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A PARALLALIZABLE SCHEME FOR WIDE BAND BEAM-FORMING

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ABSTRACT

This paper describes a Signal Processing Scheme for computing classically resolved acoustical beams form wide band array signals. The method is based on the execution of a Radon like Transform of the measured field; it can be seen as a reformulation in the field of D.S.P. of the delay and sum method.
An important feature that one meets in designing a digital beam-former is the high amount of computations that must be devoted to interpolate in time the measured data strings in order to avoid beam distortion. The beam-former structure is usually split into three stages, a)A/D conversion of the transducer signal, b)interpolation and delay, c)coherent summing; the second stage, dealing with interpolated strings, that are equivalent to signals sampled at a higher frequency, must work at a higher rate than the first and the third one.
Here we show that, even though computational effort is inevitable it can be shared among a family of reduced, fixed increment interpolation filters (FIF) working in parallel.
The use of these FIFs allows us to introduce two simplifications: a)we can suppress multi-rate delay lines, b)we can trim the branches of our computational tree corresponding to those samples that are not used for coherent summing.
An interesting aspect arises with this trimming mechanism: some special array sizes are particulary well-fitted to the method, in that the prunig procedure gives origin to a considerable reduction in the number of filters to be used. A discussion of this aspect is performed and a particular realization is shown.

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WIDE BAND BEAM-FORMING

Narrow band beam-forming, that is stearing an M element array toward a direction characterizad by an angle θ off broadsifde, is done by performing the line integral:

$$B(\theta,w) = \int_{-D}^{D} \exp(i \cdot 2 \cdot \pi \cdot x \cdot \sin\theta / l(w)) \cdot f(x,w)\, dx \qquad (1)$$

over the measured space-frequency field F(x,w).
In discrete notation the formula looks like:

$$B(\theta,w) = \sum_{j=-N}^{N} \exp(i \cdot 2 \cdot \pi \cdot j \cdot dx \cdot \sin\theta / l(w)) \cdot f(j dx, w) \qquad (2)$$

where:
M = 2*N+1 = array size
j = transducer position index
dx = transducer spacing
w = field pulsation
l(w) = field wave length

When the field is not monochromatic, but just band-limited with Fo as its cut-off frequency, the time evolution of the beams is computed by taking the inversr Fourier transform of of eq.(2).

$$B(\theta,t) = \int_{-\infty}^{\infty} \exp(-i \cdot w \cdot t) \sum_{j=-N}^{N} \exp(iw \cdot j \cdot dx \cdot \sin\theta / c) \cdot f(j \cdot dx, w)\, dw \qquad (3)$$

Performing the above integral leads to the final espresson:

$$B(\theta,t) = \int_{-\infty}^{\infty} dw \sum_{j=-N}^{N} \exp(i \cdot w \cdot (t - (j \cdot dx \cdot \sin\theta / c))) \cdot f(j \cdot dx, w) \qquad (4)$$

or, in discrete version:

$$B(\theta,t) = \sum_{j=-N}^{N} f(dx \cdot j, t - j \cdot dx \cdot \sin\theta / c) \qquad (5)$$

The field is usually sampled at a rate that is twice it band width, both in space and in time, so that:

$$B(\theta,n) = \sum_{j=-N}^{N} f(j \cdot c/2Fo, (n - j \cdot \sin\theta)/2Fo) \qquad (6)$$

Introducing normalized units so that:

C/2Fo = 1 for space

and

1/2Fo = 1 for time

eq.(6) reads:

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$$B(\theta,n) = \sum_{j=-N}^{N} f(j,n-j\sin\theta) \qquad (7)$$

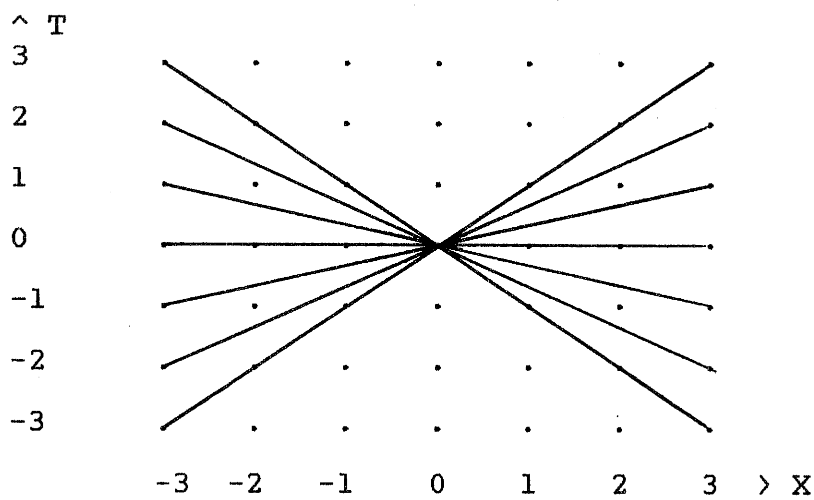
The directions of sight (beam directions) are quantized too according to the law:

$$\sin(\theta) = k / N \qquad -N < k < N \qquad (8)$$

so that:

$$B(k,n) = \sum_{j=-N}^{N} f(j,n-jk/N) \qquad (9)$$

Eq.(9) describes the discrete version of a family of line integrals that can be viewed as a portion of the RADON TRANSFORM of the 2-dim function F(x,t). Figure 1 shows the summation paths in the case of a 7 element array.



M = 7 = number of transducers = number of independent beams
$M = 2 * N + 1 \Rightarrow N = 3$
k = beam index $\qquad -N \le k \le N$
x = space sampling period $\qquad x = 1/2 * c/Fo$
t = time sampling period $\qquad t = 1/2 * 1/Fo$
Fo= upper bound of the signal frequency band

The value of the k-th beam at time n is :

$$B(k,n) = \sum_{j=-N}^{N} F(j,n+jk/N)$$

Fig.1

The computation of B(k,n) by eq.(9), requires samples of the continuous function F(x,t) corresponding to fractionary values of the normalized coordinates. These samples are not available since the function F(x,t) is sampled only at integer values of the normalized coordinetes, and therefore they must be computed in order to avoid aliasing effects in the steered beams; this implies that one has to perform an interpolation of a factor N (at most) of the sample strings of each transducer, exept the central and the extreme ones that are left unchanged.

Interpolation rates

Equation (9) says that each data string, representing the time evolution of the field at each transducer, must be interpolated at a rate at most equal to N. Each acquisition channel should therefore be equipped like in Fig.2:

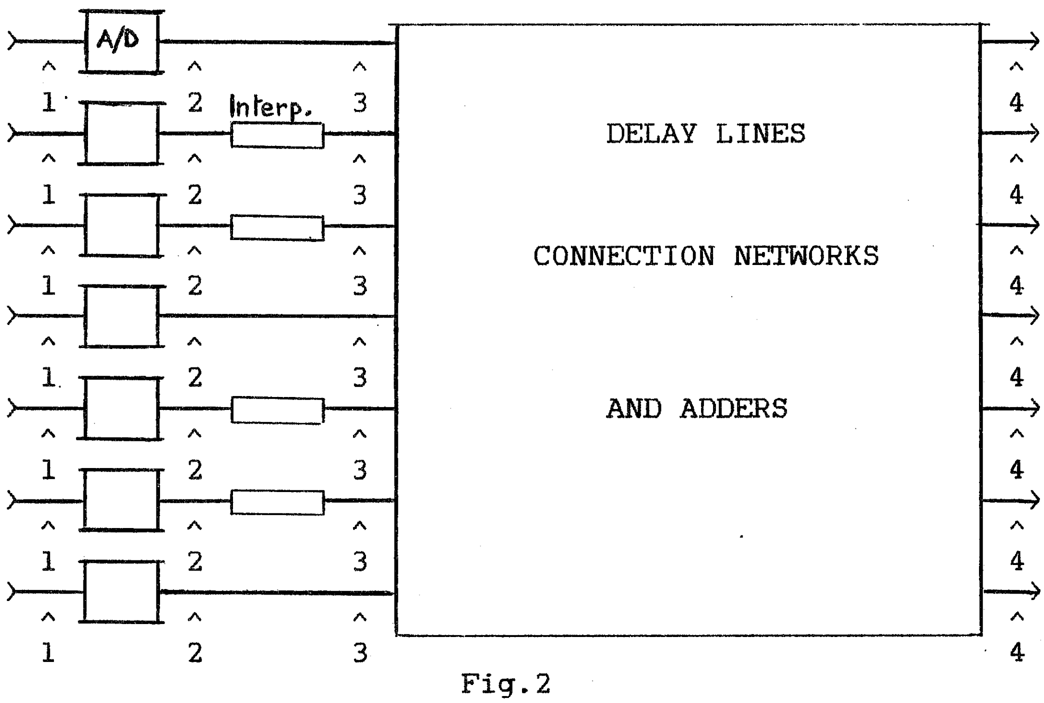


Fig.2

Level 1 => analog signal ;
Level 2 => sampled signal ;
Level 3 => beam-former input signal; bit rate variing as a function of position;
Level 4 => output steered beams; original bit rate

The measured data string should be fed into an interpolation filter whose output is then pipelined into a chain of delay lines and summed;

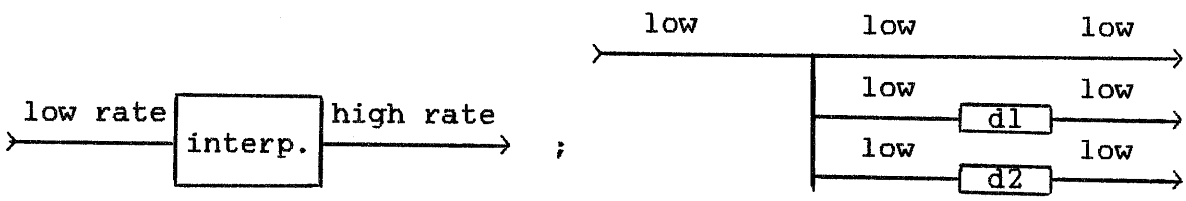
But let us consider the fractionary part of the time index of function F(j,n)in Eq.9, that gives origin to the interpolation rate, as a function of the string space position j:

$$Frc(j) = j / N \tag{10}$$

We can see that while the position index j ranges from -N to N, every time it has a divisor in common with the array number N the fraction (10) can be sempilfied and the interpolation rate can be correspondingly reduced. This means that if one decides to interpolate each channel at the same top rate N, there will generally be strings that are uselessly oversampled and a corresponding amount of computation power and memory will be vasted. On the other hand if one decides to use position dependent interpolation rates one obtains a proliferation of lines working at different speeds.

The parallel implementation

One way for overcoming both problems is given by the property of the interpolation filters of order N that can be split into N reduced filters, each one computing a single sample of the interpolated sequence between two measured values, and working in parallel.
In this way every aquisition channel of fig.3.a can take the new aspect shown in fig.3.b:



(a) fig.3 (b)

Now if we go back to fig.1 we can see that, for each channel, samples corresponding to different fractionary delays are used by different beams. This means that we can obtain all the beams in parallel just connecting each reduced interpolator output to the corresponding beam summing unit, after the insertion of the appropriate delay; in this way every delay will be an integer multiple of the fundamental sampling period. On the whole this processing scheme requires:

$$Ntot = 2*(N-1)^2 \qquad (11)$$

reduced interpolators and 2N+1 summing units.

Interpolation pruning

As we saw earlier if the array number N has some divisors, the interpolation rete of those signals coming from transducers whose position index j has divisors in common with N can be reduced.

M	N	Ntot	Nrid	Reduction rate
5	2	2	2	1
7	3	8	8	1
9	4	18	14	1.28
11	5	32	32	1
13	6	50	30	1.66
15	7	72	72	1
17	8	98	70	1.4
19	9	128	104	1.23
21	10	164	106	1.54
23	11	200	200	1
25	12	242	130	1.86
27	13	288	288	1
29	14	338	230	1.46
31	15	392	264	1.48
33	16	450	310	1.45
35	17	512	512	1
37	18	578	330	1.75
39	19	648	648	1
41	20	722	422	1.71
43	21	800	560	1.43
45	22	822	622	1.41
47	23	968	968	1
49	24	1058	552	1.91

Tab.1

This means that the interpolators corresponding to those fractionary delays that are not used can be omitted. Such mechanism can give origin to a drastic reduction of the computational effort if the set of numbers having divisors in common with N is considerably large. An analysis of this phenomenon shows that the number of interpolators required by an array of order M=(2N+1) is not a monotonic function of N. In Table 1 some values of this function are reported. The consequence of this fact is that in some cases a larger array will take less effort to be handled than a smaller one. For instance a 49 element array is almost two times simpler than a 47 element one since it takes 552 interpolators instead of 968. Nevertheless in an array of order N the output of each transducer must be used 2N+1 times by the 2N+1 beams even if the number of parallel outputs has been reduced by the pruning mechanism. Due to this fact the output samples of each interpolator must be hold for a certain time interval in order to be utilized by more than one beam; this is easily done by inserting them into an appropriate chain of delay lines.

The 13 element beam-former
As an example let us consider the 13 element array. Its array number N is equal to 6 that has divisors in common with 2 and 3. The total amount of interpolators needed is 30 instead of 50 and the reduction rate is quite fair 1.66. In fig.4 the integration paths for an array of this order are drawn

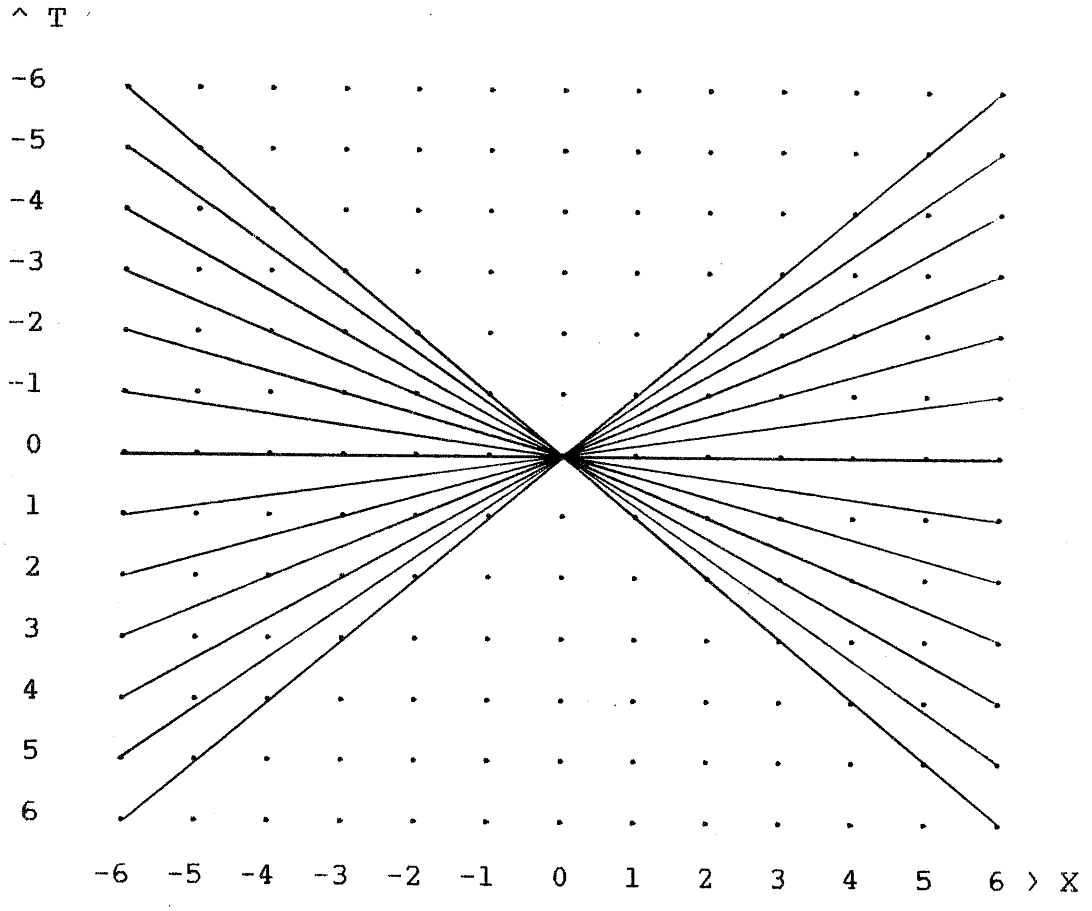


fig.4
The resulting organization of each input channel in the parallel reduced computation scheme and the connection law with each beam-summer are drawn in fig.5.

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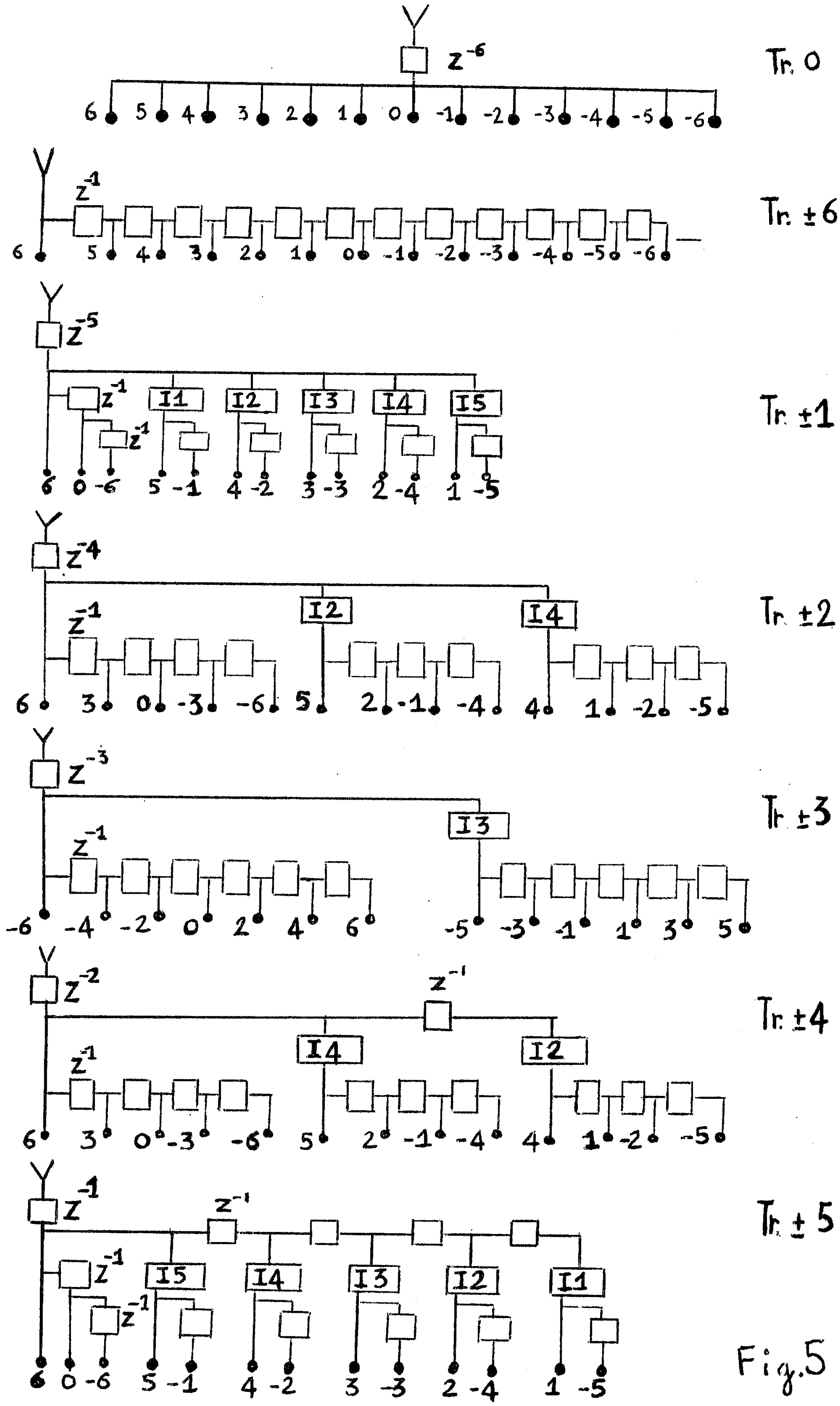

Fig.5

where:

z^{-1} = unitary delay
z^{-N} = N-fold delay
Ij = reduced interpol. of order j, i.e. the filer computing the j-th sample of the 6 times interpolated sequence.

Aa we can see every input channel is processed in different way by means of a particular organization of its interpolators and delay lines.
The burden of computation is limited to the interpolations and can be parallelized down to the level of single filter.

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