \Theta_0)}\, H_0^{(1)}(k_1 r \sin\vartheta) e^{-ik_1 r \sin\vartheta} \cdot r(\vartheta) \cdot \sin\vartheta \, d\vartheta = \frac{i}{2} I_0 \tag{26}$$

The product of the 2nd and 3rd factor under the integral then can be computed from

$$\begin{aligned} H_0^{(1)}(u) e^{-iu} &= \frac{1-i}{\sqrt{\pi}} \frac{1}{\sqrt{u}} [P(u) + i \cdot Q(u)] \\ P(u) &= 1 - \frac{1\cdot 9}{2!(8u)^2} + \frac{1\cdot 9\cdot 25\cdot 49}{4!(8u)^4} - + \ldots \\ Q(u) &= -\frac{1}{8u} + \frac{1\cdot 9\cdot 25}{3!(8u)^3} - \frac{1\cdot 9\cdot 25\cdot 49\cdot 81}{5!(8u)^5} + - \ldots \end{aligned} \tag{27}$$

It is a rather monotonous function for large arguments.

The integral I_0 in eq.(26) is of the type

$$I = \int_C e^{x \cdot f(\vartheta)} F(\vartheta)\, d\vartheta \qquad (28)$$

which is appropriate for pass integration if x (in our case $k_1 R_2$) is a large real number, and if $F(\vartheta)$ is a sufficiently steady and monotonous function (that is the reason for the above identical transformation). The exponential factor has a saddle point (pass) in the complex ϑ-plane which can be crossed by a pass way so that the magnitude of the exponential factor is large only near the saddle point, and that it decreases rapidly on both sides of it along the pass way, and, further, that it shows no oscillations along the pass way. All these are qualities which we want for an easy numerical integration.

These qualities make the pass way not only attractive for numerical integrations along it, but also for analytical approximations to the exact integral, because one can assume that only the neighbourhood of the saddle point will give significant contributions to the value of the intagral. For such analytical approximations, one therefore develops the exponent and the function $F(\vartheta)$ by a power series around the saddle point and takes only the leading terms for which the integral can be solved analytically. In fact, many of the contributions in the literature have such approximate developments as their subject.

In order to exploit the favourable qualities of the pass way, which will be named P here, it is necessary to transform the path of integration C in eq.(28), or C_2 in our integral of eq.(26), into the pass way. According to Cauchy's theoreme this is possible without change of the integral if the path of integration under way in this transformation does not cross singular points such as poles or branch cuts of the integrand.

The discussion of whether or not such singularities are crossed, exactly corresponds to the discussion whether or not surface waves will appear. Many of the controversal statements in the literature (BREKHOVSKIKH, HABAULT/FILIPPI, MARCUVITZ/FELSEN et al.:"Surface waves are no problem because they do not exist"; SOMMERFELD, THOMASSON et al.:"The existence of surface waves is proved not only by analysis but also by experiments") have their explanation in unwarranted generalisations from one type of absorbers to another type. It is necessary, therefore, to discuss this question carefully.

We later shall come back to this question. Before, we will make use of the favourable qualities of the pass way for an exact numerical integration.

6. EXACT PASS INTEGRATION

The integration path C_2 is transformed in the complex plane $\vartheta = \vartheta' + j\vartheta''$ to the pass way P which goes through the saddle point ϑ_s. This is at the place of the maximum magnitude of the exponential factor, which is determined from $df(\vartheta)/d\vartheta = 0$, which in our case is $\vartheta_s = \Theta_0$. The pass way is determined from the requirement of constant phase, i.e. from $\mathrm{Im}\{f(\vartheta_P)\} = \mathrm{const} = \mathrm{Im}\{f(\vartheta_s)\}$ where the index P means "on the pass way". Its equation in our case is:

$$\cos(\vartheta'_p - \Theta_0)\cosh\vartheta''_p = 1 \tag{29}$$

This, with the addition theoreme for trigonometric functions, can be solved for:

$$\sin\vartheta'_p = \frac{\sin\Theta_0 - \cos\Theta_0 \sinh\vartheta''_p}{\cosh\vartheta''_p} \;;\; \cos\vartheta'_p = \frac{\cos\Theta_0 + \sin\Theta_0 \sinh\vartheta''_p}{\cosh\vartheta''_p} \tag{30}$$

So the function $f(\vartheta)$ in eq.(28) becomes on the pass way:

$$f(\vartheta_p) = -\tanh\vartheta''_p \sinh\vartheta''_p + i \xrightarrow[\vartheta''_p \to 0]{} -(\vartheta''_p)^2 + i \tag{31}$$

The saddle point is at $\vartheta_p''=0$, that is on the real axis at $\vartheta_p'=\Theta_0$, it is shifted towards the right-hand side with increasing Θ_0, its outmost position is at $\vartheta_p'=\pi/2$ for grazing sound incidence. The shape of the pass way is not modified by a variation of Θ_0; it is shown schematically in **Fig.8**.

All functions of ϑ in eq.(26) – more precisely of ϑ_p on the pass way – now can be expressed as functions of ϑ_p''. The reflection factor for a *locally reacting absorber plane* (the first in eq.(5)) becomes: (32)

$$r(\vartheta_p) = \frac{Z\cdot[\cos\Theta_0 + \sin\Theta_0 \sinh\vartheta''_p - i\tanh\vartheta''_p\cdot(\sin\Theta_0 - \cos\Theta_0\sinh\vartheta''_p)] - 1}{Z\cdot[\ldots \qquad \ldots] + 1} =: r(\vartheta''_p)$$

and:

$$\begin{aligned}\sin\vartheta_p &= \sin\Theta_0 - \cos\Theta_0\sinh\vartheta''_p + i\tanh\vartheta''_p\cdot(\cos\Theta_0 + \sin\Theta_0\sinh\vartheta''_p) =: \sin(\vartheta''_p)\\ \cos\vartheta_p &= \cos\Theta_0 + \sin\Theta_0\sinh\vartheta''_p - i\tanh\vartheta''_p\cdot(\sin\Theta_0 - \cos\Theta_0\sinh\vartheta''_p) =: \cos(\vartheta''_p)\end{aligned} \tag{33}$$

The transition from ϑ to ϑ_p'' is a substitution of the variable of integration. The integral in eq.(28) now reads:

$$I = \int_{+\infty}^{-\infty} e^{x\, g(\vartheta''_p)}\, G(\vartheta''_p)\, d\vartheta''_p \tag{34}$$

with

$$g(\vartheta''_p) = i - \tanh\vartheta''_p \cdot \sinh\vartheta''_p \tag{35}$$

and

$$G(\vartheta''_p) = F(\vartheta(\vartheta''_p))\frac{d\vartheta}{d\vartheta''_p} = F(\vartheta(\vartheta''_p))\,\frac{d\,g(\vartheta''_p)/d\,\vartheta''_p}{d\,f(\vartheta)/d\,\vartheta} \tag{36}$$

The last factor is:

$$\frac{d\,g(\vartheta''_p)/d\,\vartheta''_p}{d\,f(\vartheta)/d\,\vartheta} = -\frac{\sinh\vartheta''_p\cdot(2-\tanh^2\vartheta''_p)}{\tanh\vartheta''_p + i\sinh\vartheta''_p} = -\frac{2-\tanh^2\vartheta''_p}{i + 1/\cosh\vartheta''_p} \tag{37}$$

The wanted integral I_0 of eq.(26) finally becomes (if we replace $\varphi=\vartheta_p''$ for ease of writing) in the case of a *locally reacting absorber plane* :

$$I_0 = e^{ik_1R_2}\int_0^{\infty} e^{-k_1R_2\tanh\varphi\sinh\varphi}\,\frac{2-\tanh^2\varphi}{i+1/\cosh\varphi}[r(\varphi)\sin(\varphi)H_0^{(1)}(k_1r\sin(\varphi))e^{-ik_1r\sin(\varphi)} +$$

$$+\, r(-\varphi)\sin(-\varphi)H_0^{(1)}(k_1r\sin(-\varphi))e^{-ik_1r\sin(-\varphi)}]\,d\varphi \qquad (38)$$

The terms in the brackets are rather monotonous, and the exponential decreases rapidly with increasing φ. So the number of steps is reduced to about 1/10 of the number needed for the former integrations. Even with the more complex integrands the computing time goes down to about 1/5. The integral is exact under the conditions of the next sections (singularities). One essentially pays for the reduction of step number by a now complex argument of the Hankel function. It can be generated either by the development of eq.(27) or by other algorithms (see e.g. [12]).

Anticipating the result of the next section, it should be mentioned that, in case the transformation of the path of integration will encompass a pole of the integrand, the following term (the surface wave term) should be added to p_r/P_0 for a *locally reacting absorber plane* :

$$\frac{p_{rp}}{P_0} = -\frac{2\pi}{Z}H_0^{(1)}(k_1r\sqrt{1-1/Z^2})e^{-k_1H/Z} \quad ; \ \mathrm{Re}\sqrt{1-1/Z^2} > 0 \qquad (39)$$

7. SINGULARITIES WITH BULK REACTING ABSORBERS

In the case of *bulk reacting absorbers*, the reflection factor $r(\vartheta)$ under the integral of eq.(26) is the first of eq.(4). The integrand must be checked for singularities.

First, there is the branch cut of the Hankel function. It is outside the range of the complex ϑ-plane between the original integration path C_2 and the pass way P. So it is not important here.

Next, there are the branch cuts of the roots in the reflection factor. The sign of the square root must be selected so that the wave in the medium no.2 does not increase during propagation. It follows from this requirement that the imaginary component of the root must not become negative. This gives an equation between the components ϑ' and ϑ'' on the branch cut:

$$\vartheta' = \frac{1}{2}\arcsin\frac{4k'k''}{\sinh 2\vartheta''} \qquad (40)$$

The two branch cuts are entered schematically as Q_2 into **Fig.8**. The branch points ϑ_b are at

$$\vartheta_b = \pm\arcsin k\ . \qquad (41)$$

So, even if the pass way P meets the upper branch of the cut Q_2, it does not change the sides of the cut (i.e. P crosses Q_2 twice if at all). The branch cuts, therefore, give no separate contribution.

Finally, there are poles ϑ_p of the reflection factor where the denominator $D(\vartheta)$ has zeroes. One immediately sees that, for a solution ϑ_p of the equation $D(\vartheta)=0$, also $-\vartheta_p$ is a solution. And with a solution ϑ_p, also $\vartheta_p \pm \pi$ is a solution. With all signs of the root in $r(\vartheta)$ permitted, one gets from

$$\cos(\pm\,\vartheta_{p\pm}) = \pm\sqrt{\frac{k^2-1}{(kZ)^2-1}} =: \pm\, U \qquad (42)$$

four algebraic solutions. Their positions are indicated in **Fig.8**. Evidently, the two on the left-hand side (symmetrical to $\vartheta=-\pi/2$) are unimportant.

Apparently it was overseen sometimes in the literature that two of the four solutions bring not only the denominator of the reflection factor to zero but also its nominator. Since the orders of the zeroes are the same, these solutions cannot be poles. This becomes important if one does not solve $D(\vartheta)=0$ for $\cos\vartheta_p$ as in eq.(42) but for $\sin\vartheta_p$ as it is done sometimes in the literature:

$$\sin(\pm\,\vartheta_p) = \pm\sqrt{1-U^2} = \pm\sqrt{\frac{(kZ)^2-k^2}{(kZ)^2-1}} =: \pm\, V \qquad (43)$$

with the solutions $\vartheta_p = \pm \arcsin V$. Taking the principal value of arcsin, one only finds the zeroes of the nominator, not of the denominator! This can explain a part of the contradicting statements.

Instead of a general discussion of the position of the poles (see [2]), we plot their locii quantitatively for mineral fibre absorbers with their characteristic values computed according to [10]. The results are shown in **Fig.9**. This diagram shows – in the complex ϑ-plane – some pass ways P for different angles of incidence Θ_0 (see Fig.1) as computed from eq.(30). The diagram further shows some upper branch cuts Q_2 as computed from eq.(40) for some absorber parameters E. The diagram next shows the auxiliary functions kZ and $\pm kZ\cdot U$ from which $U=\cos\vartheta_p$ is constructed. Finally, the diagram shows the locus of the poles for a continuous variation of E where the arrows show into the direction of increasing E.

One sees that even in the extreme situation $\Theta_0=\pi/2$, i.e. source and receiver in the plane $z=0$, the pass way – for a *bulk reacting mineral fibre absorber* – does not encompass the pole. The pass way under this condition only approaches the pole for low values of E, i.e. for low frequencies and/or high flow resistivity. Because under the same conditions the zero of the nominator (the locus of which is the mirror curve of the pole locus relative to the point $\vartheta=\pi/2$) approaches the pass way also, one of the conditions for an approximate pass integration is violated, namely a rather steady function $F(\vartheta)$ in eq.(28). It is no wonder, therefore, that many approximations have problems under these conditions. Applying the exact pass integration, one must not add an extra term for the pole contribution.

Two further remarks concerning the literature have their right place here. The first is, that SOMMERFELD, dealing with bulk reacting absorbers only, has used as a pole the solution with positive imaginary argument (i.e. the zero of the nominator for our type of the absorber). Therefore his results cannot be adopted immediately to a porous sound absorber. That is why one cannot call SOMMERFELD to come forward as a witness for the existence of a pole contribution (i.e. surface waves) with bulk reacting porous sound absorbers.

The next observation is, that sound absorbing grounds in the literature often are modeled as bulk reacting mineral fibre absorbers and the absorber parameter E is determined by fitting measured curves to computed ones. Small values of E are found, corresponding to large values of the equivalent flow resistivity of the ground model. A problem then arises if the original DELANY/BAZLEY parameters, [13], are used. As was shown in [14], these original parameters give rise to problems for small values of E (e.g. negative resistance of an absorber layer). If one applies the DELANY/BAZLEY formulas with their parameters, the locus of the pole in **Fig.9** would indeed cross the pass way for large angles Θ_0 and for low values of E (for details see [2]). So a separate pole contribution should be taken into account.

One conclusion thereof is: whether or not a pole contribution with *bulk reacting absorbers* exists depends also on the absorber model which is used in the computations. The answer may be different for electromagnetic absorbers (and there depending on the polarization of the wave), for underwater absorbers and for different kinds of bulk reacting absorbers for air-borne sound.

If a pole contribution would be needed, it is obtained by encircling the pole with a small lobe, starting and ending at the pass way and, after application of the residuum theoreme, it can easily be computed to be:

$$\frac{p_{rp}}{P_0} = -2\pi i \frac{(kZ)^2}{1-(kZ)^2} U e^{ik_1 HU} H_0^{(1)}(k_1 r V) \tag{44}$$

Because of Im(U)>0 (see **Fig.9**), the sound pressure field decreases exponentially with increasing distance of the receiver (and of the source) from the absorber plane. This contribution therefore is a surface wave.

8. SINGULARITIES WITH LOCALLY REACTING ABSORBERS

It should be possible to give an answer to the question of the importance of singularities in connection with the pass integration for *locally reacting absorbers* which is independent from the absorber model, because the only significant quantity for the description of the absorber, its surface impedance Z , can assume any values with non-negative real components.

Branch cuts are only those of the Hankel function. They are unimportant because already the original path of integration C_2 avoids them. A pole of first order (as long as not both Z=1 and $\Theta_0=\pi/2$) is at the zero of the denominator of the reflection factor $r(\vartheta)$ from eq.(5) which is located at ϑ_p with

$$\cos\vartheta_p = -1/Z \tag{45}$$

We apply **Fig.10** for a discussion of this question. There the pass way P is shown in its rightmost position for $\Theta_0=\pi/2$. First we consider only (for reasons of a simple representation) surface impedances Z within the first quadrant (with a diagonal hatching), and here we restrict the range (for the same reason) to the range outside the unit circle. This range will be transformed by 1/Z into the right-hand lower section of the unit circle (with diagonal hatching) and by the change of sign into the left-hand upper sector of the unit circle. This is the range for values of $\cos\vartheta_p$. Using

$$\cos\vartheta = \cos\vartheta'\cosh\vartheta'' - i\sin\vartheta'\sinh\vartheta'' ,$$

one sees that ϑ_p' must be in the range between $\pi/2$ and π and that ϑ_p'' must be negative. The three corner points of the unit circle sector, i.e. 0 , -1 and i , are encircled in Fig.10 and are transposed to the corresponding range of ϑ_p , which also has a diagonal hatching. One sees, that the partial area with the vertical hatching below the pass way will give a pole contribution. If one traces back this range of pole contributions (marked by a vertical hatching) into the Z-plane, one realizes that it is close to the positive imaginary axis.

It is easy to show that the impedances Z of the first quadrant within the unit circle will fill up the strip in the ϑ_p-plane with $\pi/2 \le \vartheta_p' \le \pi$ and $\vartheta_p''<0$. A part of these impedances can give pole contributions, also, and it is not surprising that these impedances are near to the imaginary Z-axis.

If the impedance Z is from the fourth quadrant, one can show by a quite similar argumentation that the corresponding values of ϑ_p will fill up a strip with $\pi/2 \le \vartheta_p' \le \pi$, but now with positive ϑ_p'' , so that they can never be reached by the pass way P .

The results of these discussions are:

- there are surface impedances for which an extra pole contribution of eq.(39) must be added,
- these impedances all have a negative reactance (in the $\exp(-i\omega t)$ convention), the reactance is of the type of a spring,
- the range of the impedances is close to the reactance axis,
- its size is the larger the more the angle Θ_0 approaches $\pi/2$.

Using the description of eq.(30) for the pass way, it is not difficult to derive a quantitative condition for the application of the pole contribution of eq.(39). It reads:

$$(1/Z)'' < -\frac{1}{\sin\Theta_0}\frac{[\cos\Theta_0 + (1/Z)'][1+\cos\Theta_0\cdot(1/Z)']}{\sqrt{1+2\cos\Theta_0\cdot(1/Z)' + (1/Z)'^2}} \qquad (46)$$

With the sign rule for the rectance of Z for a pole contibution, one immediately sees from eq.(39) that the sound field of this contribution decreases exponentially with increasing distance from the absorber plane. The pole contribution is a surface wave.

Some remarks should be made in this context. First: nobody, to the author's knowledge, up to now has realized that the range of the pole contribution for *locally reacting surface pllanes* coincides exactly with the range of the existence of free two-dimensional surface waves over such planes (see [2]). Second: if surface waves shall exist with a significant strength, they must be excited by the spherical wave, i.e. the free spherical wave must "offer" this wave type to a plane. BREKHOVSKIKH has shown that the decomposition of a spherical wave into plane waves above a plane is not possible with ordinary plane waves only, one has to add waves of the surface wave type parallel to that plane. So the excitation is no problem. Third: most locally reacting absorbers (and especially absorbing grounds) start at low frequencies with a spring type reactance.

So it remains only the second part of SOMMERFELD's question, whether the strength of the surface waves can be relevant at all.

9. NUMERICAL EXAMPLES

Fig. 11 shows distance curves of the relative level of p_1/p_h for surface impedances with constant resistance $Z'=2$ and with different reactances Z'' which change their sign. Source and receiver are in the plane $z=0$. One clearly sees that the sound pressure level above the absorber for spring type reactances can be much higher at medium distances than the sound pressure level in front of a rigid plane. Behind that range of high pressure levels there is a steep slope of the level down to the usual long-distance behaviour which is indicated by the distance curves with negative reactances.

Even more instructive are contour diagrams of the sound pressure levels. **Fig. 12** is an example thereof. Isobars are plotted in a vertical plane containing the source. The sound pressure p_1 above the absorber now is relative to the sound pressure p_s of a free spherical wave.The vertical axis is logarithmic for $z/\lambda_0 \geq 1$ and linear for $z/\lambda_0 \leq 1$. The source is in the absorber plane for this figure; the reactance is mass type. Only the sign of the reactance is changed in the next diagram, **Fig.13**. One clearly sees the range of the predominant surface waves close to the surface, followed by a steep descent. There is no special observation at larger heights of the receiver. The pair of **Fig.14** and **Fig.15** are for larger magnitudes of the impedance components. The range of predominant surface waves has shifted to larger distances. **Fig.16** finally gives an example for a non-zero height of the source. A minimum of interference betweeen the direct and the reflected waves is visible. The situation close to the surface has not changed very much.

10. APPROXIMATIONS IN THE LITERATURE

We now have a number of exact integrations available. Their results are in perfect agreement with each other. So one can check existing approximations. There exists a large number of approximations for the exact integrals in the literature. Many of them are discussed in [2]. The quality with respect to precision and to amount of prgramming can be quite different. Complicated approximations do not deliver always better results.

An approximation was derived by BREKHOSKIKH, [8], which is very instructive. It reads:

$$\frac{p_r}{P_0} = \frac{e^{ik_1R_2}}{k_1R_2}[r(\Theta_0) - \frac{i}{2k_1R_2}(r''(\Theta_0) + r'(\Theta_0)\cot\Theta_0)] \qquad (47)$$

where $r'(\Theta_0)$ and $r''(\Theta_0)$ is the first and the second derivative, repectively, of the reflection factor with respect to Θ_0. It shows the transition to the geometrical acoustics for reflection factors with no angular dependence and/or at large distances k_1R_2. It also makes clear that the range of nearly grazing incidence, Θ_0 about $\pi/2$, is critical becaus there the reflection factor is known to be strongly dependent on the angle of incidence. This approximation can be used for both locally and bulk reacting absorbers with the corresponding reflection factors. Possibly a pole contribution must be added for a locally reacting absorber.

The best approximation for *locally reacting absorbers* with respect to precision seems to be that of THOMASSON, [7]. Quite good also is the approximation given by DELANY/BAZLEY, [15] and that of NOBILE, [16].

11. REMARKS TO EXPERIMENTAL STUDIES

The experimental studies in the literature deserve some comments also, because there sometimes happen strange things. The main interest in experimental studies lies in the sound propagation over absorbing ground.

The technique mostly applied is that of "parameter fitting". Experimental distance curves or frequency curves are compared to numerical results which were generated on the basis of an approximation to the exact solution and of an absorber model. Then the "free parameters" of that model are tuned to give a best agreement.

A favourite model for ground is a half-space of bulk-reacting mineral fibres. The equivalent flow resistivity is the final quantity which is wanted as a characteristic for the ground. It is a strange thing, that the bulk reacting model often is compared to an approximation which was derived for a locally reacting absorber – and vice versa !

The inventive genius of some authors for ground models seems to be inexhaustible. A model of an elastic plate was compared with this paremeter fitting technique to an anlysis for a locally reacting plane. The author was not aware that he would have to compare with the theory of spherical waves over an elastic plate which is quite different.

Authors deliberately use numerical results for grazing incidence as soon as they place their loudspeaker and their microphone at low heights. They are not aware that already minor deviations from grazing incidence would change the numerical results drastically. Even if the source and the receiver are placed on the gound, would it not be reasonable to make first a sensitivity analysis of the final result of the parameter fitting technique with respect to finite angles of incidence which effectively can be produced by temeperature gradients and by wind gradients ?

And a final question: why do experimenters who compare their experimental results against analytical results not go the straigjht way ? Just determine those quantities which are required by the theory, namely surface impedance (impedance for plane waves at normal incidence) for locally reacting models and characteristic constants for bulk reacting models !

12. LITERATURE

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ANALYSIS OF SPHERICAL WAVE PROPAGATION OVER ABSORBING GROUND

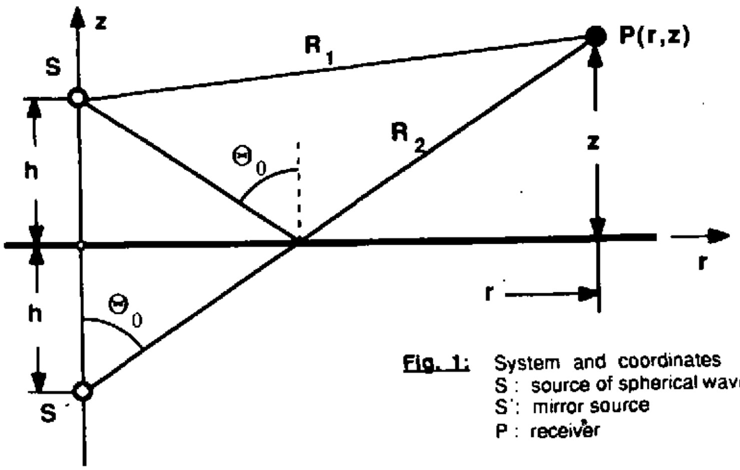

Fig. 1: System and coordinates
S : source of spherical waves
S': mirror source
P : receiver

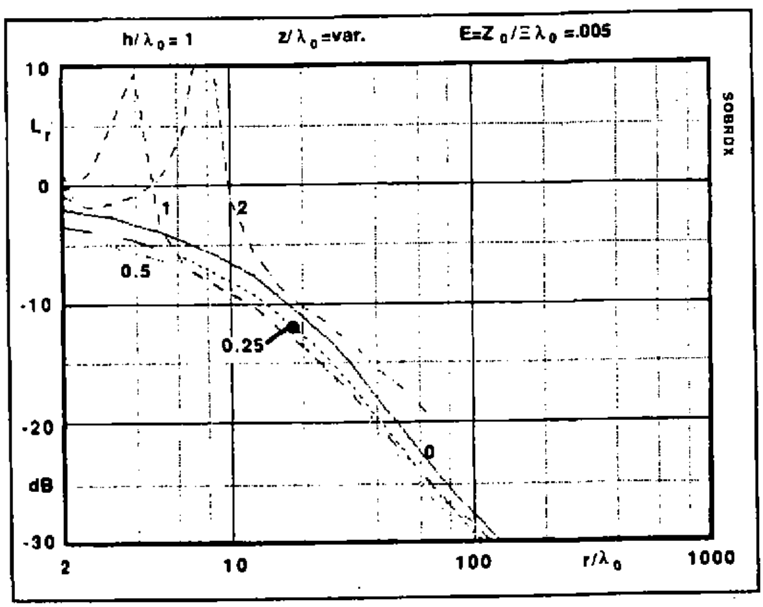

Fig.2: Distance curves of sound pressure level above bulk reacting absorber relative to field above rigid plane, computed by numerical integration of analytical exact integral.

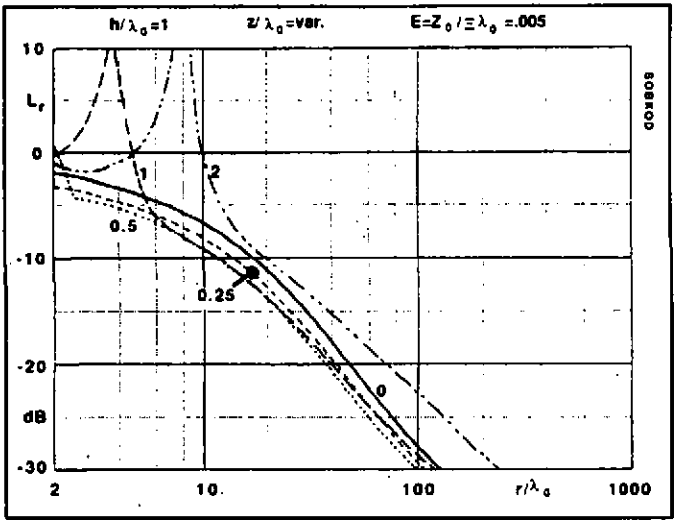

Fig.3: Distance curves of sound pressure level above bulk reacting absorber relative to field above rigid plane, computed by numerical integration of analytical exact integral after acceleration of convergence.

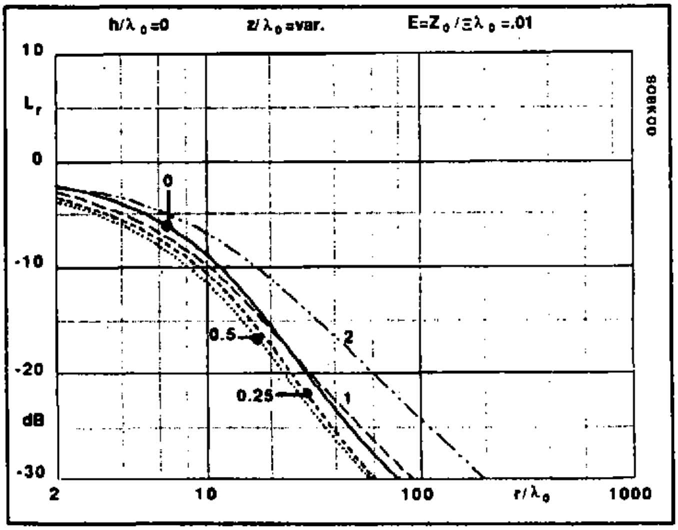

Fig.4: Distance curves of sound pressure level above bulk reacting absorber relative to field above rigid plane, computed by numerical integration of analytical exact integral after convergence acceleration.

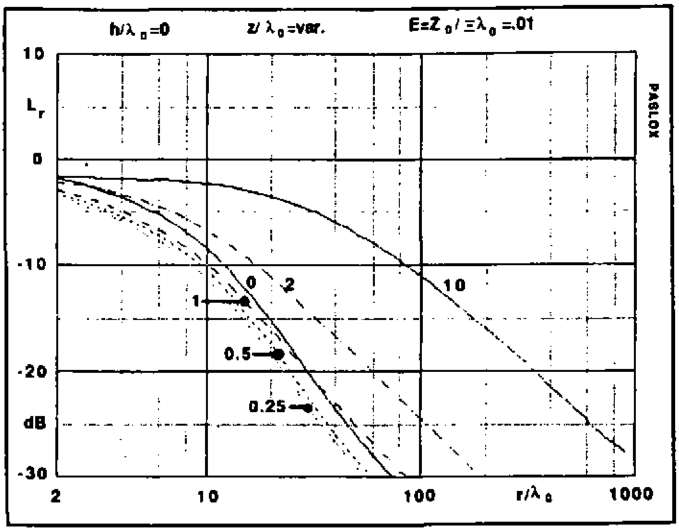

Fig.5: Distance curves of sound pressure level above locally reacting absorber relative to field above rigid plane, computed by numerical pass integration of analytical exact integral.

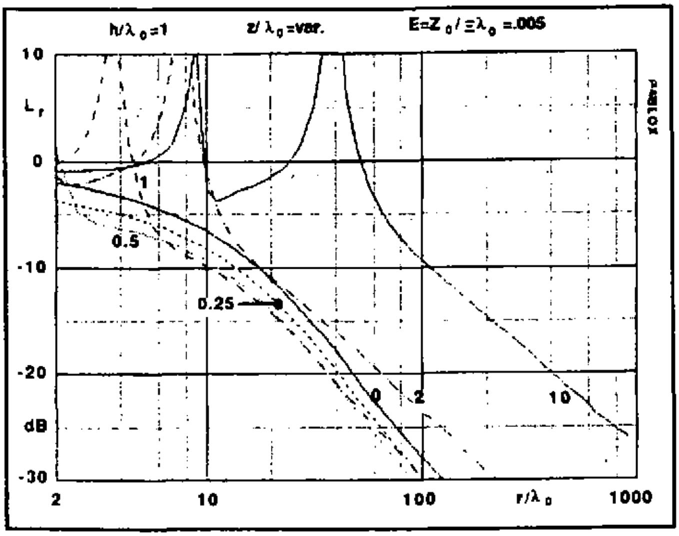

Fig.6: Distance curves of sound pressure level above locally reacting absorber relative to field above rigid plane, computed by numerical pass integration of analytical exact integral.

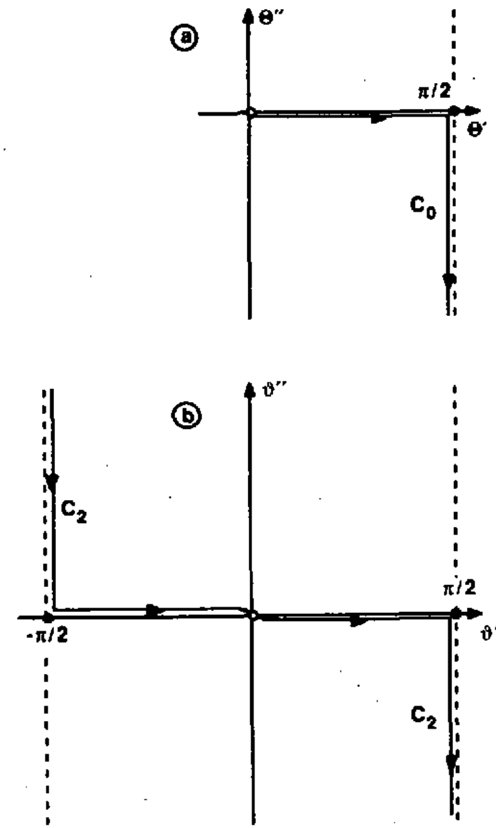

Fig. 7: Integration paths
a: to eq.(23)
b: to eq.(25) .

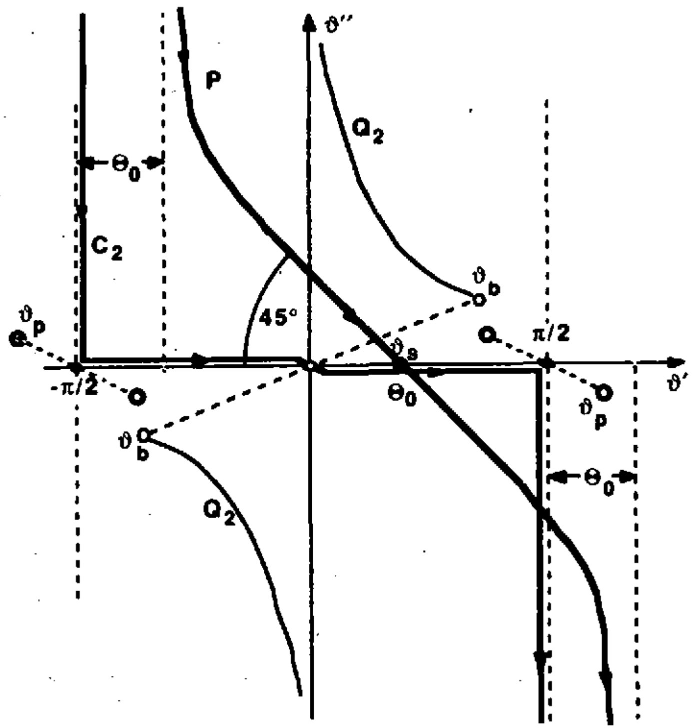

Fig. 8: Integration path for pass integration
C_2 : original path, P : pass way, C_2 : branch cuts,
$ϑ_s$: saddle point, $ϑ_b$: branch points, $ϑ_p$: poles,
$Θ_0$: angle of incidence

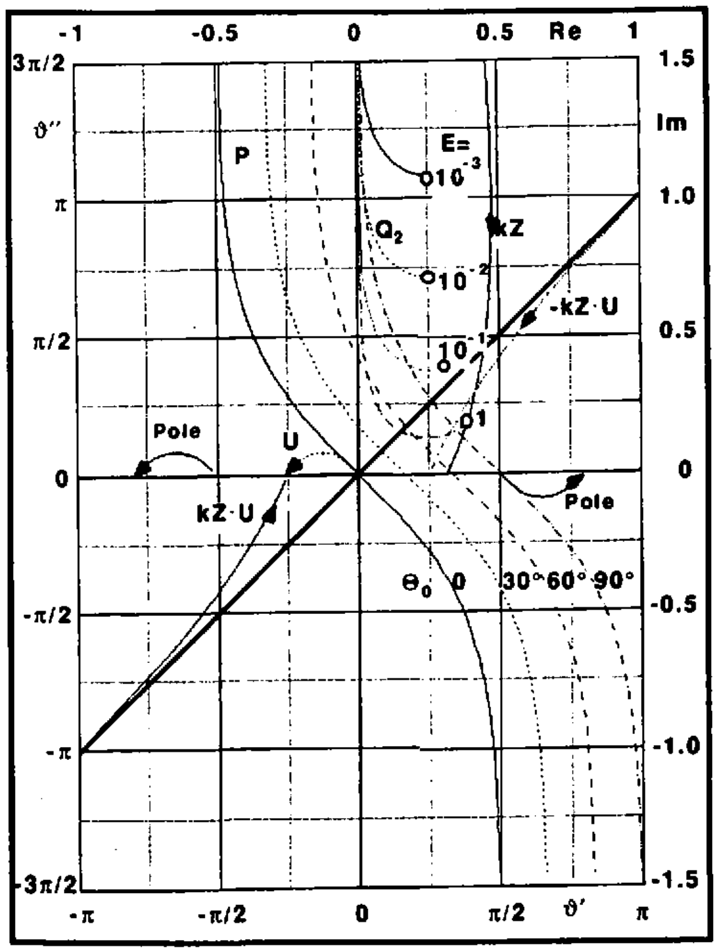

Fig.9 : Integration paths for pass integration and singularities for different absorber parameters E of a bulk reacting mineral fibre absorber at different angles of incidence.

ANALYSIS OF SPHERICAL WAVE PROPAGATION OVER ABSORBING GROUND

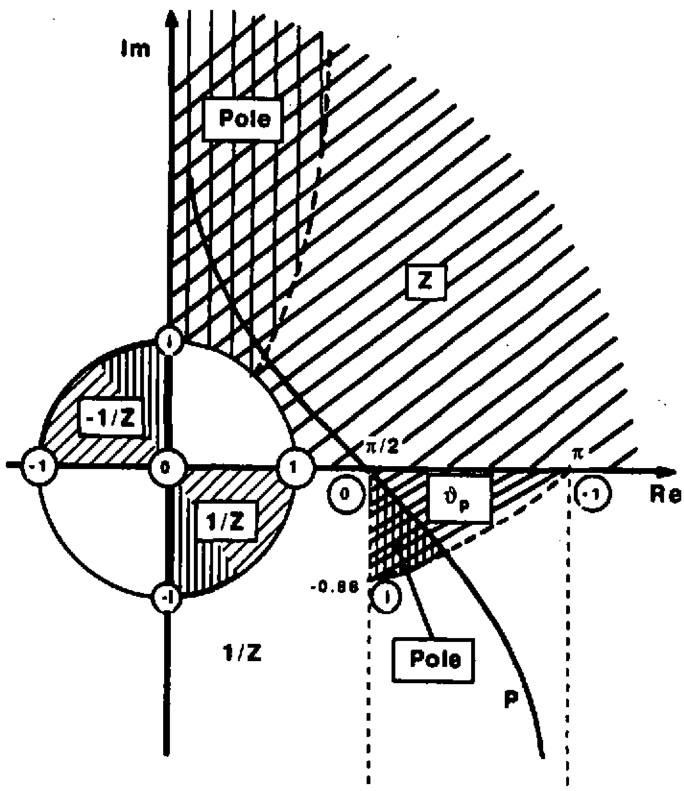

Fig. 10: To the conditions for a pole contribution ("Pole") with a locally reacting absorber plane.

ANALYSIS OF SPHERICAL WAVE PROPAGATION OVER ABSORBING GROUND

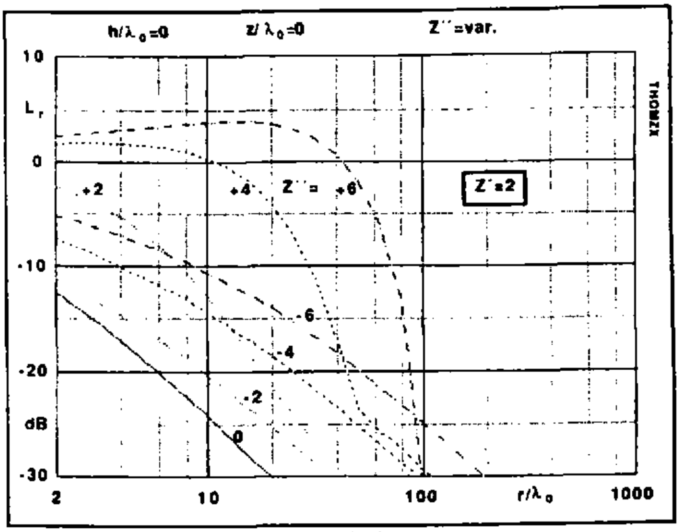

Fig.11: Distance curves of sound pressure level above locally reacting absorber relative to field above rigid plane, computed by numerical pass integration of analytical exact integral, for surface impedances with different sign of reactance.

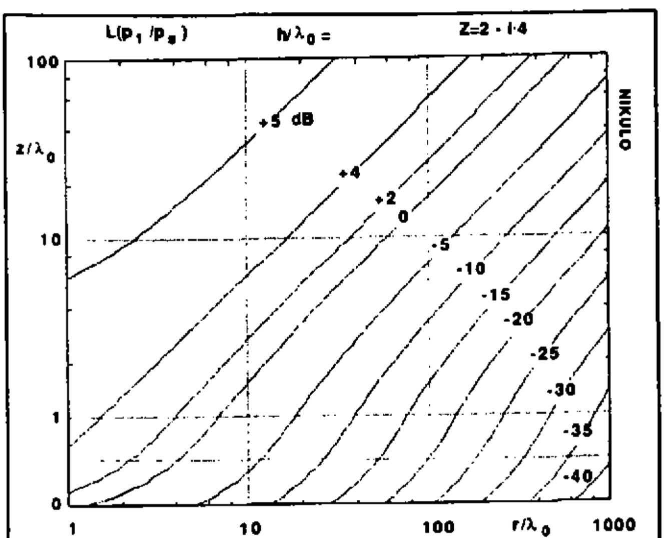

Fig. 12: Contour plot of sound pressure level relative to sound pressure of free spherical wave over a locally reacting absorber plane with given surface impedance Z.

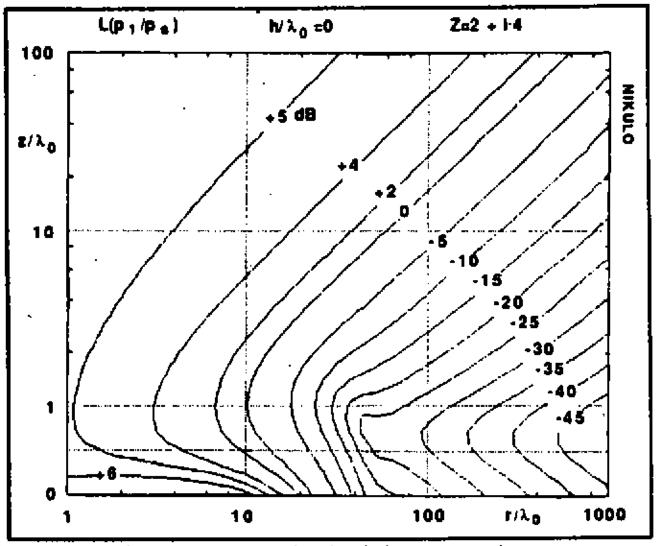

Fig. 13: Contour plot of sound pressure level relative to sound pressure of free spherical wave over a locally reacting absorber plane with given surface impedance Z with positive reactance.

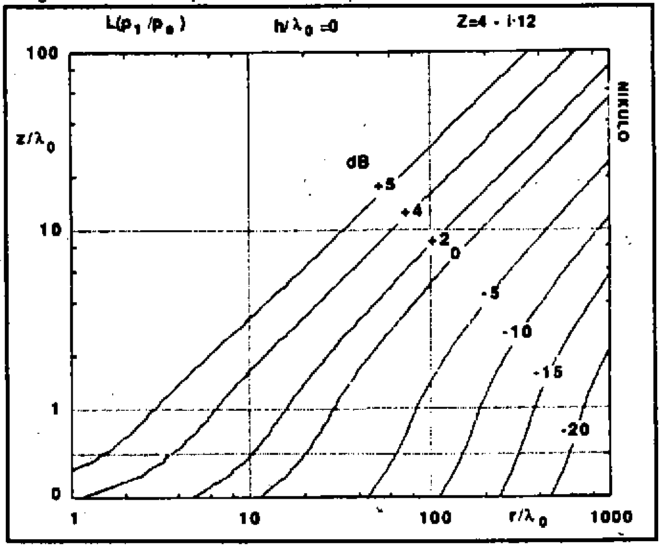

Fig. 14: Contour plot of sound pressure level relative to sound pressure of free spherical wave over a locally reacting absorber plane with given surface impedance Z.

ANALYSIS OF SPHERICAL WAVE PROPAGATION OVER ABSORBING GROUND

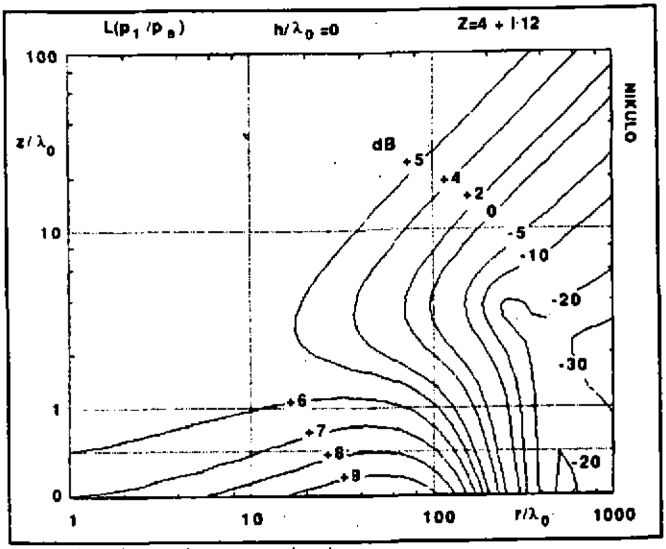

Fig. 15: Contour plot of sound pressure level relative to sound pressure of free spherical wave over a locally reacting absorber plane with given surface impedance Z of positive reactance.

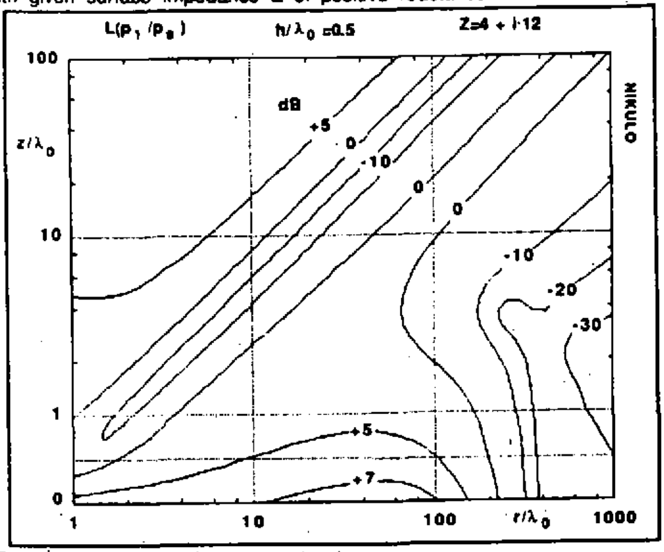

Fig. 16: Contour plot of sound pressure level relative to sound pressure of free spherical wave over a locally reacting absorber plane with given surface impedance Z with positive reactance.