Proceedings of The Institute of Acoustics

A DISCRETE-TIME, LINEAR SYSTEMS APPROACH TO THE MODELLING AND SIMULATION OF PIEZOELECTRIC TRANSDUCER STRUCTURES.

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1. INTRODUCTION

Piezoelectric transducers have found widespread application in a variety of fields, including ultrasonic non-destructive testing, medical diagnosis and underwater sonar. As a result, a variety of techniques have been developed for modelling transducer behaviour, thereby permitting critical system parameters to be optimised at the design stage. In the case of electromechanical transduction devices, it has been found convenient in the past to model the system by an equivalent electrical circuit. This is not without good reason, since the utilisation of force-voltage and velocity-current analogies permits mechanical behaviour to be conveniently evaluated by means of relatively familiar electrical network concepts. Consequently, characteristic expressions such as series and parallel resonance have developed to describe transducer behaviour from a purely electrical viewpoint. Examples of such modelling techniques include the transmission line approaches of Mason[1] and Leedom et al[2], both of which permit the study of multi-layered, unidimensional transducer assemblies.

However, electrical analogies suffer from some inherent disadvantages which can limit their flexibility when applied to certain aspects of piezoelectic transducer behaviour.[3] For example, the negative capacitance in the Mason model is unlike any real circuit element and in many instances, the physical nature behind the transduction process is masked by electric circuit topology. In addition, most equivalent circuit approaches are confined to the analysis of unidimensional transducer behaviour and as a result, cannot predict device performance when more than one dominant vibrational mode is present in the system.

More recently, a range of variational based methods[4] have been developed to overcome the problem of modelling multi-dimensional behaviour. However, such finite element approaches are computationally expensive and offer little insight into the nature of the transduction process. As a result, cause and effect relationships are often difficult to define and when coupled with the computational expense involved, the methods are often unattractive for many applications. However, for those instances where vibrational behaviour in a multi-dimensional, multi-layered, piezoelectric system requires study, variational based methods may offer the only feasible approach for simulation based design.

This paper describes an alternative strategy for modelling the behaviour of piezoelectric transducers. The technique, which utilises a linear systems, block diagram approach, is considered to overcome many of the disadvantages associated with the physical interpretation of some equivalent circuit treatments. Furthermore, the resultant transfer functions are amenable to discrete-time implementation, in the form of a recursive digital filter.

This results in a comprehensive simulation package for which input data may be synthesised with relative ease. It is also computationally inexpensive.

Before continuing, the following points should be noted. Firstly, the discrete-time, linear systems approach cannot conveniently cater for strongly coupled multi-dimensional behaviour or when significant shear forces are present. Consequently it cannot be viewed as a complete alternative to finite element methods. Secondly, for reasons of space, the model derivations are not presented here. The interested reader may consult the appropriate reference material.

2. THE THICKNESS-MODE TRANSDUCER

In this case, the device under consideration is a thin, piezoelectric slab subject to the physical constraints of linear, planar, lossless and uni-dimensional wave motion. Consequently, mechanical and electrical quantities such as stress (Γ), electrical field (E), strain (S) and electrical displacement (D) are assumed to vary only in a direction normal to the plane surface of the transducer, with each quantity governed by the following piezoelectric relationships[1].

$$\begin{aligned} \Gamma &= C^D - h_{33}D \\ E &= -h_{33}S + D/\epsilon^S \end{aligned} \qquad (1)$$

Where all quantities have their usual meaning.

Assuming that the device radiates directly into semi-infinite, real media positioned at each face, the piezoelectric relationships may be combined with the equations of motion to generate a wave equation which can be solved by Laplace Transform techniques. The mechanical boundary conditions are then applied to provide a series of transfer functions in the Laplace domain, representing transmission, reception and operational impedance[(3)]. These are then implemented in a linear systems, block diagram format.

As an illustration, consider the operational impedance of a thickness-mode device. This is defined as the ratio of voltage across the transducer to total current through the transducer and is described by the following transfer-function relationship. [(3,5)]

$$\begin{aligned} \bar{Z}_T &= \bar{V}_T / \bar{I}_T \\ &= \frac{1}{sCo} \cdot \left[1 - \frac{h^2 Co}{sZc} \cdot \left(\bar{K}_F \; T_F/2 + \bar{K}_B \; T_B/2 \right) \right] \end{aligned} \qquad (2)$$

Where Co is the device static capacitance; Zc is the device acoustic impedance; T_F and T_B are internal transmission coefficients for waves of force generated at the front and back faces respectively and s is the Laplace complex variable. Laplace transformation is denoted by the bar symbol. The parameters K_F and K_B describe mechanical reverberation occuring at each face of the device. As such they comprise reflection coefficients and delay terms.

That is,

$$\bar{K}_F = (1 - e^{-sT})\ (1 - R_B\ e^{-sT})/(1 - R_F\ R_B\ e^{-2sT})$$

$$\bar{K}_B = (1 - e^{-sT})\ (1 - R_F\ e^{-sT})/(1 - R_F\ R_B\ e^{-2sT}) \qquad (3)$$

Where,

$$R_F = (Z_c - Z_1)/(Z_c + Z_1)$$

$$R_B = (Z_c - Z_2)/(Z_c + Z_2)$$

R_F and R_B correspond to reflection coefficients for waves of force incident upon the front and rear faces respectively, while T is the transit time for mechanical waves to propagate across the transducer thickness.

Equation (2) may be represented by the admittance block diagram shown in Fig 1 from which the various electro-mechanical interactions may be summarised as follows.[5]

On application of a voltage (V_T) to the transducer electrodes, a current (I) flows through the bulk capacitance of the device. As a result of primary piezoelectric action, two forces (F_1 and F_2) are produced at the front and rear faces of the transducer respectively. A fraction of each force is transmitted into the surrounding media and the remainder back into the transducer, hence producing, as a result of secondary piezoelectric action, the feedback currents I_F and I_B at the input summing point. Since the feedback is shown as being positive, the resultant current through the transducer is the vector sum of an input current flowing through the bulk capacitance and the feedback currents produced by front and rear face displacements. The process is outlined in Fig 1 where the various relationships between force, particle displacement and current are clearly defined.

Block diagram representations of transmission, reception and multi-layered structures [6] may be derived and studied in a similar manner. Although this approach is essentially a frequency domain technique, time domain data may be obtained via a suitable transformation. This is described in a subsequent section.

3. TWO DIMENSIONAL TRANSDUCER STRUCTURES

The block diagram method may be extended to the study of transducer structures which exhibit two, loosely coupled modes of vibration. [7,8,9] A typical example is outlined in Fig 2 which describes a long tall, thin, piezoelectric parallepiped, typical of many piezoelectric array structures. For this configuration, the fundamental piezoelectric relationships may be given by the following matrix expression:-

$$\begin{bmatrix} \Gamma_1 \\ \Gamma_3 \\ E_3 \end{bmatrix} = \begin{bmatrix} c^D_{11} & c^D_{13} & -h_{31} \\ c^D_{13} & c^D_{33} & -h_{33} \\ -h_{31} & -h_{33} & \beta^S_{33} \end{bmatrix} \cdot \begin{bmatrix} S_1 \\ S_3 \\ D_3 \end{bmatrix} \qquad (4)$$

Where all symbols have their usual meaning and the suffixes 1 and 3 refer to the lateral (width) and thickness (height) dimensions respectively.

The operational impedance transfer function for this configuration is represented by the linear systems, admittance block diagram shown in Fig 3. In this figure, the parameters Z_{c1} and Z_{c3} correspond to mechanical impedances for the lateral and thickness dimensions respectively; while Ψ_{13} and Ψ_{31} are mechanical conversion factors which relate particle displacement in one dimension to force in the other. As such, they may be regarded as mechanical cross-coupling factors. [7,9] The parameter Φ is a piezoelectric conversion factor relating electrode voltage to force (or displacement to charge) in the lateral dimension Mechanical wave reverberation in each dimension is described by the functions K_1 and K_3. For example:-

$$\bar{K}_3 = T_F\bar{K}_F/2 + T_B\bar{K}_{B/2} \qquad (5)$$

The model thus represents the admittance function of a piezoelectric element sustaining two principal modes of vibration which are loosely coupled via mechanical and piezoelectric interaction. This may be illustrated by the following physical analysis. [10]

A quantity of charge is initially deposited on the device electrodes, as a result of the applied voltage and static capacitance Co. This primary charge then generates two functions of force as a result of piezoelectric action. Firstly, a function of primary thickness force occurs via the piezoelectric constant h and secondly, a lateral force is generated via the conversion factor Φ/Co. Consider now the forces which are initially generated in the thickness direction via the block h. These are converted to reverberating functions of particle displacement by means of the block $\bar{K}/sZc_3$. This in turn is responsible for the generation of secondary and tertiary charge components, represented by the feedback portions of the block diagram. For example, the block hCo converts the particle displacements to secondary functions of charge, in an identical manner to that of the thickness-mode device. In addition, the thickness particle displacements generate, via mechanical cross coupling, (shown by the block $\Psi_{31}\ \bar{K}_1\ /\ sZc_1$) a reverberating function of particle displacement in the lateral dimension. This in turn, generates a tertiary component of charge at the input summing point by means of the piezoelectric conversion factor Φ and furthermore, generates additional components of force in the thickness dimension via the mechanical conversion factor Ψ_{13}.

The primary lateral forces also undergo a similar interactive procedure to generate secondary and tertiary charge components as indicated in the diagram. It may be observed that for no lateral interaction ($\Phi = 0, \Psi_{31} = 0$), the behaviour corresponds to that of a purely thickness mode device, as described in Fig 1.

Similar models for transmission and reception in multi-layered media have been derived,[7,8,9] and when coupled with a suitable simulation package, are extremely useful for the analysis and study of array element behaviour. Moreover, the factors which contribute to secondary/tertiary charge generation (and hence electro-mechanical coupling) are readily isolated and provided that the appropriate constants are available, permit the rigid assessment of different piezoelectric materials.

4. DISCRETE-TIME IMPLEMENTATION OF THE SYSTEM TRANSFER FUNCTIONS

Many practical transducer applications require a detailed knowledge of the time-domain transient response characteristics. As there is often great difficulty in obtaining an inverse Laplace transform solution for the complex transcendental transfer functions associated with piezoelectric transduction, the technique most often employed in obtaining time-domain response is the application of an IFFT routine to suitably sampled and sorted frequency domain data. Although effective, such methods can be computationally expensive at high sampling frequencies (which are often required) and moreover, all input data must be specified in the frequency domain. This is not always convenient, especially where complex input functions are involved.[11]

An alternative technique involves implementation of the system transfer functions directly in the time domain. This is achieved by the application of Z-transform and digital filter theory to obtain a discrete time model of the transducer transfer function. In general, piezoelectric transducer transfer functions comprise a complex mixture of delay operations and high order s-domain continuous functions. The delay operators, which represent mechanical reverberation, can be conveniently represented in the Z-domain by the straightforward impulse - invariance mapping $Z = e^{sT_p}$, where T_p is the sampling period. However, the continuous s-domain functions, which arise mainly from electrical loading, require more care if the time domain response is to be preserved accurately. This may be achieved by adopting a modified impulse invariance approximation, (the Z-form method) originally derived by Boxer and Thaler.[12] The Z-form method involves a substitution for the various powers of s as indicated in Table 1. As a result, the transducer transfer functions may be implemented in the Z-domain and after inverse transformation, realised as a recursive digital filter.

To illustrate the technique, consider the reception transfer function for a thickness mode transducer which is subject to an input function of force (F_1) and has an arbitrary electrical load (z_E) connected across the electrodes. That is,

$$\frac{\bar{V}}{\bar{F}_1} = \frac{-T_F\bar{U}\bar{k}_F h/sz_c}{1 - h^2 Co(T_F\bar{k}_F + T_B\bar{k}_B)/2sz_c(1 + sCo\bar{z}_E)} \qquad (6)$$

where $\bar{U} = sCo\bar{z}_E/(1 + sCo\bar{z}_E)$.

Eq.(6) may be re-written as follows:-

$$\frac{\bar{V}}{\bar{F}_1} = \frac{-hT_F/z_c)(\bar{U}/s)\bar{k}_T}{1 + X_k\bar{G}\,\bar{D}} \qquad (7)$$

where $\bar{D} = T_F\bar{k}_F + T_B\bar{k}_B$. This function comprises solely of scalar constants and a series of delay terms:

$$X_k = -h^2Co/2Zc$$

$$\bar{G} = 1/\ s(1 + sC_o\bar{z}_E)\ ,$$

This function is comprised solely of scalar constants and continuous terms in s.

Consequently, the transfer function may be expressed in the following general format:

$$\bar{A} = \frac{\sum_{m=o}^{M} \sum_{n=o}^{N} a_{mn}s^n \,.\, e^{-msT}}{\sum_{l=o}^{L} \sum_{n=o}^{N} b_{ln}s^n \,.\, e^{-lsT}} \qquad (8)$$

where a and b are scalar coefficients; N is the highest index of s occurring in either numerator or denominator; M is the highest delay term index occurring in the denominator.

In order to implement the Boxer and Thaler method, (8) must be expressed in negative powers of s. That is,

$$\bar{A} = \frac{\sum_{m=o}^{M} \sum_{n=o}^{N} a_{mn}s^{-(N-n)} \,.\, e^{-msT}}{\sum_{l=o}^{L} \sum_{n=o}^{N} b_{ln}s^{-(N-n)} \,.\, e^{-lsT}} \qquad (9)$$

Eq.(9) may be transformed directly into the Z-domain by means of a suitable Z-form, Q-matrix mapping [13] which may be implemented automatically via a digital computer.

This yields the following Z-domain transfer function

$$H(z) = \frac{\sum_{n=o}^{M} c_n z^{-n}}{\sum_{n=o}^{N} d_n z^{-n}} \qquad (10)$$

The inverse response is obtained by directly programming (10) to produce a result which is in the form of a recursive digital filter. That is,

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{n=o}^{N} e_n z^{-n}}{1 + \sum_{n=1}^{N} f_n z^{-n}}$$

Inverse z-transforming yields

$$y(i) = \sum_{n=o}^{N} e_n x(i-n) - \sum_{n=1}^{N} f_n y(i-n) \qquad (11)$$

5. EXPERIMENTAL RESULTS

The linear systems approach to transducer modelling has been comprehensively evaluated and a wide range of experimental and simulated data has been published. (See for example references 3, 6, 9 and 11). In consequence, this section is restricted to a selection of experimental results based around two-dimensional array type structures.

Consider firstly Figure 4 which illustrates the operational impedance spectral magnitude and phase characteristics of a 3-layered transducer structure, immersed in an oil bath and comprising two PZT-5A devices separated by an epoxy bondline. All other material parameters are as shown in the figure, with the active transducer possessing a width/thickness configuration ratio of 0.45. The simulated and experiments are observed to be in good agreement, illustrating the general accuracy of the modelling technique.

As another example, consider Figure 5 which illustrates the time and frequency domain response of an isolated element operating into water. The device was driven via a switching mosfet pulser circuit and the response measured using a PVDF membrane hydrophone, both of which were included in the overall simulation. Experimental and simulated data are observed to be in close agreement. Similarly, Figure 6 illustrates the pulse-echo response of a single-element within an ultrasonic array. Once again, good agreement may be observed.

Finally, it is interesting to compare the linear systems approach with an alternative method of piezoelectric simulation. Figure 7 shows the dependence of the electro-mechanical coupling factor on configuration ratio using the linear systems model of Figure 2 and the finite element approach adopted by SATO[14]. For convenience, the experimental results for PZT-5A material are shown on the same diagram. The linear systems approach isagain shown to produce good results.

6. CONCLUDING REMARKS

Some applications of the linear systems approach to the modelling of piezoelectric transducer structures have been presented. This method is considered to offer improved insight into the physical mechanisms behind the transduction process and when implemented in discrete-time format, is computationally inexpensive. It should be stressed however, that in the present formulation the approach relies on linear, loosely coupled, mechanical wave propagation where shear forces are largely absent. Although the theory may be extended to cover other modes of propagation, the resultant transfer functions become so complex that numerical techniques are required for their solution. In such instances, there is often little advantage in adopting the present approach.

The various modelling packages developed of Strathclyde have found widespread application within the Ultrasonics Research Group. This work includes; computer aided design of ultrasonic systems for biomedicine, sonar and non-destructive testing; the development and testing of digital signal processing algorithms for deconvolution, inversion and correlation-based filtering; the development of array and image processing techniques for ultrasonic systems. Current research activity in the field of transducer modelling includes the simulation of multi-layered active piezoelectric devices, monolithic arrays and composite transducer/array assemblies.

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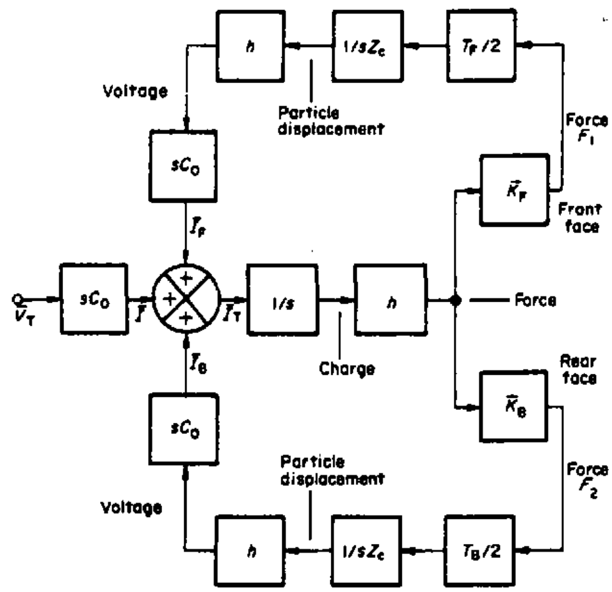


Figure 1

TABLE I
z-FORM RELATIONSHIPS

s^{-k}	z-Form
s^{-1}	$(T_p/2)\left(\frac{1+z^{-1}}{1-z^{-1}}\right)$
s^{-2}	$(T_p^2/12)\left(\frac{1+10z^{-1}+z^{-2}}{(1-z^{-1})^2}\right)$
s^{-3}	$(T_p^3/2)\left(\frac{z^{-1}+z^{-2}}{(1-z^{-1})^3}\right)$
⋮	⋮

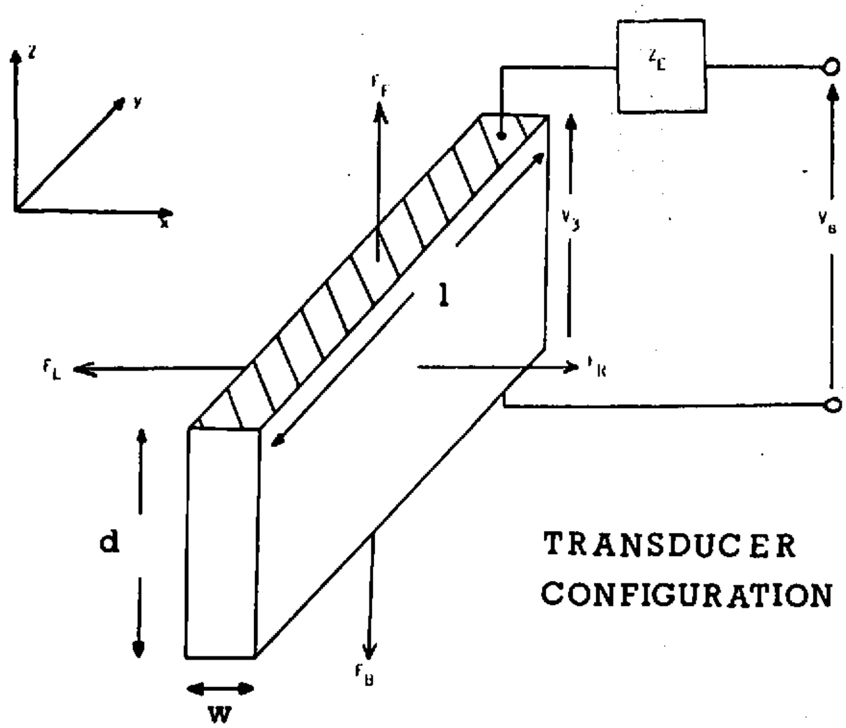


Figure 2

TRANSDUCER CONFIGURATION

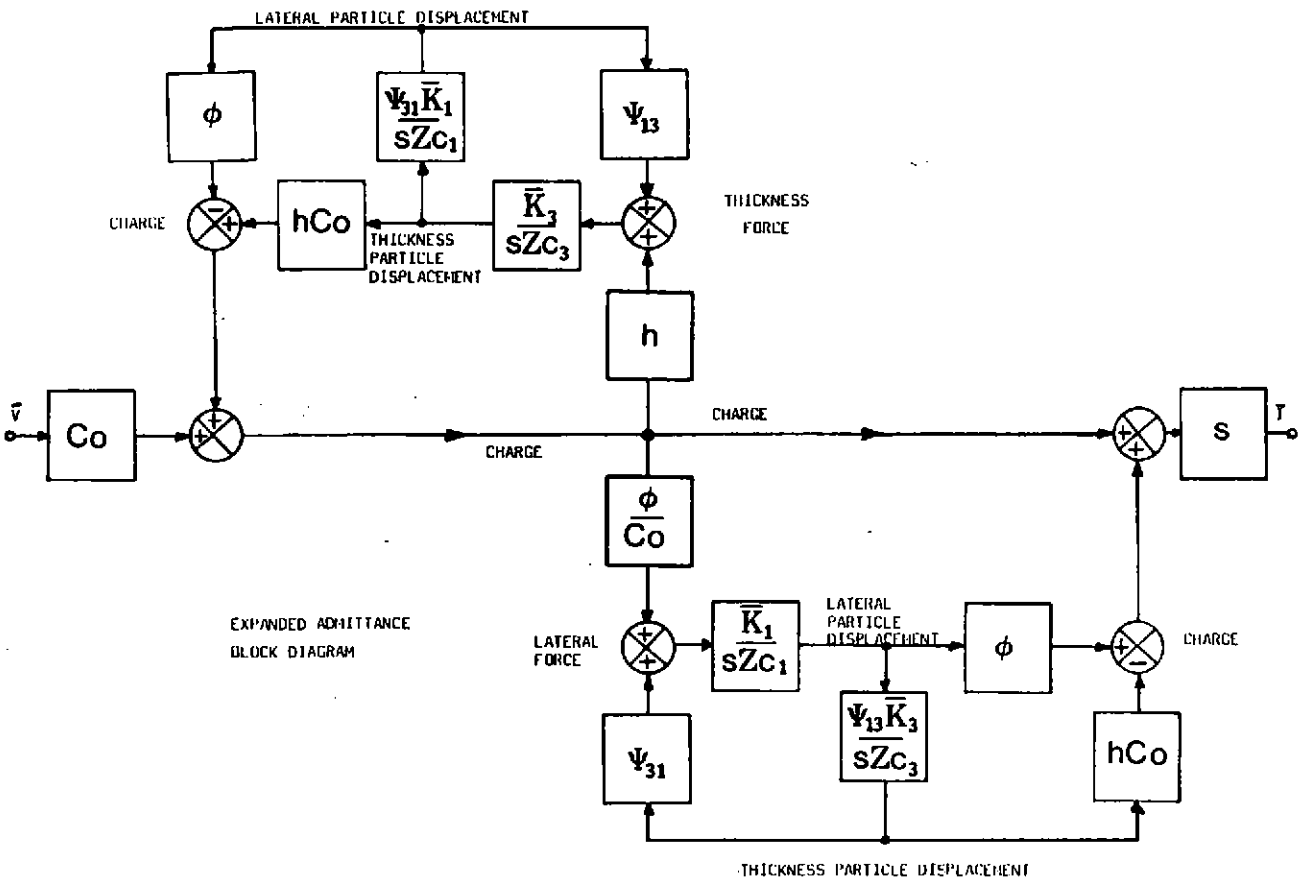
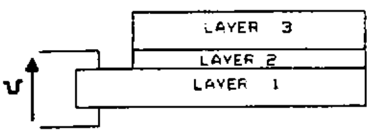

LATERAL PARTICLE DISPLACEMENT
φ
Ψ31 K̄1 / sZc1
Ψ13
CHARGE
hCo
THICKNESS PARTICLE DISPLACEMENT
K̄3 / sZc3
THICKNESS FORCE
h
V̄
Co
CHARGE
CHARGE
s
Ī
φ / Co
EXPANDED ADMITTANCE BLOCK DIAGRAM
LATERAL FORCE
K̄1 / sZc1
LATERAL PARTICLE DISPLACEMENT
φ
CHARGE
Ψ31
Ψ13 K̄3 / sZc3
hCo
THICKNESS PARTICLE DISPLACEMENT

Figure 3

SIMULATED AND MEASURED OPERATIONAL IMPEDANCE SPECTRA
FOR A THREE-LAYERED STRUCTURE IN THE THICKNESS DIRECTION

CONFIGURATION PARAMETERS		
LAYER 1		
MATERIAL	:	PZT-5A
HEIGHT	:	0.95mm
WIDTH	:	0.69mm
LENGTH	:	12mm
LAYER 2		
MATERIAL	:	EPOXY
HEIGHT	:	60μm
WIDTH	:	0.69mm
LENGTH	:	12mm
LAYER 3		
MATERIAL	:	PZT-5A
HEIGHT	:	1.54mm
WIDTH	:	0.69mm
LENGTH	:	10mm
BACKING	:	OIL (all faces)



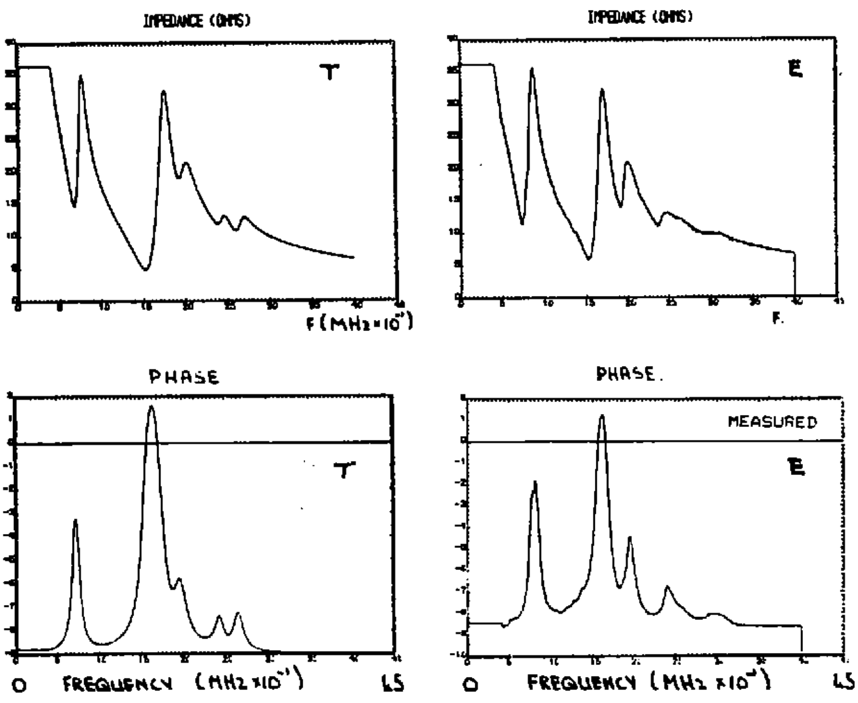
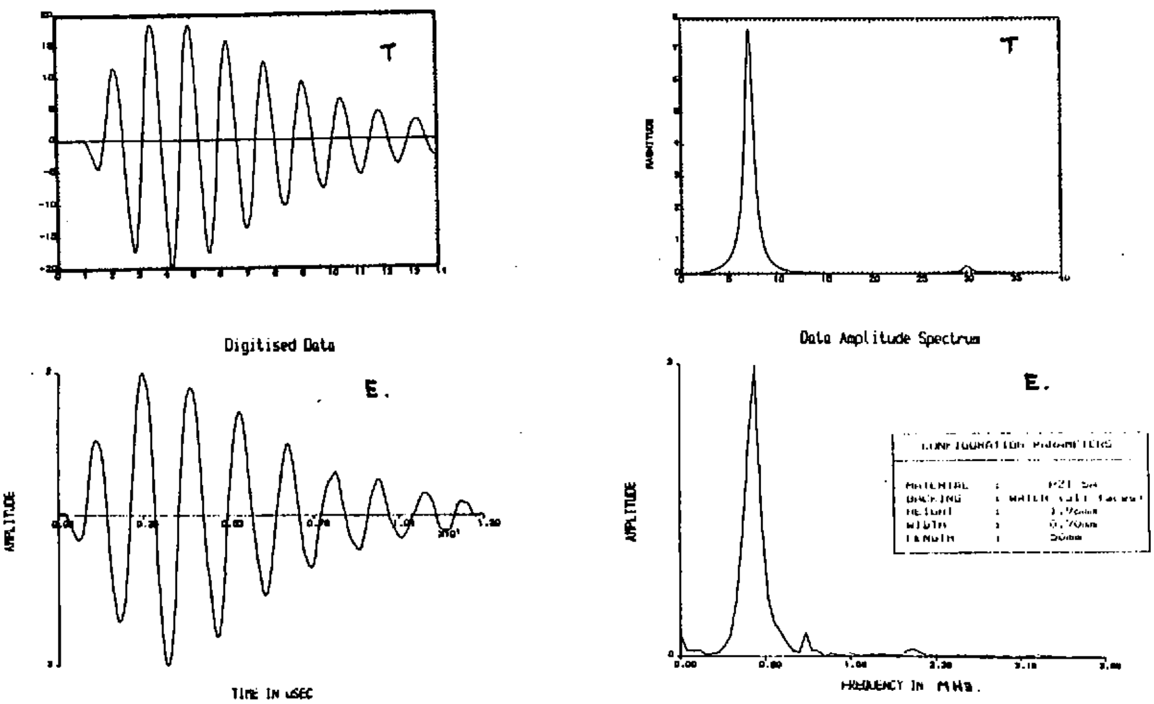


Figure 4

Figure 5 SIMULATED AND MEASURED FORCE OUTPUT FROM AN ISOLATED TALL, THIN TRANSDUCER OPERATING INTO WATER



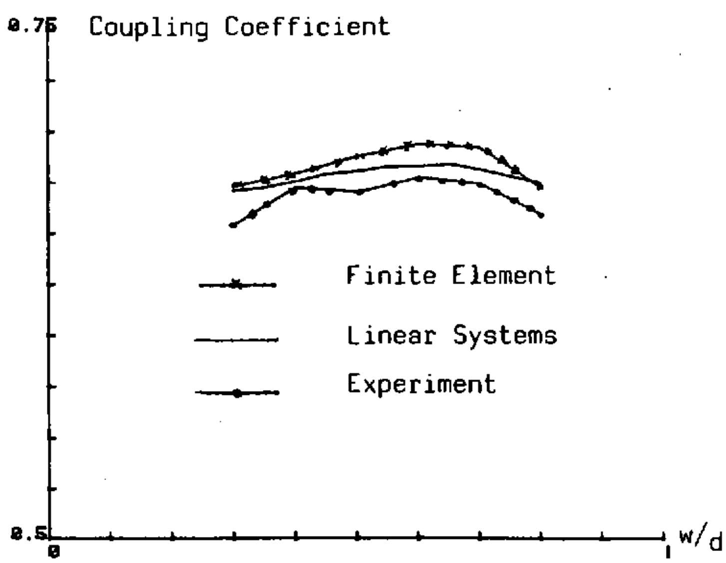


Figure 7

SIMULATED AND MEASURED PULSE ECHO RESPONSE FOR A MOUNTED ARRAY TRANSDUCER, OPERATING INTO WATER VIA A FRONT FACE LAYER

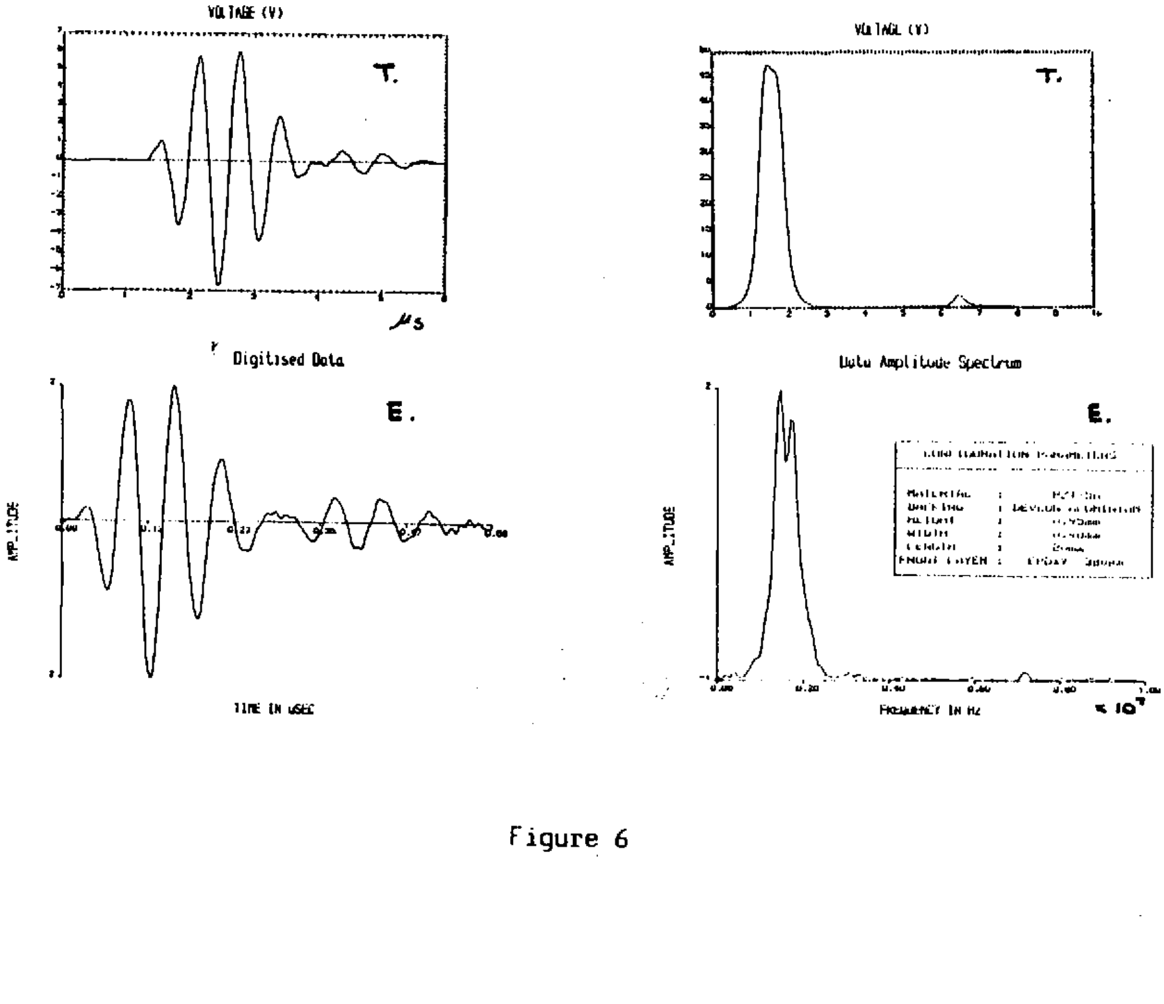


Figure 6