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ERRORS OF TIME DELAY BEAMFORMING WITH INTERPOLATED SIGNALS

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1. INTRODUCTION

If an antenna beam system with complete azimuth coverage is to be constructed in a technically feasible form, it must be possible to select the time displacements between two adjacent sensor-signals to be as small as possible. Time-discrete signal processing then requires an infinitely high sampling frequency f_s , which is not technically possible.

A solution is offerend by time-linear interpolation of the sensor signals. The sampling frequency only has to fulfil the SHANNON Theorem. The data rate is correspondingly minimal. The time-linear interpolation deforms the antenna signal. The deformation generally manifests itself as a reduction in the antenna gain. In the small number of special cases in which only sampling values are used for beam forming the antenna signal exhibits no deformation.

If the beam forming process is dealt with in such a way that there is no upper limit to the sampling frequency, then the antenna steering angle Θ and the target angle ϕ are commutative with regard to the beam forming. On the other hand, if the sampling frequency f_s has values which lie within the order of magnitude of the signal frequency f_c , the commutation is noticeable in the beam pattern.

The aim of this investigation is to portray the phenomena which occur in the beam forming of time-linearly interpolated signals, to describe the phenomena mathematically, and to point out the technical consequences. Initial theoretical investigations on this topic have been presented in [1].

2. TIME-LINEAR INTERPOLATION

Of the possible types of interpolation, time-linear signal interpolation between two adjacent sampling values is selected. The time-dependent signal s(t) is sampled at the normalised times $t' = 0,1,2...$ (Fig.1), where $t' = t \cdot f_s$. The signal value of the interpolation signal $s(t'+\Delta)$ is obtained by linear interpolation between the signal values $s(t')$ and $s(t'+1)$. The interpolation error - shown as "Error" in Fig. 1 - is a function of Δ. It is immediately obvious that the interpolation error tends towards zero with increasing sampling frequency f_s if the signal frequency is kept constant. It should be noted that non-linear types of interpolation reduce the interpolation error but significantly increase the amount of signal processing required.

3. BEAMFORMING WITH TIME-INTERPOLATED SIGNALS

If it is assumed that only sampling values of the N sensor signals may be used for the beam forming, only an finite number of beams can be constructed.

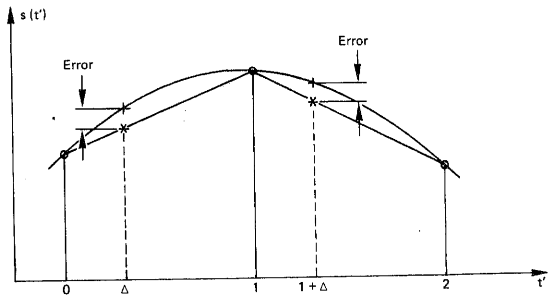

Fig. 1: Time-linear interpolation.

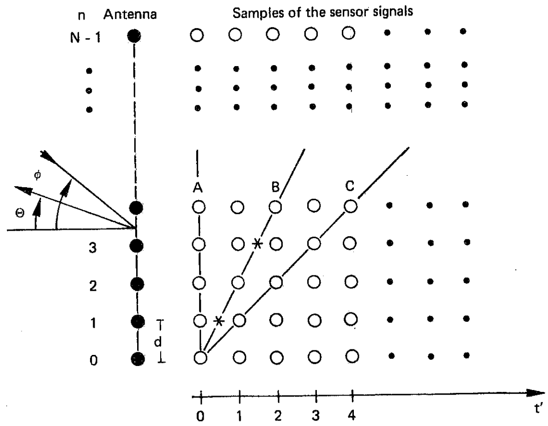

Fig. 2: Sketch showing the principle of time-delay beamforming.

The time delays between adjacent sensor signals are multiples of the sampling period $1/f_s$. The left-hand side of Fig. 2 shows a linear antenna with N equidistant sensors.

The levels of the zeroth and first natural beams are obtained by summation of the sensor signals along the curves A and C, followed by squaring, averaging and logarithmation. Targets which lie in the sector between the "broadside" channel and the first natural beam , for example, cannot be acquired by main lobes. The word "broadside" refers to the direction of the normal to the antenna. To increase the target level for a target at the bearing of the beam B, time-linear signal interpolation is performed; the interpolation values of the individual sensor signals are computed at the times marked by asterisks, and are then processed further in accordance with the beam forming algorithm.

In our case, it is necessary to consider two types of beam pattern which are normallys identical and which differ only in the measurement technique used to record them. The two types are the antenna pattern and the steering pattern. To record the former, the steering angle of the antenna is kept constant and the target is made to move over the azimuth. In the case of the steering pattern, on the other hand, the target remains at a fixed position and the steering angle is varied. During this process, "pseudo grating lobes" occur if interpolated signals are used.

3.1 Antenna Pattern

When the target moves over the azimuth, the target signals are incident upon the linear antenna from various bearings. The antenna parameters are the number of sensors N and the uniform spacing d of the sensors. These parameters apply both to Fig. 3 and to all subsequent illustrations which show beam patterns. Fig. 3 shows the antenna pattern for an antenna steering angle $\theta = 0°$ and target movement from 0° to 90°, with a ratio of narrow-band signal frequency to sampling frequency $f_c' = f_c/f_s = 0.256$ and a signal propagation velocity $c = 1500$ m/s. The antenna pattern is the same as for non-interpolated signals.

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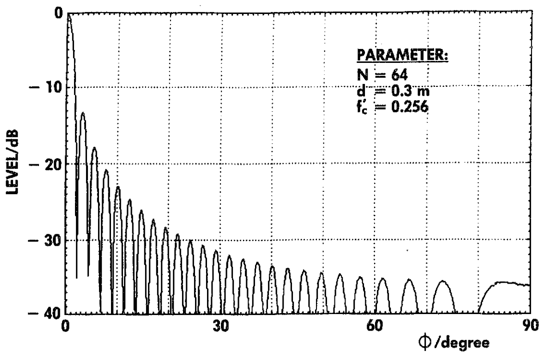

Fig. 3: Antenna pattern, θ = 0°.

To make this situation easier to understand, the following illustration can be used. If the target is situated in the broadside direction in the far field, the received signals are identical at all sensors of the linear antenna if SNR $\rightarrow \infty$. If the target now moves over the azimuth, the signals reach the different sensors at different times. For beam forming at all target positions, only sampling values are used because, as agreed, the antenna is steered in the broadside direction. The steering angle is achieved by non-delayed summation of the sensor signals.

3.2 Steering Pattern

The steering pattern in Fig. 4 is obtained when the target is in the broadside direction. The following differences exist in comparison to the antenna shown in Fig. 3:

- o The main lobe is sharper,
- o the side lobes have an additional structure and
- o there are additional maxima which are called "pseudo grating lobes" (PGLs) here.

Further phenomena can be seen in Fig. 5, which shows a steering pattern for

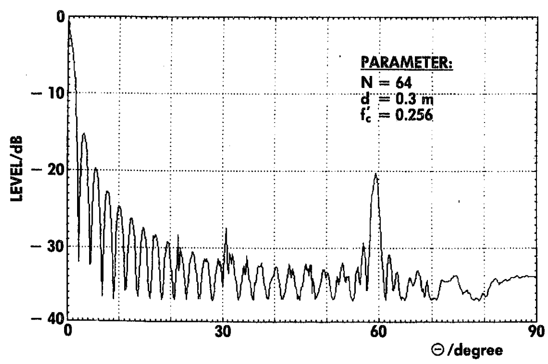

Fig. 4: Steering pattern, Ø = 0°, f'_c = 0.256.

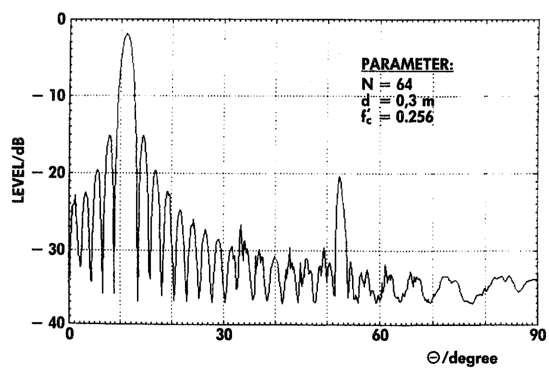

Fig. 5: Steering pattern for Ø=11.1°.

$\emptyset$ = 11.1°. When the target moves away from the broadside direction, not only the main lobe but also the PGLs move. For example, the largest PGL moves from about 59° to 52° while the level remains unchanged.

A change in the normalised signal frequency f_c' causes a shift and a change in level of the PGLs, as documented by Figs. 4 and 6.

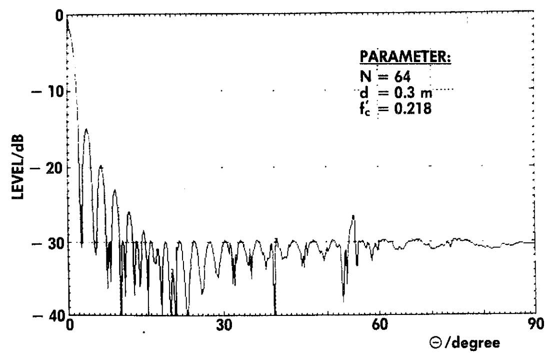

Fig. 6: Steering pattern, $\emptyset$ = 0°, f_c' = 0.218.

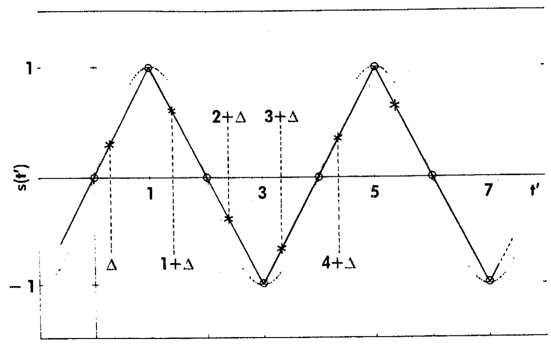

Fig. 7: How the PGLs are generated.

3.3 <u>Pseudo Grating Lobes (PGLs)</u>

An intuitive explanation of the occurrence of PGLs can be given with the aid of Fig. 7. In a special case, it is assumed that f_c' = 0.25, that the target is situated in the broadside direction, and that the signal function and sampling function correspond to one another in such a way that the sampling occurs at the zero points and at the extremes of the sinusoidal signal.

This sampling scheme applies to all sensor signals. The setting of the time delays takes place with interpolation in the chords between sampling values - see Fig. 1. All of the chords together, Fig. 7, form a triangular function which corresponds in a way to the new antenna signal. The beam forming is now carried out not on the basis of the signal values but on the basis of function values of the triangular function. It is important to note that the sampling frequency f_s is not changed by the interpolation procedures, but the narrow-band signal has become a broad-band pseudo signal. For sufficiently high frequency components of the triangular function, grating lobes therefore occur in the steering pattern. The triangular function is proportional to the following series expansion:

$$f_\Delta(\omega_c t) = \text{cnst}\cdot\sum_{i=1}^{\infty}(-1)^{(i-1)}\frac{\sin[(2i-1)\omega_c t]}{(2i-1)^2}\quad,\ \omega_c = 2\pi f_c \qquad (1)$$

The pseudo greating lobes in Fig. 4 at the angles $\theta = 59°$, $31°$ and $21.5°$ can thus be interpreted directly as the 1st, 2nd and 3rd harmonics of the triangular function. The "sharp point" of the main lobe for $\theta = 0°$ in Fig. 4 is caused by contributions form the pseudo grating lobe components.

3.3.1 PGLs and Normalised Signal Frequency

As shown in Figs. 4, 5 and 7, the level and position of the pseudo grating lobes depend on the target bearing ϕ, on the signal frequency f_c', normalised to the sampling frequency, and on the antenna parameters. To a good approximation, the maximum level value of the PGLs can be stated as:

$$L(f_c', m) \cong 10 \cdot \log \left[\frac{\sin(\pi f_c')}{\pi(f_c' + m)} \right]^4 \qquad (2)$$

The corresponding position of the PGLs is computed from the following:

$$\Theta_{m,n} = \sin^{-1} \left[\frac{f_c' \cdot \sin\phi - \frac{c}{d f_s} n}{f_c' + m} \right] \qquad (3)$$

The number pairs $\{m, n\}$ - both of them integers - describe the individual PGLs. Thus, for example, $\{-1,+1\}$ corresponds to the 1st harmonic of the triangular function shown in Fig. 7, $\{+1,-1\}$ corresponds to the second harmonic, and $\{-2,+1\}$ corresponds to the 3rd harmonic. In general, it can be said that m denotes the harmonics of the signal function, whereas n indicates, the order of the periodic continuation of the beam pattern in the virtual space. The connection to equation (1) is made by substituting $f_c' = 0.25$ in equation (2). The maximum level values L are thus proportional to $(4\ m + 1)^{-4}$. By substituting integers for m, we obtain the squares of the coefficients in the series stated in equation (1). Fig. 8 shows the level and position of the individual PGLs for a target bearing of $\phi = 0°$ according to equations (1) and (3). The normalised signal frequency f_c' has been selected as the parameter; it varies within the band $0 \leq f_c' < 0.5$, and leads to solutions in limited angle-intervals only. The increase in f_c' is indicated by an arrow. With decreasing f_c'-values the levels of the PGLs tend to $-\infty$, i.e. the PGLs are eliminated from the steering pattern.

From the point of view of side lobe suppression, the existence of the PGLs must be regarded as undesirable. The PGL level maxima are represented by the dashed curve. This curve can thus be considered as an angle dependent upper limit when $\phi = 0°$ and $f_c' \rightarrow 0.5$. If ϕ varies, i.e. $0 \leq \phi \leq 90°$, this limit varies as well - see Fig. 9; it always remains below -13dB.

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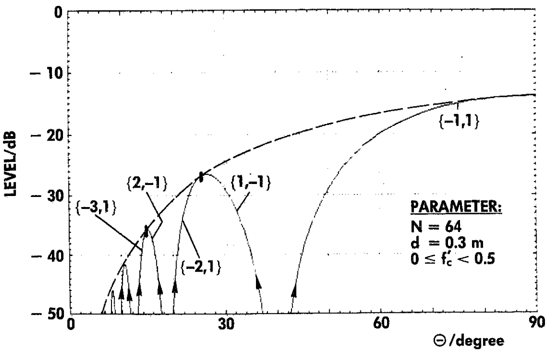

Fig. 8: Level, position and f_c'-dependence of the PGLs.

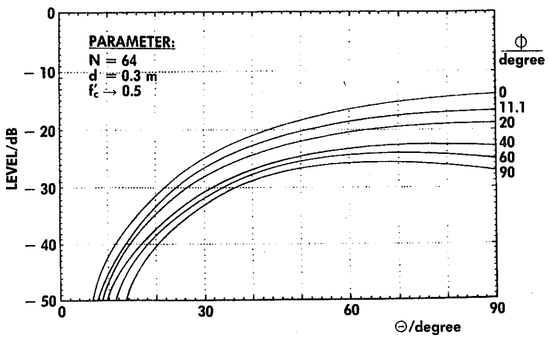

Fig. 9: Level, position, Ø and f_c'-dependence of the PGLs.

4. SUMMARY

Linear interpolation of time-discrete signals permits beam forming on a linear antenna for any desired direction with a minimal data rate. The deformation of the antenna signal leads to errors in the beam pattern. These errors manifest themselves in the form of pseudo grating lobes and impair the target detection process because they give the false impression that additional targets are present.

The level and position of the pseudo grating lobes depend on the antenna parameters, on the target bearing angle and - especially - on the signal frequency normalised to the sampling frequency. The selection of the sampling frequency f_s is based on the relative target levels in the case of a multi-target situation. It must be selected in such a way that, in a particular application, the pseudo grating lobes do not have a disruptive effect; in each individual case, it must be decided whether the dominant factor is the level problem or the resolution problem.

5. REFERENCES

[1] D. Rathjen, G. Bödecker, M. Siegel, "Omnidirectional Beam Forming for Linear Antennas by Means of Interpolated Signals", IEEE J. Ocean. Eng., vol. OE-10, pp. 260 - 268, July 1985.