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EXPERIMENTAL NUMERICAL CORRELATION OF SUBSYSTEM CONTRIBUTIONS IN THE ADVANCED TRANSFER PATH ANALYSIS FRAMEWORK

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In a complex vibroacoustic system the overall noise or vibration in a given location is the sum of multiple subsystem contributions. From an experimental perspective, the total noise can be directly measured but not the contributions. Methods based in transmissivity measurements, as ATPA, allow to find these contributions experimentally and understand the system behaviour through the path concept.

Two different contributions to the ATPA method are included here. On the one hand, a numerical model that simulates a simple vibroacoustic problem is shown. This is a closed cuboid-shaped box with air cavity inside. The ATPA experimental procedure is reproduced numerically in order to gain knowledge on some aspects of the method. On the other hand, a technique for the automatic identification of the subsystems which is based on the path concept and transfer matrices is applied to the acoustic problem of coupled rooms. The proper definition of subsystems influences very much the reliability of ATPA results.
Keywords: ATPA, TPA, path, subsystem, transfer

Introduction

A common way to understand the vibroacoustic problems is through the path concept. One formal definitions of 'path' can be found in [1]. More recently, it was shown in [2] that the solution of a mechanical problem can be expressed in terms of paths. An application example is to characterise the transmission of vibration of sound along the automotive parts. It is usually generated in the wheels and engines and travels through the chassis, axes and insulating layers till the passenger cavity passing through.

The final goal is always to characterise the response of each system part (measured in terms of the acceleration of a vibrating element or the acoustic pressure in a zone of interest) caused by a specific excitation. A large amount of experimental methods have been historically developed. We can distinguish, in a quite general classification between: Transfer Path Analysis (TPA) and Advanced TPA (ATPA). The main difference between the methods grouped under the name TPA and the ATPA is that TPA characterises the contributions from the inputs to some outputs by means of a model inversion (measuring the transference between forces and accelerations). On the contrary, ATPA can characterise the topology of the mechanical system and thus, the paths.

If the studied subsystem is understood as a black box with n inputs and m outputs interconnected through the box, the TPA can predict for each output which is the influence of each input. This means that TPA is able to decompose the output signal in parts coming from every input signal. However, it is unable to describe how the input and outputs are connected. ATPA is able to characterise, in addition, how the input and output signals are connected inside the black box, discover which is the intrinsic structure of the mechanical system, which and how are the paths. For this reason when a detailed analysis of the mechanical system is needed, the use of ATPA is mandatory.

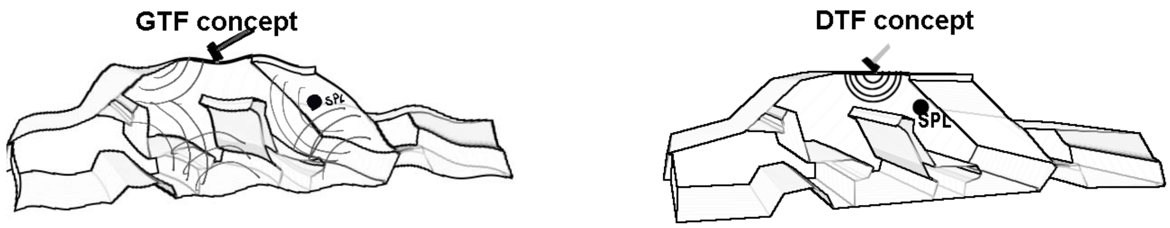
The difference between TPA and ATPA is that TPA measures global transfers between subsystems while ATPA measures (as the experiment output) the direct transfers.

The coefficients of the global transfer matrix $\mathbf{T}^G$ are defined as

$$\mathbf{T}^G{}_{ij} = \frac{x_j}{x_i} \tag{1}$$

where x_j is the signal at j.

The global transfer is the usual situation found when there is an excitation, a receiver (micro or accelerometer) and the receiver signal is due to all possible transmission paths. This is the case of the car in Fig. 1(a) where the sound pressure level (SPL) in the micro is due to the vibration of all the car subsystems (and direct sounds if any).



(a) (b)

Figure 1: Sketch of the global transfer and direct transfer concepts applied to an ATPA test of a car. The excitation is a hammer impact applied on a car subsystem. The target is the sound pressure level (SPL) on a micro placed inside the passengers cabin. The two variants are: (a)Global transfer, no subsystem is blocked and all possible paths from the excitated and non-excited subsystems to the target are possible. The target signal have contributions from all subsystems passing through all the paths; (b)Direct transfer, where only one transmission path linking the excited subsystem and the target microphone is allowed.

A different situation is when all the subsystems are blocked and the transmission from the excitation to the receiver can only be done through a single path. This is never found in real life and can only be reproduced with many difficulties under laboratory conditions. An example is the strip method [3]. See for example Fig. 1(b). This is a car where only the excited subsystem is able to vibrate and radiate sound into the passenger cavity. Consequently, only one path, between the excited subsystem and the receiver can exist (all other paths are blocked).

Paths are characterised by means of the direct transfer matrix $\mathbf{T}^D$. The coefficient $\mathbf{T}^D{}_{iT}$ has information about the path between the position i and the target point T and it defined as

$$\mathbf{T}^D{}_{iT} = \frac{p_T}{x_i} \tag{2}$$

where p_T is the signal at the target point T when an excitation is applied at i and all the nodes $j \neq i$ are blocked.

Even if this situation could be reproduced and all the paths characterised one by one, it would be time-consuming and costly. ATPA overcomes this difficulty as explained in Section 2.

At this point, we can see that global transfers are related with contributions (quantitative) while direct transfers are related with the path concept (qualitative and also quantitative). The global transfers can be more easily measured, while the direct transfers require more elaborated procedures.

The main goal of the present project is to develop a numerical model that reproduces a simple ATPA experimental setup. Once calibrated, the numerical model will allow for a faster realisation of virtual experiments, the possibility of doing parametric analysis or a more visual representation of the results and physical phenomena, among others.

In the remainder of the document, the ATPA method is explained in Section 2 and the aspects in which the numerical model can be helpful briefly mentioned. The numerical model is described in Section 3 and some preliminary results shown in Section 4. Section 5 deals with the automatic identification of subsystems by means of the path concept and the transfer matrices. It is an extension of the method presented in [2] to the case of connected rooms. The conclusions at the current stage of the project and, more important, the future developments are summarised in Section 6.

The ATPA method

In the ATPA method, the mechanical system is tested under the condition of Fig. 1(a) where global transfer can be measured. It can be formulated as [1, 4]

$$p_T = \sum_{i=1}^{N} x_i \mathbf{T}^D{}_{iT} \tag{3}$$

where p_T is a signal in the target (i.e. pressure in a microphone placed inside the passenger's cavity of a car), x_i is the measured signal in the subsystem i (i.e. acceleration of a vibrating panel), $\mathbf{T}^D{}_{iT}$ are the direct transfers between the subsystem i and the target and N is the number of subsystems in which the mechanical system has been divided.

In addition, a relationship between the global transfer defined in Eq. (1) and the direct transfer defined in Eq. (2) can be obtained

$$\mathbf{T}^G{}_{ij} \mathbf{T}^D{}_{jT} = \mathbf{T}^G{}_{iT} \text{ for } i = 1, 2, \ldots, N \tag{4}$$

Eq. (3) can be understood as a balance equation that must be satisfied for any considered excitation (i.e. point forces of different magnitude and applied at different positions). And this is important because we can obtain as many equations of the type of Eq. (3) as experiments we realise. Apart from measurement errors, the coefficients $\mathbf{T}^D{}_{iT}$ must be the same for every realisation of the experiment (changing the excitation) because they depend on the mechanical system which has not been modified. On the contrary, the signals x_i and p_T differ at each realisation of the experiment.

The characterisation of the paths is then reduced to the mathematical problem of determination of the coefficients $\mathbf{T}^D{}_{iT}$. This can be done, for example, by means of the solution of the linear system of just N equations like Eq. (3) (for the case of exactly N realisations of the experiment) or a least square fitting if more than N realisations of the laboratory experiment are done.

In ATPA, a different subsystem is excited at every realisation of the experiment. This guarantees the linear independence of the N equations. In other words, more different behaviours of the mechanical system are included in their characterisation and the information is more representative. In addition, the understanding of the coefficients in Eq. (3) as direct transfers, allows the experimental procedure to be split and use a reduced number of channels at the same time if needed.

A very important aspect in an ATPA analysis is the proper definition of the subsystems: sets of degrees of freedom represented by only one sensor (i.e. an accelerometer that must be a representative measure of the movement for the whole subsystem). The final quality of the results highly depends on this aspect. In practise a sensor is used in order to characterise every subsystem.

The ATPA test has two different variants: coherent and energetic. In the coherent case, the contributions are taken into account in module and phase (in the strict sense of Eq. (3)). On the contrary, the energetic variant which can only be used at high frequencies, deals with outputs that are representative of the subsystem energy. Eq. (3) needs to be rewritten as

$$(p_{RMS})_T = \sum_{i=1}^{N} (x_{RMS})_i \mathbf{T}^D{}_{iT} \quad (5)$$

where RMS means root mean square or some other energetic measure of the subsystem energy. Some differences exist in the testing and post-process procedures of both variants.

The present research tries to improve the ATPA method by developing a numerical model that can reproduce a simple laboratory experiment. Once calibrated, the numerical model could be used in order to study, analyse and gain understanding of aspects that are very difficult to visualise and control in the laboratory or real test. Among others these aspects are: improvement of strategies to deal with background noise when it can not be neglected, automatic identification of the subsystems, optimisation of the sensor position inside each subsystem, combination of more than one sensor per subsystem, study the influence of the excitation type (point force, rain-on-the roof, acoustic wave, etc.) and the spectrum of the excitation, establish the conditions that lead to have only positive combinations coefficients in the energetic version of ATPA.

Description of the Numerical model and experimental setup

A first prototype of a virtual experimental setup is developed. It is based on the finite element method (FEM) and simulates a cuboid box made of six concrete walls or sides. The coupled vibroacoustic problem is considered with the pressure (fluid) -displacement (solid) formulation and by means of the software Code-Aster [5]. Point and surface forces can be applied on each wall. Their direction is orthogonal to the box surface and they are applied at more or less random positions. One control point per side (the accelerations are measured there) and one control point inside the box cavity (measurement of the pressure) are used.

The following simulations have been considered:

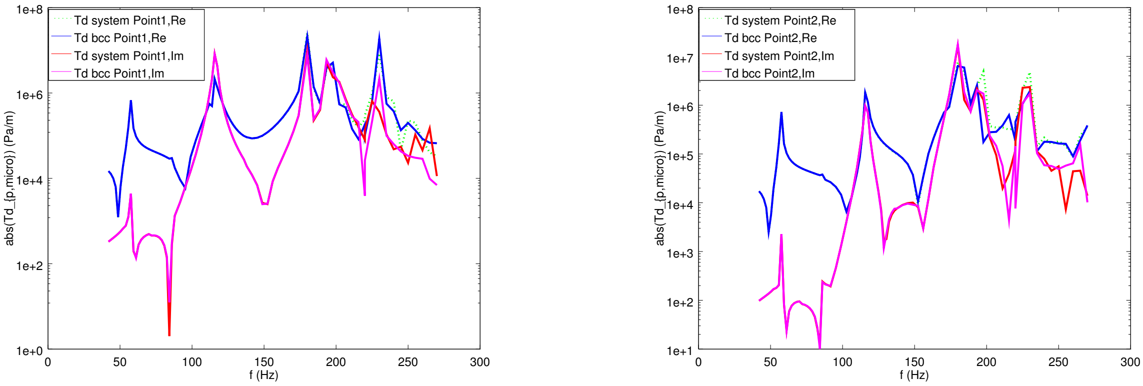
1. *Excitation of the box at the points* $i = 1, 2, \ldots, N$. Each excitation applied to a node i provides a row of the matrix $\mathbf{T}^G$ and the coefficient $\mathbf{T}^G{}_{iT}$. The direct transfer matrix can be obtained from Eq. (4). Afterwards, N simulations per frequency are required in order to generate the linear system of equations Eq. (3).
2. *Excitation of the box at the points i with all the other nodes $j \neq i$ blocked.* With this simulation type, the coefficients $\mathbf{T}^D{}_{iT}$ are obtained. This situation is difficult to consider in the laboratory because in requires time to block all the paths except the one that is studied.
3. *Simulation with a different excitation type.*

Preliminary results

The numerical model can be used to compute the direct transfer coefficients by means of two different procedures obtaining almost the same result. On the one hand a 'real configuration' where six realisations of the experiment with a point force applied in a different surface every time in order to obtain a system of six linear equations like Eq. (4). The direct transfer coefficients can then be obtained by inversion of this linear system. On the other hand the second configuration where all the paths except one are blocked is considered. This allows the straightforward computation of the direct transfer coefficients.

Fig. 2 shows a comparison of both options and the results are almost equivalent. Since this is the coherent version of the method, good agreement in real and imaginary parts can be seen. This is a numerical evidence, at least for the tested frequency range, that the current realisation of the experiment (one sensor per subsystem, definition of sides as subsystems etc) is adequate.

Another relevant aspect is the influence of the excitation type. As mentioned before, ATPA uses a different excitation in order to generate every equation and in general only one subsystem is excited



(a) (b)

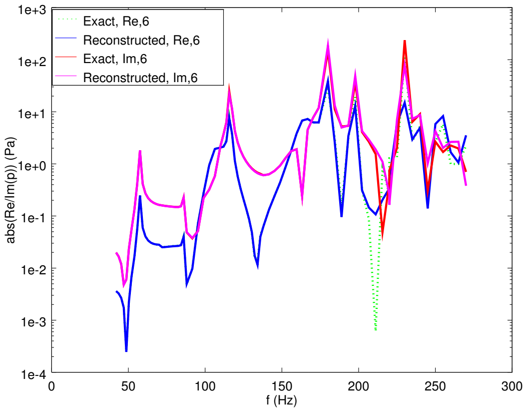
Figure 2: Computation of the direct transfer. Every plot represents a point in each of the walls. ‘system’ means that the direct transfer coefficients has been obtained by means of Eq. (4), while ‘bcc’ means that they have been directly obtained by means of a FEM model with proper boundary conditions.

every time. This differs from the excitations acting on the mechanical system (car, train etc) during the service state. The target signal, which in the box experimental setup is the pressure in the micro placed inside the cavity, is reproduced by means of a precomputed direct transfer matrix when different excitations are considered. It means that $\mathbf{T}^D$ is precomputed as shown in Fig. 2. Since the box is characterised by means of six subsystems (the six faces of the cuboid), the transfer matrix $\mathbf{T}^D$ is a frequency dependent matrix of dimension six. It is computed by considering six different excitation states, with unit point forces applied on the control point of each box surface. Afterwards, the box and cavity setup is excited with a different force. In this new realisation of the experiment seven signals are registered: the six accelerations on the box surfaces and the acoustic pressure inside the cavity (the target signal). But this pressure can also be computed by means of Eq. (3) because $\mathbf{T}^D$ is precomputed and x_i are now known signals. If the target signal is the same by means of both procedures, it means that the mechanical system is properly characterised by means of $\mathbf{T}^D$ matrix (proper subsystem definition and path understanding) and that there exist an independence of the excitation type.

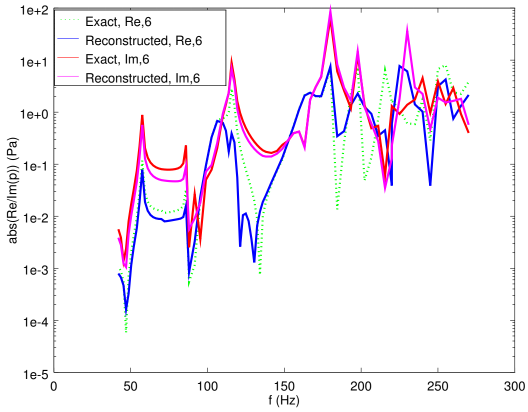
Fig. 3 shows three different situations where the excitation has been modified. In the first one (a), the box is excited with six point forces acting at the same time in the same positions considered for the determination of the matrix $\mathbf{T}^D$. In the second one (b), the six point forces are acting in a random position inside the box face. Finally in the third situation (c), an oscillating uniform pressure is acting in one of the box faces. It can be seen how the agreement is in general quite correct in the whole frequency range. It is of course better in the case (a) becuse the excitation used to determine the matrix $\mathbf{T}^D$ is the same as the excitation used in order to reconstruct the target signal. However, some discrepancies appear. The task is now to discover which is the reason and find the way to correct them (maybe by changing the position of the sensors in every face or using another definition of the subsystems).

Subsystem identification by means of the transmission paths

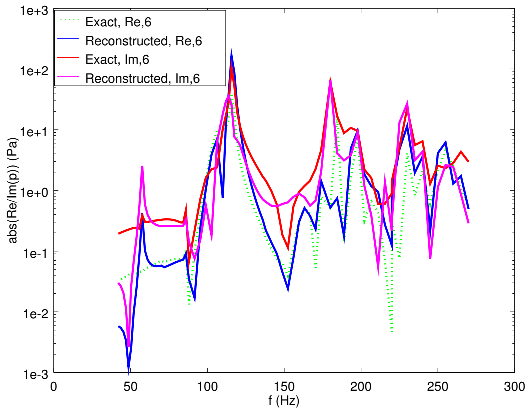
As mentioned above, a key aspect of the method is the correct identification and definition of the subsystems. Previous techniques has been developed with this purpose [6, 7, 2]. What is shown here is the application of the method developed in [2] to an acoustic problem: two rectangular rooms



(a)



(b)



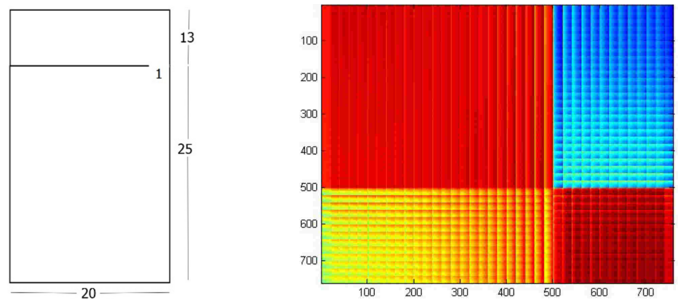
(c)

Figure 3: Reconstruction of the signal in the micro inside the box for different excitations: (a)point forces in the same positions used to compute the direct transfer coefficients; (b) point forces in 'random' positions; (c)Uniform pressure in one of the box faces.

connected through a one dimensional hole, see Fig. 4(a). The problem (Helmholtz equation) is solved by means of finite differences and the hole is modelled as a linear constrain between two nodes. The thickness of the hole is a variable parameter which is related to the wavelength.

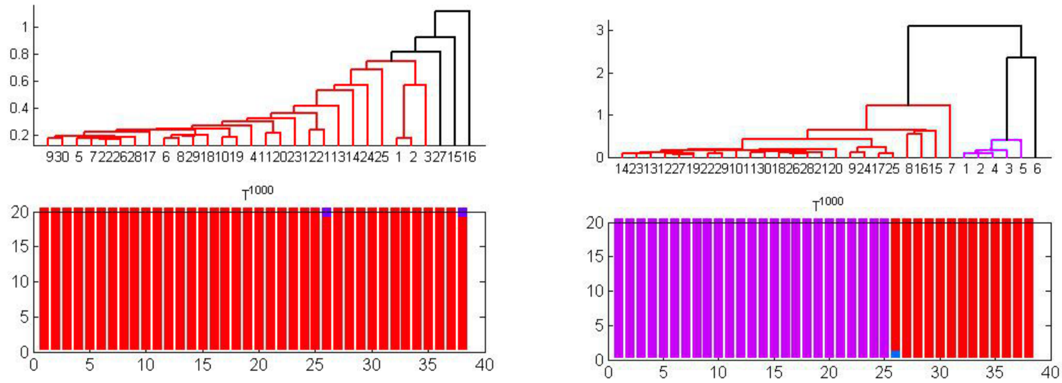
The identification method is based on the powers of the direct transfer matrix. The colours in Fig. 4(b) show the absolute values of the coefficients in dB scale. The rows of this matrix are the input data of a cluster analysis that provides as output dendrograms (see Fig. 5).

The dendrograms are a graphical representation of the node groupings. A criterion must be established in order to define an optimal subsystem definition. As explained in [7] the option leading to larger dendrogram branches is chosen and this leads to a subsystem identification satisfying nice physical properties from the vibroacoustic point of view. In that case, Fig. 5(a) is a case of two rooms coupled through a hole size is closer to the problem wavelength. This leads to strongly coupled situation. The dendrogram identifies all the nodes under the same group (no branch is much larger than the others in order to suggest another grouping option). Fig. 5(b) corresponds to a case of a small hole when compared to the acoustic wavelength. This leads to weakly coupled rooms. Two different groups can be clearly identified in the dendrogram.



(a) (b)

Figure 4: (a)Sketch of the two rectangular rooms connected by means of a hole. (b) Power of the direct transfer matrix $\mathbf{T}^{D^{1000}}$ for the case of weak coupling.



(a) (b)

Figure 5: Dendrograms of two rooms linked by means of a hole. Different ratio between the wavelength and the hole thickness: (a)10, strong coupling; (b) 98, weak coupling.

Conclusions and future work

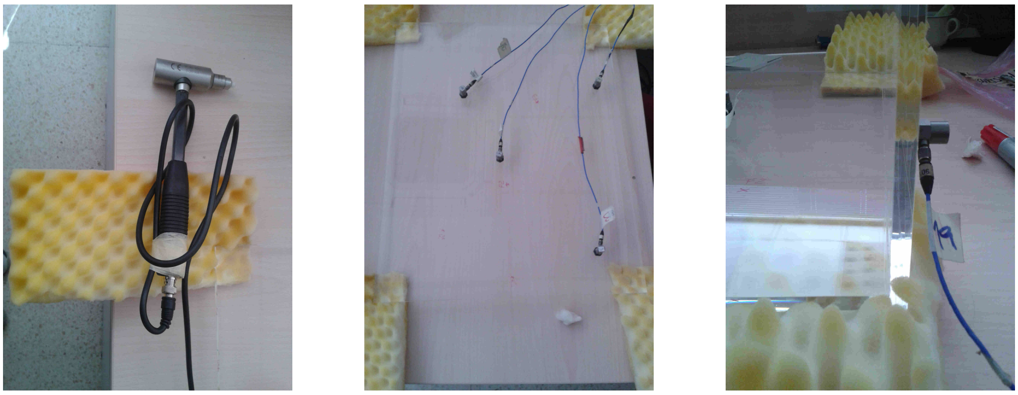
The provided numerical results are quite preliminary. However, they already illustrate the potentialities of the model in order to gain knowledge on the ATPA method. On the one hand, it is shown that the two alternative setups to compute the direct transfer matrix lead to almost equivalent results. One setup is more easy to test in practise while the other (blocking paths) requires a more elaborated and time-consuming experiment. The practical realisation of the experimental setups (one sensor per box face, excitation with point forces at only one face simultaneously, etc.) allowed to reproduce the exact identity expressed by Eq. (3). On the other hand, the numerical model has also been used in order to study the influence of the excitation type on the target signal reconstruction. Some not very large differences are found.

The potentiality of the numerical model still needs to be exploited. Other aspects to analyse are:

- Quantify the influence on the excitation type. Determine the possible differences on the response of the mechanical system when it is excited by means of forces acting during a service situation (which differ from the forces used in the laboratory test).
- Improve the process of subsystem identification
- Optimise the position and number of sensors used in each subsystem in order to improve the reconstruction of the target signal

- To develop an energetic model of the box experiment in order to test the energetic version of the ATPA method.
- Improvement of strategies to deal with background noise when it can not be neglected

In addition, the numerical model is being calibrated and adapted to a reduced scale model where the box is made of methacrylate. Some images of this ongoing process are shown in Fig. 6.



(a) (b) (c)

Figure 6: Images of the reduced scale model of the box that is being tested in the laboratory: (a) impact hammer used to excite each subsystem (face); (b) Simple plate used to calibrate the material properties of the methacrylate; (c) Detail of a box corner, the plates are welded with chemical glue in order to obtain a junction that is as homogeneous as possible.

Acknowledgments

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