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RESEARCH ON THE DYNAMIC BEHEAVIOR OF FLEXIBLE CONNECTION ROTOR SYSTEM UNDER SEISMIC EXCITATION

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Rotary machine has been widely applied in engineering field. However, seismic excitation may induce a relatively serious mechanical failure on the rotary machine and cause damage to property due to the high rotational kinetic energy of rotary machine in operation. Therefore, the stability of rotary machine under seismic excitation has attracted more and more attention. Vibration response characteristics of rotary machine under the seismic excitation can directly reflect its anti-seismic performance, and generally flexible connection rotor system can effectively improve stability of the movement system. Aiming at the flexible connected two-section rotor system in this study, quality discretization method has been used to build the physical model of rotor system. The motion differential equation of system was established based on Lagrange equation with generalized coordinates, and the modal characteristics of the system was analyzed. According to the anti-seismic resistance requirement, the improved Kanai-Tajimi seismic model proposed by Ou jinping was adopted to simulate the non-stationary random process of the earthquake, and then the *Whilson-θ* method was utilized for numerical solution of the equation. Finally, the vibration response characteristics of system under the seismic excitation, vibration shape and whirl orbit of system, and dynamic response characteristics of flexible connection piece were obtained, consequently, the quantitative research on anti-seismic performance of flexible connection system whirl orbit was realized.
Keywords: flexible connection rotor, vibration response, seismic excitation

1. Introduction

Rotary machine are widely used in engineering field, which may encounter some external base excitation during the long running period (seismic interference, around machine failure) [1], therefore, the stability under seismic excitation has attracted more and more attention. In recent years, the structure dynamics of stiffness rotor has been analyzed extensively. Flexible connectors shall be used on all piping connected to rotating equipment to reduce the transmission of noise and vibration, and to eliminate stresses in systems due to misalignment and thermal movement [2]. Aiming at a kind of flexible connection rotor system, the stability of this system under seismic excitation has been investigated in this study. From the structure diagram of flexible connection rotor system shown in Fig. 1, the rotor is composed of two rigid cylinders connected by the flexible connection, and two sides of the rotor are fixed by two bearings, respectively. When the external environment vibration happens, it affects the flexible connection rotor system in the form of base excitation and induces the harmful vibration of rotor, eventually, poses a threat to the normal operation of the rotating machinery.

This study aims to calculate and analyze the vibration response of flexible connection rotor system under seismic excitation, explore corresponding response characteristics, finally, provide support to the optimized design scheme of rotor seismic performance.

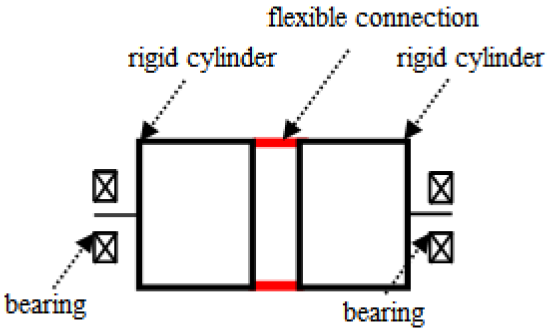


Figure 1: Structure diagram of flexible connection rotor system.

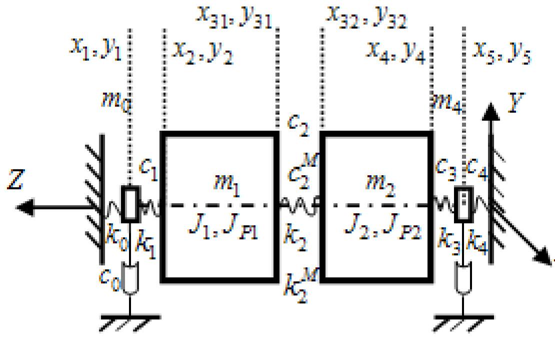


Figure 2: Physical model.

2. Dynamics model of flexible connection system

2.1 Physical model

The model was discretized by lumped mass method. Due to the bending rigidity of flexible connection piece is much less than that of barrel, the rotor cylinder was discreted into three parts, the right and left steady bearings were simplified as spring - damper - quality unit which can only move horizontally. The elastic part of flexible connection piece and steady bearing of rotor are considered as elastic components, the quality of which can be ignored. Taking the micro-vibration of rotor into account, the simplified physical model is shown as Fig. 2, in which the equivalent stiffness of bearing are k_0 and k_4, viscous damping coefficient are c_0 and c_4, quality are m_0 and m_4; quality of rigid cylinders are m_1 and m_2, axial lengths are l_1 and l_2, ratios of length between left barrel and mass center and total length are λ_1 and λ_2, polar moment of inertia are J_{P1} and J_{P2}, equator moment of inertia are J_1 and J_2, the rigidity of left and right supporting shafts are k_1 *and* k_3, length is l_z, structure damping coefficient are c_1 and c_3, transverse shear stiffness of flexible connection piece is k_2, bending stiffness is k_2^M, transverse shear damping coefficient is c_2, bending damping coefficient is c_2^M. System state is described by generalized coordinates of x_1-x_5, y_1-y_5 shown in Fig. 2.

2.2 Mathmatics model

Based on above physical model, the motion equation of the rotor system is derived using generalized Lagrange Eq. (1) [3]. Because the length of flexible connection piece is far less than length of barrel, its transverse shear stiffness is much more than bending stiffness. For simplicity, the shear stiffness of flexible connection piece is set to be infinite and the flexible connection piece is considered to produce angular rotation only without transverse relative motion, namely, the flexible is deemed as spherical hinge with torsion spring.

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right)-\frac{\partial T}{\partial q_i}+\frac{\partial U}{\partial q_i}+\frac{\partial Z}{\partial \dot{q}_i}=Q_i \tag{1}$$

Simplified motion equations can be written as the following matrix (2).

$$[M]\{\ddot{r}\}+([C]-i\omega[H])\{\dot{r}\}+([K]-i\omega[C_I])\{r\}=-[M_o]\{\ddot{r}_b\} \tag{2}$$

In Eq. (2): $[M]$ =

$$\begin{bmatrix} m_0 & 0 & 0 & 0 & 0 \\ 0 & (1-\lambda_1)^2 m_1 + \frac{J_1}{l_1^2} & \lambda_1(1-\lambda_1)m_1 - \frac{J_1}{l_1^2} & 0 & 0 \\ 0 & \lambda_1(1-\lambda_1)m_1 - \frac{J_1}{l_1^2} & \lambda_1^2 m_1 + \frac{J_1}{l_1^2} + (1-\lambda_2)^2 m_2 + \frac{J_2}{l_2^2} & \lambda_2(1-\lambda_2)m_2 - \frac{J_2}{l_2^2} & 0 \\ 0 & 0 & \lambda_2(1-\lambda_2)m_2 - \frac{J_2}{l_2^2} & \lambda_2^2 m_2 + \frac{J_2}{l_2^2} & 0 \\ 0 & 0 & 0 & 0 & m_4 \end{bmatrix};$$

$$[C_E] = \begin{bmatrix} c_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & c_4 \end{bmatrix}; \qquad [H] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{J_1}{l_1^2} & -\frac{J_1}{l_1^2} & 0 & 0 \\ 0 & -\frac{J_1}{l_1^2} & \frac{J_2}{l_2^2} & -\frac{J_2}{l_2^2} & 0 \\ 0 & 0 & -\frac{J_2}{l_2^2} & \frac{J_2}{l_2^2} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$[K] = \begin{bmatrix} k_0 + k_1 & -k_1 & 0 & 0 & 0 \\ -k_1 & k_1 + \frac{k_2^M}{l_1^2} & -k_2^M\left(\frac{1}{l_1^2} + \frac{1}{l_1 l_2}\right) & \frac{k_2^M}{l_1 l_2} & 0 \\ 0 & -k_2^M\left(\frac{1}{l_1^2} + \frac{1}{l_1 l_2}\right) & k_2^M\left(\frac{1}{l_1} + \frac{1}{l_2}\right)^2 + k_3\frac{l_Z^2}{l_2^2} & -k_2^M\left(\frac{1}{l_2^2} + \frac{1}{l_1 l_2}\right) - k_3\left(\frac{l_Z}{l_2} + \frac{l_Z^2}{l_2^2}\right) & k_3\frac{l_Z}{l_2} \\ 0 & \frac{k_2^M}{l_1 l_2} & -k_2^M\left(\frac{1}{l_2^2} + \frac{1}{l_1 l_2}\right) - k_3\left(\frac{l_Z}{l_2} + \frac{l_Z^2}{l_2^2}\right) & \frac{k_2^M}{l_2^2} + k_3\left(1 + \frac{l_Z}{l_2}\right)^2 & -k_3\left(1 + \frac{l_Z}{l_2}\right) \\ 0 & 0 & k_3\frac{l_Z}{l_2} & -k_3\left(1 + \frac{l_Z}{l_2}\right) & k_3 + k_4 \end{bmatrix};$$

$$[C_I] = \begin{bmatrix} c_1 & -c_1 & 0 & 0 & 0 \\ 0 & c_1 + \frac{c_2^M}{L_1^2} & -c_2^M\left(\frac{1}{l_1^2} + \frac{1}{l_1 l_2}\right) & c_2^M\frac{1}{l_1 l_2} & 0 \\ 0 & -c_2^M\left(\frac{1}{l_1^2} + \frac{1}{l_1 l_2}\right) & c_2^M\left(\frac{1}{l_1} + \frac{1}{l_2}\right)^2 & -c_2^M\left(\frac{1}{l_2^2} + \frac{1}{l_1 l_2}\right) & 0 \\ 0 & c_2^M\frac{1}{l_1 l_2} & -c_2^M\left(\frac{1}{l_2^2} + \frac{1}{l_1 l_2}\right) & c_2^M\frac{1}{l_2^2} + c_3 & -c_3 \\ 0 & 0 & 0 & -c_3 & c_3 \end{bmatrix};$$

$$[M_o] = [m_0 \quad m_1(1-\lambda_1) \quad m_1\lambda_1 + m_2(1-\lambda_2) \quad m_2\lambda_2 \quad m_4]^T$$

If the right end of Eq. (2) is equal to zero, the motion equation of free vibration of rotor system can be described as Eq. (3).

$$[M]\{\ddot{r}\} + ([C] - i\omega[H])\{\dot{r}\} + ([K] - i\omega[C_I])\{r\} = 0 \tag{3}$$

3. Frequency characteristic of rotor system

All of the parameters in rotor system have been listed in Table 1. Response characteristics of rotor system under seismic shock are codetermined by free vibration characteristics without seismic shock and forced vibration characteristics with seismic excitation of system. By solving the motion Eq. (3), each order modal frequency of rotor system within the whole speed range (0 ~ 300 s^{-1}) were obtained as shown in Fig. 3.

Table 1: Parameters in rotor system

Parameter	Value	Parameter	Value	Parameter	Value
m_0	1.3 kg	J_2	0.07 $kg\cdot m^2$	λ_2	0.5
m_1	5.5 kg	k_0	1e3 N/m	J_{p1}	0.7 $kg\cdot m^2$
m_2	5.5 kg	k_1	1e2 N/m	J_{p2}	0.7 $kg\cdot m^2$
m_4	1.3 kg	k_2	1e5 N/m	J_1	0.07 $kg\cdot m^2$
l_1	0.1 m	k_2^M	5e2 N/rad	c_2	10 N·s/m
l_2	0.1 m	k_3	1.5e3 N/m	c_2^M	0.1 N·s/rad
l_z	0.01 m	k_4	1e2 N/m	c_3	1 N·s/m
h	0 m	c_0	1 N·s/m	c_4	1 N·s/m
λ_1	0.5	c_1	1 N·s/m		

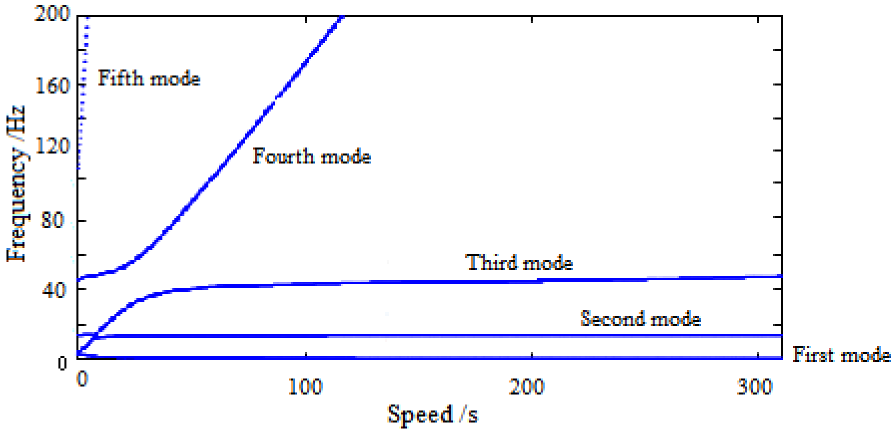


Figure 3: Each order modal frequency of rotor system.

Calculations indicate that the first two order modal frequencies of rotor system are all within the range of 0～30Hz, also the frequency of earthquake impact energy is mainly in this range. Therefore, it is necessary to take effective seismic fortification measures for flexible connection rotor system to ensure it safe operation under seismic shock.

4. Nonstationary random seismic model

As the general rotating machinery, it must meet the severe earthquake resistance requirements in order to make sure its operational reliability. By reference to the improved Ou jinping model and a given model parameter of 7 intensity, theoretical calculation of seismic performance of flexible connection rotor was carried out. Compared with Kanai-Tajimi seismic model, seismic wave model improved by Ou jinping addes frequency spectrum parameter which reflects the bedrock features, and is more in line with seismic dynamic characteristic [4, 5], besides that, this model proposes a new method to determine the spectral intensity factor and seismic duration for different seismic intensities and different site category.

According to the improved seismic wave model by Ou jinping, time-distance curve of accelerated velocity for the stationary ground motion can be expressed as Eq. (4).

$$\ddot{x}(t) = \sqrt{S(\omega)}e^{i\omega t} \tag{4}$$

$$S(\omega) = \frac{1+4\zeta_g^{\ 2}(\omega/\omega_g)^2}{[1-(\omega/\omega_g)^2]^2+4\zeta_g^{\ 2}(\omega/\omega_g)^2} \cdot \frac{1}{1+(\omega/\omega_h)^2} \cdot S_0 \tag{5}$$

In Eqs. 4 and 5, $\ddot{x}(t)$ is smooth ground acceleration function; $S(\omega)$ is auto-spectral density; S_0 is white noise density spectrum of bedrock; ζ_g and ω_g are the damping ratio and predominant frequency of surface covering soil layer; ω_h is the spectrum parameter to reflect the bedrock feature.

The actual seismic wave is the non-stationary random process, which need to introduce the non-stationary intensity function.

$$\ddot{X}_g(t) = f(t)\ddot{x}(t) \tag{6}$$

$$f(t) = \begin{cases} (t/t_b)^2, t \le t_b \\ 1, t_b \le t \le t_c \\ e^{-c(t-t_c)}, t_c \le t \end{cases} \tag{7}$$

$\ddot{X}_g(t)$ is non-stationary ground acceleration function; $f(t)$ is non-stationary intensity function; t_b, t_c, c are the start time, end time and attenuation parameter of main seismic stage, respectively; T_d means the stationary seismic duration and $t_b = 0.5T_d; t_c = 1.2T_d; c = 2.5/T_d$.

In order to simulate the non-stationary random earthquake ground motion of 9 intensity, all the important parameter is listed in Table 2.

Table 2: Non-stationary random seismic parameters under intensity 9

S_0 /m^2/s^3	ζ_g	ω_g /rad/s	ω_h /rad/s	T_d /s	ω/rad/s
2.16*10^{-2}	0.64	31.42	8π	8.17	200π

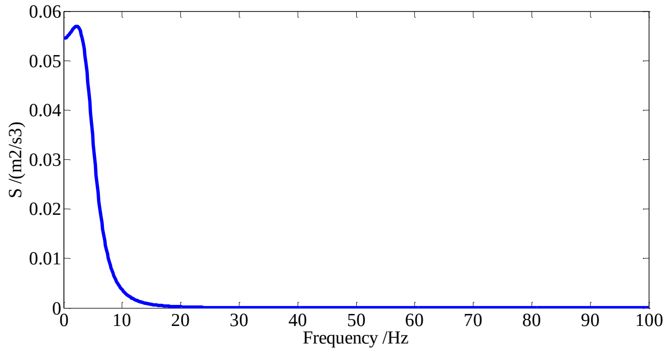


Figure 4: Seismic auto-spectral density.

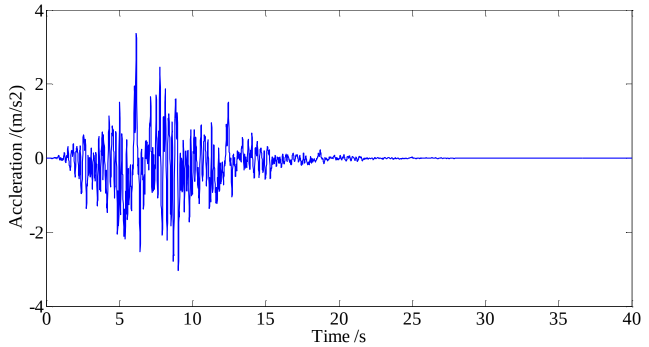


Figure 5: Time-distance curve of non-stationary random earthquake ground motion.

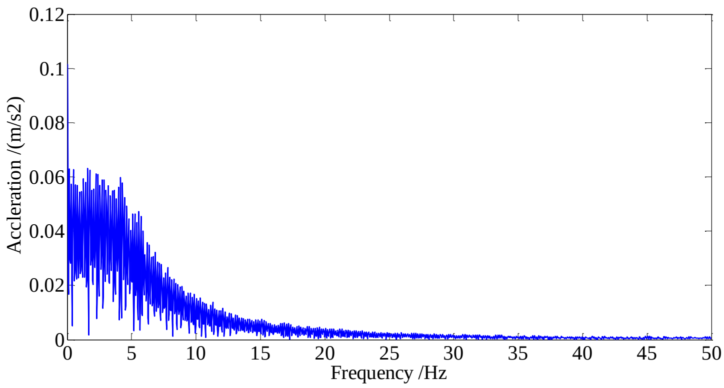


Figure 6: Frequency spectrogram of non-stationary random earthquake ground motion

The power spectrum, time-distance curve and frequency spectrogram of non-stationary random earthquake ground motion have been shown in Figs. 4, 5 and 6 below. From Fig. 6, it can be seen that the seismic energy is mainly within 20 Hz.

5. Dynamic response characteristics of rotor system under seismic shock

5.1 Vibration characteristics of rotor

Wilson method is used to calculate the dynamic response characteristics of the system under seismic impact. The advantage of *Wilson-θ* method is unconditionally stable, and the calculation accuracy is very high using smaller θ. Besides that, it can ensure that the influence of algorithm damping is very tiny once the ratio of Δt and T was smaller than 0.1[6, 7]. The time-distance history of rotor in a single direction shows the vibration characteristic. Figure 7 is the result in time domain of the rotor system vibration, and the Fig. 8 gives the result of frequency spectrogram for rotor system vibration.

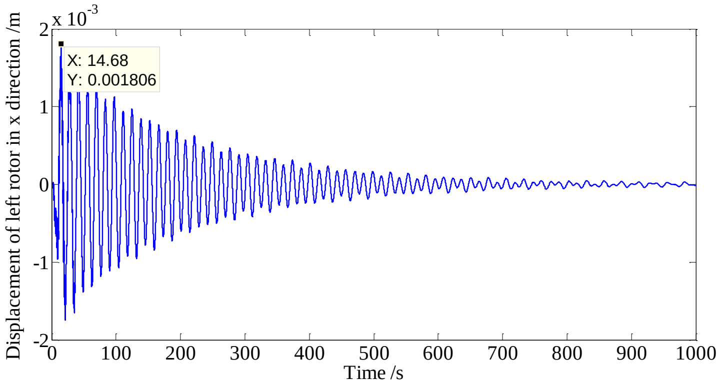


Figure 7: Result in time domain of rotor's left side vibration.

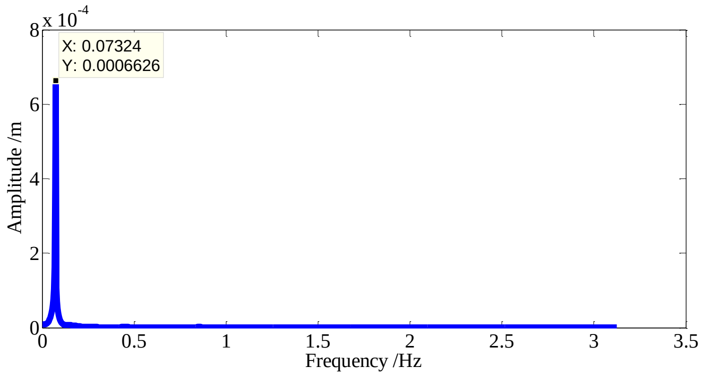
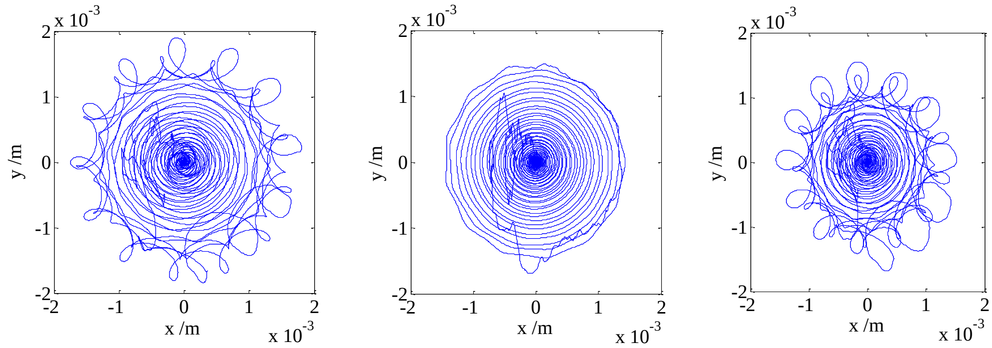


Figure 8: Frequency spectrogram of the rotor's left side vibration.

Calculation results show that the vibration amplitude of flexible connection rotor system reached its maximum of 14.68 s under seismic shock, after that the amplitude shew the tendency of free attenuation with damping, then the seismic vibration vanished gradually. The vibration frequency mainly induced by earthquake was 0.07 Hz, which was consistent with the calculated first order modal frequency of system in Eq. (3). Therefore, vibration of flexible connection rotor system caused by earthquake was given priority to the first-order vibration.

5.2 Vortex motion characteristics of rotor

Vortex motions of all the parts in rotor system under seismic impact have the same direction. At the initial stage of vibration, the vibration amplitude increased drastically, and vibration track was irregular, each part was characterized by low frequency vortex motion attenuation in the process of amplitude attenuation, amplitude of each measurement point was almost same. Compared to the amplitude in other position, vortex track of the flexible connection piece was more regular. Vortex motion curve of each part in the rotor system is shown in Fig. 9.



(a) Left end of rotor. (b) Flexible connection rotor. (c) Right end of rotor

Figure 9: Vortex motion track figure of the rotor system.

5.3 Bending vibration characteristic of flexible connection piece

Flexible connection piece is the weak part of the rotor system. Bending angle of the flexible connection piece α_j, j= 1, 2, in which 1 and 2 represent the XOZ and YOZ plane component, respectively, and its computation formula is shown in Eq. (8).

$$\alpha_j = \frac{u_2 - u_3}{l_1} + \frac{u_4 - u_3}{l_2} \tag{8}$$

Time-domain and frequency-domain graphs of bending vibration of flexible connecting pieces under seismic shock are given in Figs. 10 and 11 respectively.

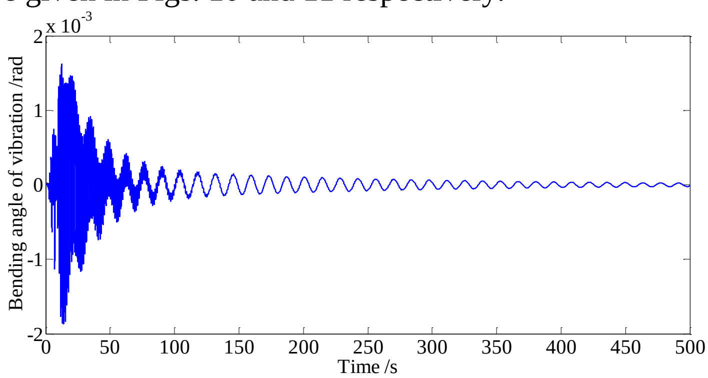


Figure 10: Time-domain figure of bending vibration of flexible connecting pieces under seismic shock.

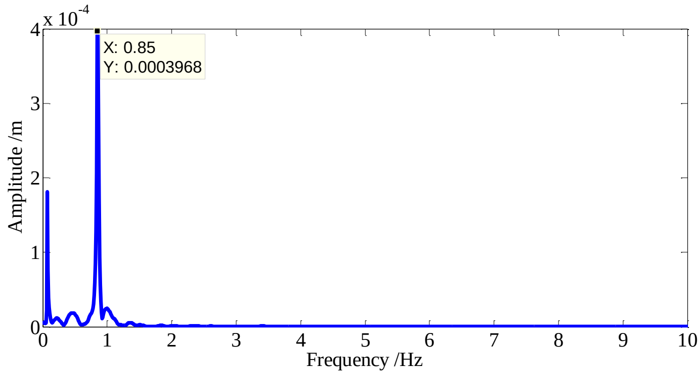


Figure 11: Frequency-domain figure of bending vibration of flexible connecting pieces under seismic shock.

Results show that the flexible connecting pieces have bending vibration, bending vibration reached the maximum within the first 10 seconds of earthquake, attenuation duration of bending vibration is about 300 s, which was shortened by more than half compared with Fig. 7. The frequency of bending vibration mainly contains the first order vibration frequency of 0.07 Hz and 0.85 Hz.

6. Conclusions

Based on the lumped mass method and Lagrangian method with generalized coordinates, motion differential equation of flexible connection rotor system under seismic impact was deduced. *Whilson-θ* numerical calculation method was used to solve the each order modal characteristics of the system. Taking the non-stationary seismic shock as the power source, vortex motion track of system and the dynamic response characteristics of the flexible connection pieces can be obtained. It is found that under the non-stationary seismic shock, vibration of the flexible connection rotor system is given priority to the first order vibration of system, the vortex motion of each part in rotor system has the same direction. Besides that, the flexible connection piece has bending vibration, and the maximum bending vibration happens in the first 10 seconds of seismic shock, the bending vibration frequency are mainly composed of the system's first order vibration frequency of 0.07 Hz and 0.85 Hz. In addition, bending vibration may cause plastic failure of the parts, which need the corresponding protective measures.

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