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ACOUSTICAL CHARACTERISTICS OF RECTANGULAR PISTONS

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ABSTRACT

Electrical circuit analogue modelling of the acoustical radiation from circular pistons is an established technique. With the recent availability of radiation impedance data for various *rectangular* pistons, this technique has been applied to compare and contrast the behaviour of these two common types of radiator.

INTRODUCTION

The acoustical characteristics of circular piston transducers have been extensively modelled using established techniques, usually employing an electrical circuit as an analogue of the lumped (electrical, mechanical and acoustical) parameters of the device under investigation. Normally the acoustical elements (radiation impedance) of the system are neglected as insignificant quantities. This assumption can be acceptable under such conditions as high piston mass and at low frequencies.

In the case of rectangular pistons, the recent publication of radiation impedance data [1] now permits an investigation and comparison of the differing acoustical behaviour of these two types of radiator. Also, the majority of rectangular pistons are ribbon-type transducers, which are generally low-mass systems operating at higher audio frequencies. Therefore any such comparison requires consideration of the acoustical elements.

Past work on rectangular sources has usually involved the assumption of constant diaphragm acceleration: a particularly informative treatise in this vein was produced by Lipshitz & Vanderkooy [2], where finite-length line sources having either monopole or dipole radiation characteristics were studied. In order to define constant acceleration, the radiation impedance was neglected and a constant input voltage assumed. In reality the unavoidable effects of the radiation impedance will preclude this, and other simple approximations, such as constant diaphragm *velocity*.

The purpose of this paper is to analyse the effects of radiation impedance on circular and rectangular pistons operating in an infinite baffle environment. We will confine our study to loudspeakers, although by reciprocity the principle also applies to microphones.

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RADIATION IMPEDANCE

The radiation resistance and radiation reactance functions for a flat circular piston in an infinite baffle are well documented [3,4,5]. They are derived using a classic analytical technique. These functions are shown in Figure 1a.

This same technique has been adapted to evaluate the radiation impedance functions of a rectangular diaphragm of any given aspect ratio, and is fully described in [1]. Briefly, the method involves calculating the reaction force on an infinitesimal area dS′ due to the change in pressure caused by the motion of all other elemental areas dS on the piston surface. This gives rise to a quadruple integral which can be evaluated by numerical methods. The results for a rectangular piston of 50:1 aspect ratio are shown in Figure 1b. Notice that, in order to accommodate the *two* critical dimensions (length and width) of the rectangular piston, the traditional normalised frequency kR (where R is the radius of the circular piston) is replaced by $k\sqrt{Sd/\pi}$ (where Sd is the piston area) and the aspect ratio (e.g. 50:1) must be specified.

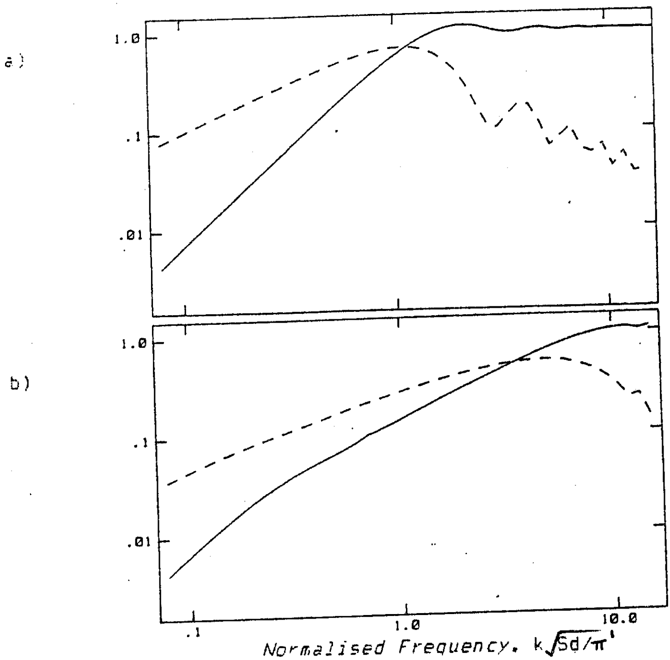
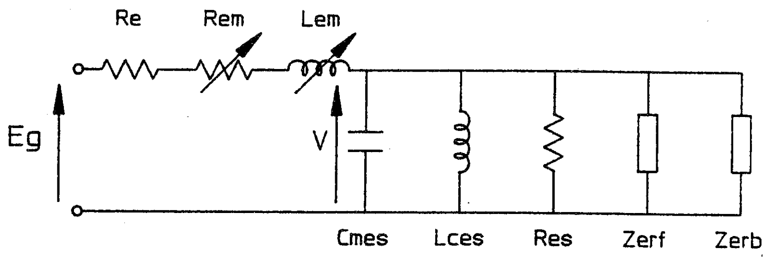


Figure 1. Normalised radiation impedance functions for a) circular piston, and b) 50:1 rectangular piston, mounted in an infinite baffle. Solid line - radiation resistance; broken line - radiation reactance.

ELECTRICAL CIRCUIT ANALOGUE MODELLING

Figure 2 shows an electrical circuit analogue for a loudspeaker. Eg is the electrical input voltage; Re is the d.c. resistance; Cmes is an electrical capacitance representing 'static' mechanical mass; Lces is an inductance representing mechanical compliance; Res represents mechanical damping. In the following analysis the frequency-dependent electrical components Rem and Lem will be neglected, as these are only significant in coil-type transducers [6]. Here we are interested in the effects of different acoustical elements (Zerf and Zerb). If we define the piston to be mounted in an infinite baffle in anechoic space, then Zerf=Zerb. So the circuit can be reduced to that of Figure 3.



V = Bl.u   where Bl is the force factor of the motor system and u is the (complex) velocity of the piston

Figure 2. Electrical circuit analogue for a loudspeaker.

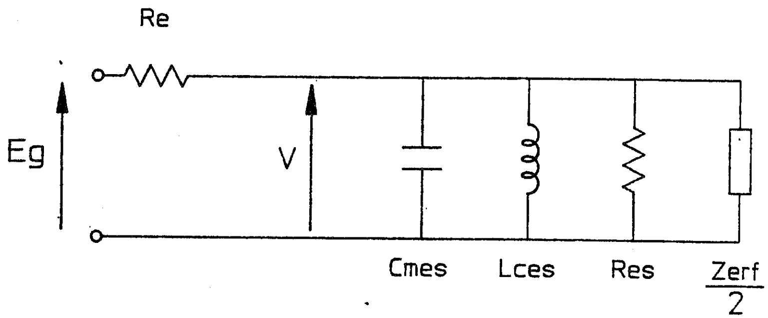


Figure 3. Simplified electrical circuit analogue for a loudspeaker mounted in an infinite baffle.

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SOUND PRESSURE, SOUND POWER

Having calculated the surface velocity using the circuit analogue, it is now possible to predict Sound Pressure Level (SPL) or Sound Power Level (SWL).

At low frequencies or at large distances from the piston, the radiation pattern can be assumed to be hemispherical. In this case we can use an approximate formula for sound pressure, P:

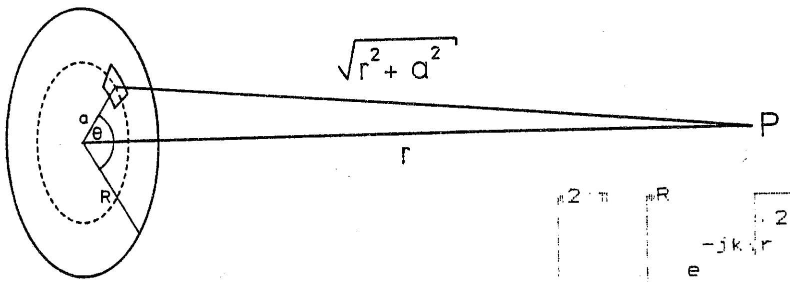
$$P = j\,\rho\, f\cdot u\, Sd\, \frac{e^{-jkr}}{r}$$

where ρ is the density of air (=1.18 kg/m^3 @22°C), f is the frequency
r is the distance from the centre of the piston to the observation point
k = 2πf/c where c is the velocity of sound in air (=345 m/s @22°C)

- Here we implicitly assume simple harmonic motion of the form exp(jkct).

However, for high frequencies or for distances close to the surface of the piston, a more rigorous approach is to divide the surface of the piston into infinitesimal hemispherical sources (as with calculations of radiation impedance) each having the same (known) velocity, and to integrate over the surface to obtain P:

a) Circular piston



$$P = j\cdot\rho\cdot f\cdot u\cdot \int_0^{2\pi}\int_0^{R} \frac{e^{-jk\sqrt{r^2+a^2}}}{\sqrt{r^2+a^2}}\, a\, da\, d\theta$$

Figure 4. Calculation of sound pressure due to a circular piston, at any point along its axis.

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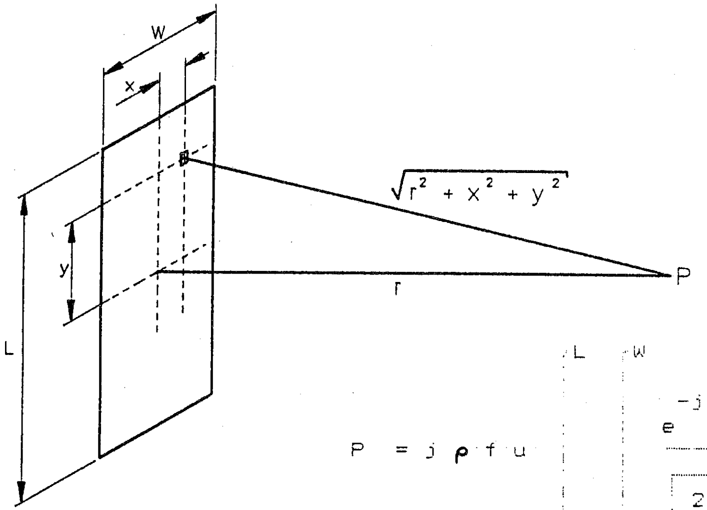
b) Rectangular piston



$$P = j \rho f u \int_0^L \int_0^W \frac{e^{-jk\sqrt{r^2 + x^2 + y^2}}}{\sqrt{r^2 + x^2 + y^2}} \, dx \, dy$$

Figure 5. Calculation of sound pressure due to a rectangular piston, at any point along its axis.

In the case of Sound Power (W) we have the straightforward expression

$$W = \frac{u^2.Rerf}{2}$$

where Zerf = Rerf + j.Xerf

-clearly knowledge of the radiation resistance is crucial to this calculation.

Now Sound Pressure Level and Sound Power Level can be calculated from

$$SPL = 20.\log(|P|/Po) \quad \text{and} \quad SWL = 10.\log(|W|/Wo)$$

where $Po = 2.10^{-5}\ N/m^2$ and $Wo = 1.10^{-12}\ W$

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RESULTS

A comparison is made between a circular piston and a rectangular piston having identical electro-mechanical parameters; the only difference being the radiation impedance data. The diaphragms are taken to have very low static mass (230mg) because the radiation impedance is of greatest significance, and therefore of greatest interest, in such circumstances. The force factor (Bl) is 0.082 Tm, and the diaphragm area is 5.83 x 10^{-3} m^2. . Compliance is taken to be very high, such that it is insignificant except at very low frequencies. The input electrical power is a nominal 1W. Note that it is most practical to assume an electrical input which is constant with frequency. It is less realistic to assume, for example, constant diaphragm acceleration, because of the complex nature of the radiation impedance.

As previously noted, if we wish to model loudspeaker behaviour at mid and high frequencies then we cannot ignore the effects of radiation impedance. Hence, all following data contains the functions shown in Figure 1 as representative of Zerf.

Figure 6 shows the difference in surface velocity magnitude of the two diaphragms. Notice the discontinuity in the circular case.

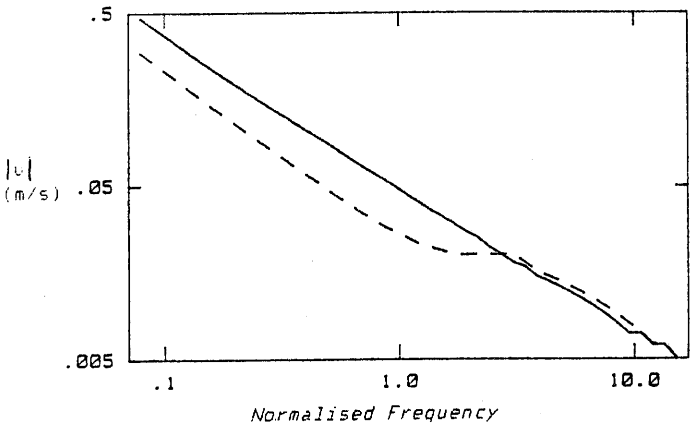

Figure 6. Modulus of surface velocity: broken line - circular diaphragm; solid line - rectangular diaphragm.

Figures 7-9 compare the SPLs of the two cases at varying axial distances from the source. Note that the high-frequency response of the rectangular diaphragm improves with increasing distance. Another way of interpreting this phenomenon is that the soundfield becomes more hemispherical in the far-field. Because the equivalent circular piston has a maximum dimension (i.e. radius) of R = 0.043m (c.f. L = 0.53m for the rectangle) radiation is already virtually hemispherical at r = 1m. Hence there is no significant change in SPL response shape at r = 4m.

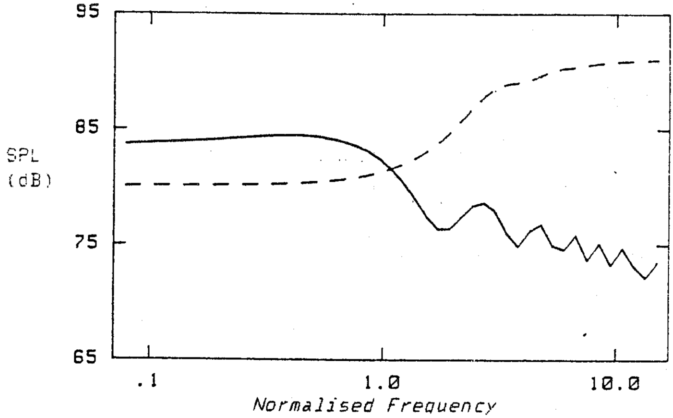

Figure 7. Comparison of the SPL models at a distance of 1m along the piston axis: broken line - circular; solid line - rectangular.

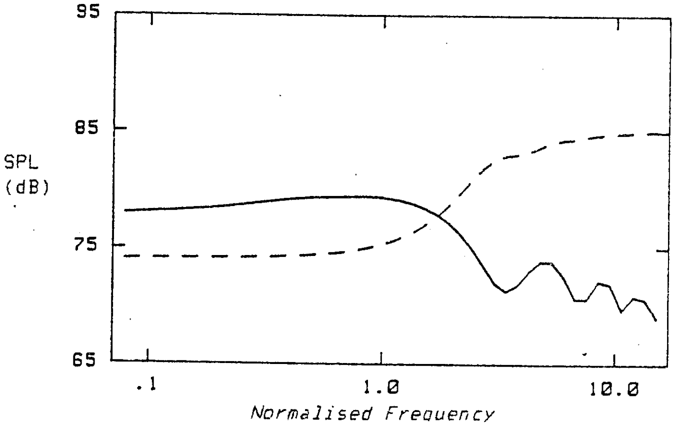

Figure 8. Comparison of the SPL models at a distance of 2m along the piston axis: broken line - circular; solid line - rectangular.

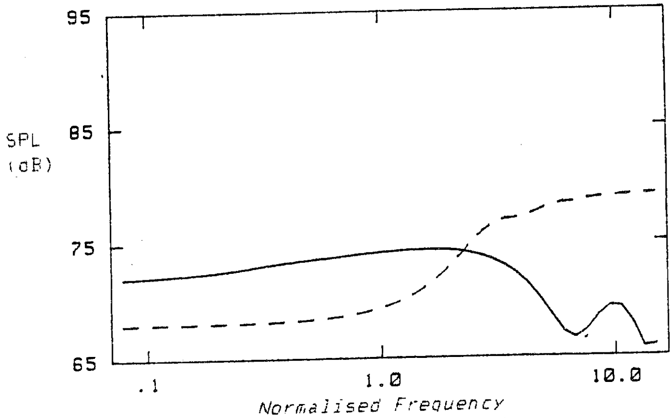

Figure 9. Comparison of the SPL models at a distance of 4m along the piston axis: broken line - circular: solid line - rectangular.

Figure 10 shows the Sound Power Level curves for the two cases. The surface velocity is dependent upon the radiation impedance (both resistance *and* reactance): hence there is a complex interaction between velocity and radiation resistance which determines the SWL. It is clearly inadequate to assume a velocity function (such as 'constant with frequency' or 'inversely proportional to frequency') when employing the radiation resistance to calculate Sound Power output. In the circular case the discontinuity in velocity is exaggerated by discontinuities in the radiation resistance. The rectangle exhibits a smooth roll-off in power response.

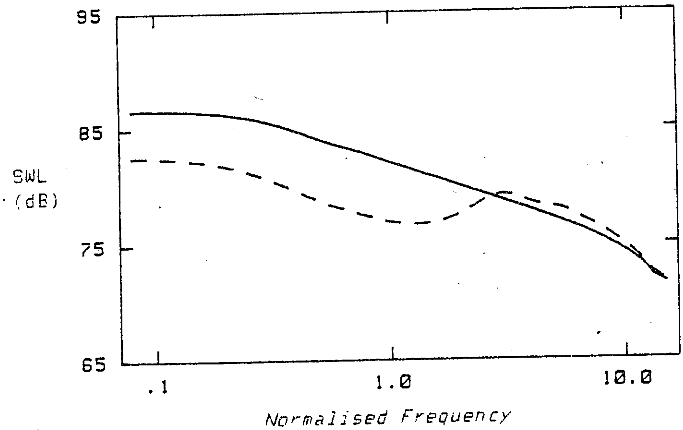

Figure 10. Sound Power Levels. Broken line - circular, solid - rectangular.

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Perhaps the most significant observation is that the rectangular diaphragm exhibits a 4dB improvement in output sensitivity in comparison to its circular counterpart. This is primarily due to the considerable reduction in radiation reactance (or 'mass loading') at low frequencies. The 'hemispherical approximation' formula is useful as a simple indicator of this difference. At low frequencies, the predicted sensitivities at 1m are 84.1dB and 80.1dB respectively.

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