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ESTIMATION ON THE PROBABILITY DISTRIBUTION OF VON MISES STRESS FOR COMPLEX STRUCTURE UNDERGOING MULTIPLE-DIMENSIONAL RANDOM EXCITATIONS

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In this paper, the probability distribution of Von Mises stress for plate-shell complex structure subjected to multiple-dimensional random excitations is studied. Firstly, for a linear system, each stress component is unrelated with each other and can be regarded to be zero mean Gaussian process. Thus the variance of each stress component can be obtained from the respective power spectral density. Secondly, it is assumed that the squared process of each stress component is also uncorrelated with each other. According to the basic principle of random process, the means and variances for squared processes of stress components can be derived. Consequently, the mean and variance of Von Mises stress can be further obtained. Thirdly, it is supposed that Von Mises stress of structure follow the two-parameters Weibull distribution and then a method is developed to determine the Weibull distribution parameters. The numerical results show the probability density distribution of Von Mises stress of complex structure follows two-parameters Weibull distribution well. The probability density of Von Mises stress presented in the work provides the theoretical basis for strength estimation of complex structure subjected to multiple-dimensional random excitations.

Keywords: multiple-dimensional random excitations, random vibration analysis, Von Mises stress, probability distributions, Weibull distribution

1. Introduction

Random vibration is a kind of primary environmental excitation affecting advanced aircraft and spacecraft structures for whole flight, such as aerodynamic noise of a rocket fairing and thrust pulsation generated by combustion instability of a rocket engine during a practical launch of spacecraft. Generally, these random excitations possess the characteristics of high level and wide frequency band. The strength is an important index in the design of spacecraft structures and influencs the capability and reliability of aircraft and spacecraft structures directly. Wada [1] showed that the equivalent peak value of stress method are closer to the actual condition rather than the equivalent peak value of acceleration method. In fact, the real environment is mostly multiple-dimensional random excitations instead of single-dimensional loadings, which can cause structures to exhibit complex response characteristics, and stresses will not be uni-axial or bi-axial, but multi-axial in the most cases.

Alternatively, it is possible and efficient to calculate an equivalent stress from the multi-axial stresses with popular used equivalent stresses, that is, Von Mises stress. Dan Gregor, Fernando Bitsie and David O. Smallwood [2] suggested the maximum Von Mises stress and the location were different for the combined axis loads versus the single axis loads. Consequently, it is improper to substitute for multiple-dimensional random environment with single-dimensional random environment and the equivalent peak value of Von Mises stress method as the significant parameter to design and to assure success in multiple-dimensional random environment is required. It is the key problem to this method to determine the probability density function (pdf) of Von Mises stress.

Preumont [3] defined the zero mean Gaussian random process value as the equivalent von Mises stress to estimate the high-cycle fatigue life of metallic structures subjected to a random multiaxial loading. But the Von Mises stress is a non-zero mean positive value, which have different characteristics from Gaussian random process, thus the factor of mean of Von Mises stress cannot be taken into account with the zero mean value of the equivalent von Mises stress method. Actually, even in linear structures under Gaussian excitations, the Von Mises stress is a non-Gaussian random process, a nonlinear function of the linear stress components, whose the probability distribution is hard to determine. In the Refs.[4-5], methods for obtaining approximations for outcrossing probabilities of non-Gaussian processes have been discussed. Grigoriu [6] obtained approximate estimations for the mean outcrossing of non-Gaussian translation processes by studying the outcrossing characteristics of a Gaussian process obtained from Nataf's transformation of the parent non-Gaussian process. Segalman [7-9] have derived an expression of the cumulative probability distribution of Von Mises stress resulting from random excitations with Gaussian of zero mean. JIN YI-shan [10] and SHA Yun-dong [11] presented the probability distribution of Von Mises stress for plate structure or thin-walled structure undergoing single-dimensional random loadings process accords approximately with two-parameter Weibull distribution.

In this paper, the probability distribution of Von Mises stress for plate-shell complex structure subjected to multiple-dimensional random excitations is studied. Analysis suggests that the probability density function of Von Mises stress process of complex structure subjected to multiple-dimensional random environment accords approximately with two-parameter Weibull distribution, the formula for calculating Weibull parameters are given. As a result, the probability density function of Von Mises process of complex structure subjected to multiple-dimensional random excitations can be determined. It lays the foundation for determining the peak probability density of Von Mises process and strength estimation of complex structure undergoing multiple-dimensional random excitations.

2. Probability distribution of Von Mises stress process

2.1 Von Mises Stress Process

In a general three dimensional stress field the Von Mises stress is given by:

$$S_v^2 = S_x^2 + S_y^2 + S_z^2 - S_x S_y - S_y S_z - S_x S_z + 3S_{xy}^2 + 3S_{yz}^2 + 3S_{xz}^2 \tag{1}$$

where S_v is Von Mises stress, S_x, S_y and S_z are the normal stress components, S_{xy}, S_{yz} and S_{xz} are the shear stress components. These stress components are zero mean Gaussian random processes. It is seen that the Von Mises stress is a non-Gaussian random process as the relationships between the Von Mises stress and the linear stress components is nonlinear.

In the reference [10], it is verified that the probability distribution of Von Mises stress obeys two-paramaters Weibull distribution for plate structure in single-dimensional random vibration. Here, it is supposed that Von Mises stress of complex structure subjected to multiple-dimensional random excitations also follows the two-parameters Weibull distribution.

The probability density function of a Weibull random variable is

$$p(s_v) = \frac{\gamma}{\alpha}\left(\frac{s_v}{\alpha}\right)^{\gamma-1} \exp\left[-\left(\frac{s_v}{\alpha}\right)^{\gamma}\right], \qquad s_v \geq 0 \tag{2}$$

The mean and variance of a Weibull random variable can be expressed as

$$E(s_v) = \alpha\Gamma\left(1+\frac{1}{\gamma}\right) \tag{3}$$

and

$$\sigma^2(s_v) = \alpha^2\left[\Gamma\left(1+\frac{2}{\gamma}\right) - \Gamma^2\left(1+\frac{1}{\gamma}\right)\right] \tag{4}$$

In order to prove the above assumption, firstly, the time histories of six stress components (zero mean Gaussian process) of plate-shell complex structure subjected to multiple-dimensional random excitations are simulated by Monte Carlo method. Secondly, the time history of Von Mises stress can be obtained with Von Mises stress criterion. Thirdly, the mean, the variance and the probability density of Von Mises stress are identified by its time history, and the parameter α and γ of Weibull distribution are identified by substituting the mean and the variance into Eq.(3) and Eq.(4). Finally, the probability density of Von Mises stress and Weibull distribution is obtained as shown in Fig.1. It is found that the statistical result of the time histories of Von Mises stress coincides with Weibull distribution curve well, thus it is reasonable to suppose Von Mises stress of plate-shell complex structure subjected to multiple-dimensional random excitations to follow the two-parameters Weibull distribution. The probability distribution of Von Mises stress can be obtained as long as the parameter α and γ of Weibull distribution are identified by this method.

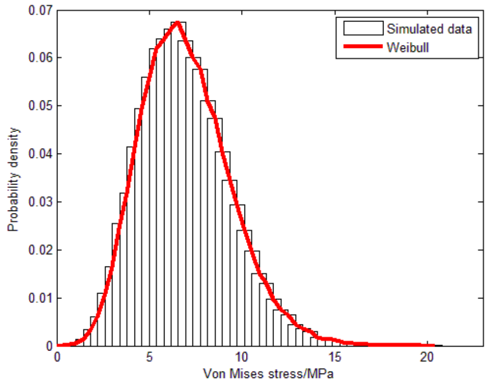


Figure 1: the probability density of Von Mises stress curve.

2.2 Squared Processes of Stress Components

It is shown that the right hand side of Eq.(1) contains the squared processes of six stress components. According to the expression of Eq.(1), the mean and the variance of the squared processes of Von Mises stress can be expressed as:

$$E(S_v^2) = E(S_x^2) + E(S_y^2) + E(S_z^2) - E(S_xS_y) - E(S_yS_z) - E(S_xS_z) + 3E(S_{xy}^2) + 3E(S_{yz}^2) + 3E(S_{xz}^2) \tag{5}$$

$$\sigma^2(S_v^2) = \sigma^2\left(S_x^2 + S_y^2 + S_z^2 - S_xS_y - S_yS_z - S_xS_z + 3S_{xy}^2 + 3S_{yz}^2 + 3S_{xz}^2\right) \tag{6}$$

It is well known that for a linear system, each stress component is uncorrelated with each other and can be regarded to be the zero mean Gaussian random process. Thus the variance of each stress component can be obtained from the respective power spectral density. Similarly, it is assumed that the squared process of each stress component is also uncorrelated with each other. Therefore, the covariance of each stress component is zero. Based on the properties of generalized covariance, the mean and the variance of random variable $X$and Y is given by:

$$E(XY) = E(X)E(X) \tag{7}$$

$$\sigma^2\left(\sum_i X_i\right) = \sum_i \sigma^2(X_i) \tag{8}$$

according to Eq.(7) and (8), Eq.(5) can be expressed as:

$$\begin{aligned} E(S_v^2) &= E(S_x^2) + E(S_y^2) + E(S_z^2) - E(S_x)E(S_y) \\ &\quad - E(S_y)E(S_z) - E(S_x)E(S_z) + 3E(S_{xy}^2) + 3E(S_{yz}^2) + 3E(S_{xz}^2) \end{aligned} \tag{9}$$

$$\sigma^2\left(S_v^2\right)=\sigma^2\left(S_x^2\right)+\sigma^2\left(S_y^2\right)+\sigma^2\left(S_z^2\right)+\sigma^2\left(S_xS_y\right) \\ +\sigma^2\left(S_yS_z\right)+\sigma^2\left(S_xS_z\right)+9\sigma^2\left(S_{xy}^2\right)+9\sigma^2\left(S_{yz}^2\right)+9\sigma^2\left(S_{xz}^2\right) \tag{10}$$

Consequently, if the means and variances for squared processes of stress components are given, the mean and variance of Von Mises stress can be further obtained.

For the zero mean Gaussian random process $X \sim N\left(0,\sigma\right)$, we have

$$E\left(X^k\right)=\begin{cases} 0 & k \text{ is odd} \\ (k-1)!!\sigma^k & k \text{ is even} \end{cases}$$

where $(k-1)!!=(k-1)\times(k-3)\times\cdots\times 3\times 1$, and

$$E\left(X^2\right)=\sigma^2 \tag{11}$$

$$E\left(X^4\right)=3\sigma^4 \tag{12}$$

when $k=2,4$,the variance is given by

$$\sigma^2\left(X^2\right)=E\left(X^4\right)-\left[E\left(X^2\right)\right]^2=2\sigma^4 \tag{13}$$

2.3 Parameter Identification of Weibull Distribution

The origin moment of random variable X is given by

$$E\left(X^k\right)=\int_{-\infty}^{\infty}x^k f\left(x\right)dx \tag{14}$$

where $f\left(x\right)$ is the probability distribution of X .

It can be obtained

$$E\left(s_v^2\right)=\alpha^2\Gamma\left(1+\frac{2}{\gamma}\right) \tag{15}$$

$$E\left(s_v^4\right)=\alpha^4\Gamma\left(1+\frac{4}{\gamma}\right) \tag{16}$$

where Γ is the gamma function.

The variance of s_v^2 can be obtained

$$\sigma^2\left(s_v^2\right)=\alpha^4\left[\Gamma\left(1+\frac{4}{\gamma}\right)-\Gamma^2\left(1+\frac{2}{\gamma}\right)\right] \tag{17}$$

In a summary, the mean and the variance of squared processes of stress components are firstly obtained by Eq.(11) and Eq.(13). Secondly, the mean and the variance of squared processes of Von Mises stress are obtained by Eq.(9) and Eq.(10). Thirdly, the parameter α and γ of Weibull distribution are identified by substituting the mean and the variance of squared processes of Von Mises stress into Eq.(15) and Eq.(17). Finally, the probability distribution of Von Mises stress can be determined.

3. Numercial Example

To check the validity of analytic methods, consider the example of a plate-shell steel complex structure shown in Fig.2. The structure is clamped around the base and subjected to an acceleration base excitation defined by Power Spectral Density (PSD) shown in Fig.3. The multiple-dimensional random excitations, that is, X, Y and Z axis inputs at the same input levels are applied. The three support accelerations are uncorrelated. The first thirty-two modes of structure are within the bandwidth of the excitation. The total of 63 shell elements have been used in the finite element discretization, and combine 14 spring element with enough stiffness is used to simulate clamped boundary condition. The thickness of the structure is 7mm. In this case, the resulting von Mises probability distribution is evaluated at one point (i.e. node 1234).

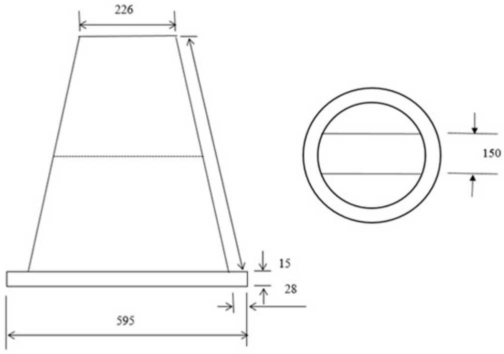


Figure 2: The front view and top view of the geometry.

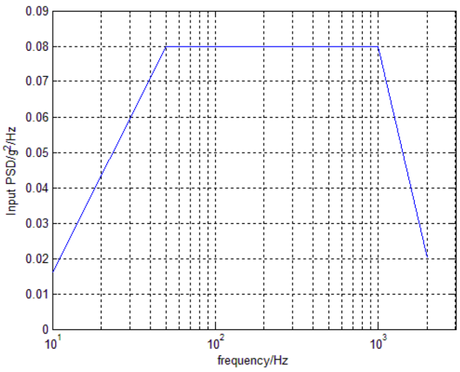


Figure 3: Acceleration PSD imposed at the base of the structure.

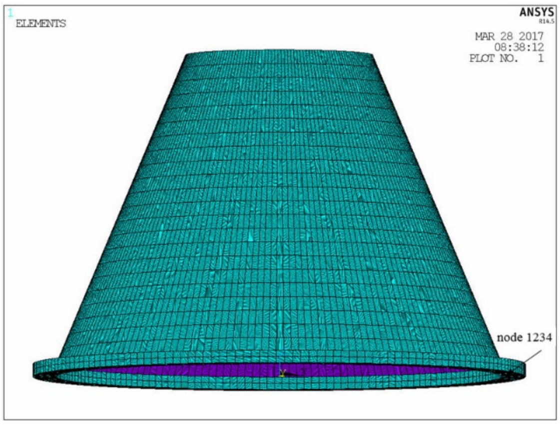


Figure 4: Finite element model of the structure.

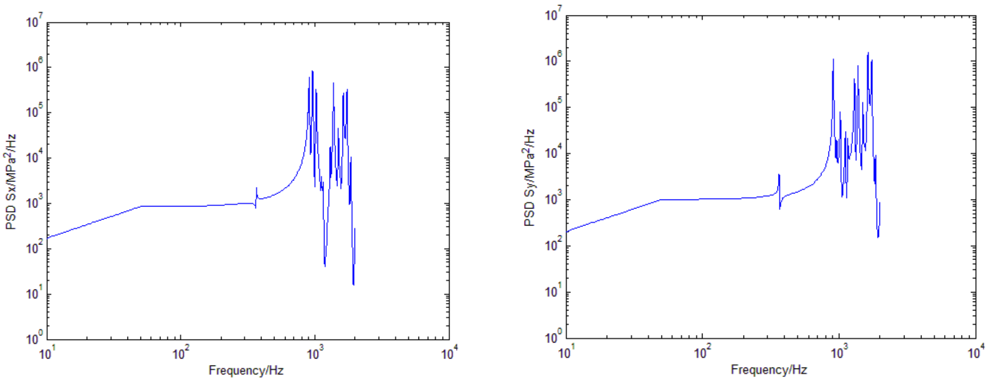
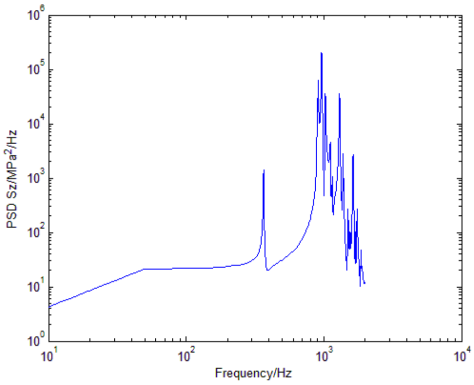


Figure 5: The PSD of S_x.

Figure 6: The PSD of S_y.



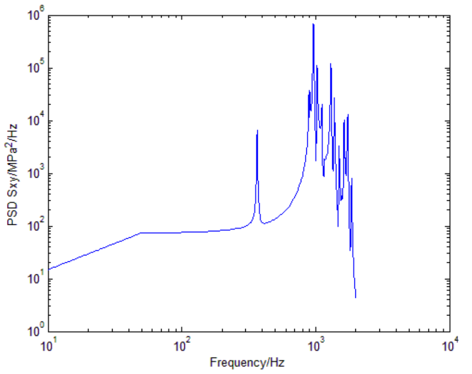
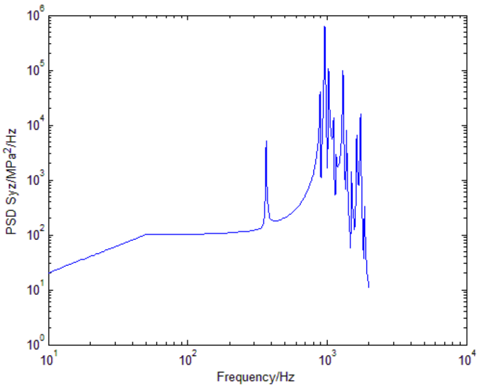


Figure 7: The PSD of S_z.

Figure 8: The PSD of S_{xy}.



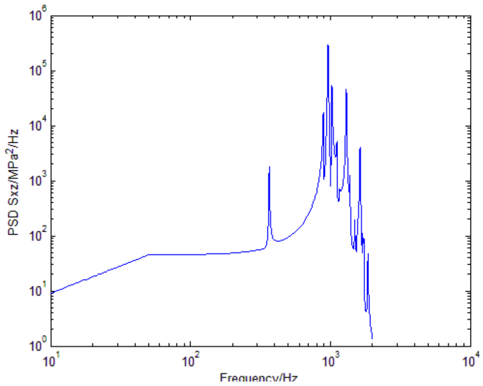


Figure 9: The PSD of S_{yz}.

Figure 10: The PSD of S_{xz}.

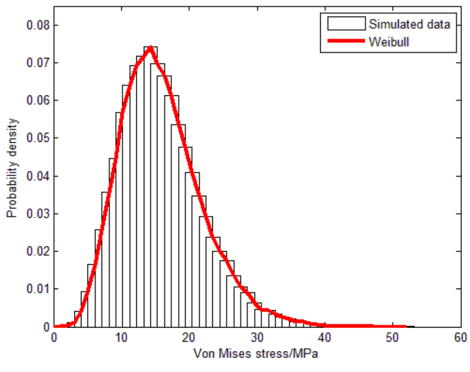


Figure 11: the probability density of Von Mises stress.

Fig.5-Fig10 shows the PSD of the six stress components obtained from ANSYS in one selected element. Fig.11 gives the result of the probability distribution of Von Mises stress. It is seen that the probability distribution of Von Mises stress coincides with two-parameters Weibull distribution curve well. The numerical simulation of the mean of squared processes of Von Mises stress is 15.88MPa, the numerical simulation of the variance of squared processes of Von Mises stress is 35.32MPa^2. The mean of supposed two-parameters Weibull distribution is 15.85 MPa. The variance of supposed two-parameters Weibull distribution is 36.84MPa^2. The numerical simulation results are in good agreement with the supposed two-parameters Weibull distribution results.

4. Conclusions

It is found that the probability density distribution of Von Mises stress of complex structure subjected to multiple-dimensional random excitations coincides with two-parameters Weibull distribution curve well. It means that the two-parameters Weibull distribution can be used to describe the probability density distribution of Von Mises stress of structure in the case of multiple-dimensional random excitations. Using this method, the parameter of Weibull distribution can be defined easily by the PSD of six stress components to obtain the probability density distribution of Von Mises stress. According to the results, the peak probability density of Von Mises process and strength estimation of complex structure undergoing multiple-dimensional random excitations can be determined further.

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REFERENCES

1 BK Wada, Historical overview of structural modelling, design loads and testing of spacecraft, *AIAA paper, 2000-1773*, (2000).

2 Dan Gregor, Fernando Bitsie and David O. Smallwood, Comparision of the response of a simple structure to single axis and multiple axis random vibration inputs, *Proceedings of the 79th Shock and Vibration Symposium*, Orlando, FL, USA, (2008).

3 X. Pitoiset, A. Preumont, Spectural methods for multiaxial random fatigue analysis of metallic structures [J], *International Journal of Fatigue*, 541-550, (2000).

4 Breitung K, Rackwitz R, Nonlinear combination of load processes [J], *Journal of Structural Mechanics*, 10(2), 145-166, (1982).

5 Pearce H T, Wen Y K, On linearization points for nonlinear combination of stochastic load processes [J], *Structural Safety*, 2(3), 169-176, (1985).

6 Grigoriu M. Crossings of non-Gaussian translation processes [J], *Journal of Engineering Mechanics*, 110(4), 610-620, (1984).

7 Segalman D, Reese G, Estimating the probability distribution of Von Mises stress for structures undergoing random excitation Part 1: derivation [R], Anaheim CA, *United States: International Mechanical Engineering Congress and Exposition*, (1998).

8 Segalman D, Reese G, Field R, Jr. ,et al. Estimating the probability distribution of Von Mises stress for structures undergoing random excitation Part 2: example calcuations [R], Anaheim CA, *United States: International Mechanical Engineering Congress and Exposition*, (1998).

9 Segalman D, Reese G, Field R, Jr. ,et al. Estimating the probability distribution of Von Mises stress for structures undergoing random excitation [J], *Journal of Vibration and Acoustics*, 122(1), 42-48, (2000).

10 JIN Yishan, LI Lin, Probability density function of Von Mises stress for structures undergoing random excitation [J], *Chinese Journal of Applied Mechanics*, 21(3), 13-16, (2004). (in Chinese)

11 SHA Yundong, GUO Xiaopeng, ZHANG Jun, Estimation method for random sonic fatigue life of thin-walled structure of a combustor liner based on stress probability distribution [J], *Journal of Aerospace Power*, 26(4), 727-734, (2011).