Proceedings of the Institute of Acoustics

INPUT POWER SUPPRESSION OF VIBRATING INFINITE BEAMS AND PLATES BY SECONDARY SOURCES

G. Pavić

CETIM, Acoustics Department, Senlis, France

1. INTRODUCTION

One of the proposed strategies of active control of acoustical sources consists in minimisation of total power output due to both the primary and the control excitations, [1]. Such type of control, applied to finite closed spaces which exhibit a distinct modal behaviour, results in a reduction of the global energy level. The control excitations are related to primary excitations by a simple relationship. In the case of mechanical systems this relationship can be put to a matrix form analogous to one established for the acoustical sources, [2]

$$F_c = - R_{cc} \cdot R_{ce} \; F_e \qquad (1)$$

where F denotes force vector, R the real part of mobility matrix, while subscripts c and e refer to positions of control and primary excitations respectively. Eq. (1) is valid for any linear system, closed or infinite.

In this paper, the power minimisation conditions for certain mechanical infinite and semi-infinite systems are analysed with the aim of investigating possibilities for a complete cancellation of total power input.

2. CONTROL OF BEAMS

Consider a straight beam, excited by a normal force at the origin $x = 0$. The beam is assumed to extend without limits to both sides from the origin. The power input in this case is the product of the excitation force of F_0 and the velocity at $x = 0$:

$$P_f = C \, F_0^2 \qquad (2)$$

where the subscript "f" stands for the "free" case, while C is a factor inversely proportional to the square root of frequency.

Suppose that it is requested to cancel energy flow in the positive direction of the beam axis.To do so, an actuator is placed at a distance l from the origin.

Using the formula for energy flow in a beam, [3], a condition for flow cancellation away from the actuator, $x > l$, can be found which yields a simple relationship between the amplitudes of the excitation force F_0 and the control force F_1 :

$$F_1 = - F_0 \cdot \exp(-jkl) , \quad k \text{ - wave number} \qquad (3)$$

Thus the control force, which prevents energy propagation on one side of the beam, matches the excitation force in amplitude but lags in phase behind it. For a trivial case when the control and the excitation positions coincide, i.e. $l = 0$, the two forces have to be exactly out of phase to produce energy cancellation. The phase shift between F_1 and F_0 is frequency dependent. The total power, i.e. the sum of the powers supplied by the primary excitation and the actuator reads

INPUT POWER SUPPRESSION OF VIBRATING INFINITE BEAMS AND PLATES BY SECONDARY SOURCES

$$P = P_0 + P_1 = 2C\, F_0^2 \sin^2 (kl) = 2\, P_f \sin^2 (kl) \quad (4)$$

It can be seen that the control excerted by a single actuator suppresses in this case the energy flow on one side of the beam but allows energy travel to the other side. The actuator acts essentially as an energy sink (negative input power), absorbing strictly the same amount of energy as the amount input by the excitation to the control side of the beam. Here the net energy flow fluctuates between zero and twice the amount of the free flow (i.e. the net flow without control applied) P_f. Since $< \sin^2 > = 1/2$, the average value of P equals exáctly P_f. The "corrective" power from the actuator approaches zero at higher frequencies where the actuator acts as a reactive load.

At certain frequencies where $kl = n\pi$, $n = 1, 2, 3...$, both input powers P_0 and P_1, and thus the total power P, reduce to zero. This phenomenon results from the mutual interference of travelling waves induced by the excitation and the control forces. Unlike for the case $l \longrightarrow 0$, here vibrations exist along the beam at $x = 0$, but these do not carry energy because the controlled part of the field contains only evanescent waves.

If the actuator's force is adjusted to its optimum value, which minimizes the total power input according to (1), it becomes equal to :

$$F_{1opt} = - \cos (kl)\; F_0 \quad (5)$$

which exactly halves the total power input from the previous case,

$$P_{opt} = P_f \sin2\; (kl) \quad (6)$$

This power is provided fully by the primary excitation. In such a case the power flows in equal proportion to both sides of the beam.

In order to cancel the energy flow completely, two actuators are needed. It can be shown that the optimum tuning of the control forces, according to (1), produces zero power input at both the primary excitation and each of the actuators. This condition holds for any distances between the actuators and the primary excitation.

Fig. 1 shows the optimum actuator forces for a case of an assymmetric set-up (-30 %, +70 %).

Depending on frequency, the optimum control forces vary in amplitude up to unlimited values. The latter case occurs at frequencies where the wavelength becomes an integral fraction of the spacing between actuators, i.e. each time the travelling waves produced by the two actuators cancel each other. Because of such a cancellation effect, no net wave from the actuators remains to suppress the travelling wave from the excitation, thus the amplitude formally has to increase to infinity. The total force required, i.e. the sum of individual control force amplitudes, can never fall below the amplitude of the primary excitation.

In a special case, when the actuators are spaced symmetrically about the primary excitation, the control forces become mutually identical and equal to :

$$P_1 = P_2 = -P_0 / 2\cos(kl) \quad (7)$$

For a one-dimensional waveguide exhibiting one type of vibration, e.g. flexural vibration only, two actuators suffice to completely cancel the total power input regardless of the number of primary excitations. In an idealistic case where the vibration damping is zero, the (two) actuators can be located anywhere and still produce total cancellation.

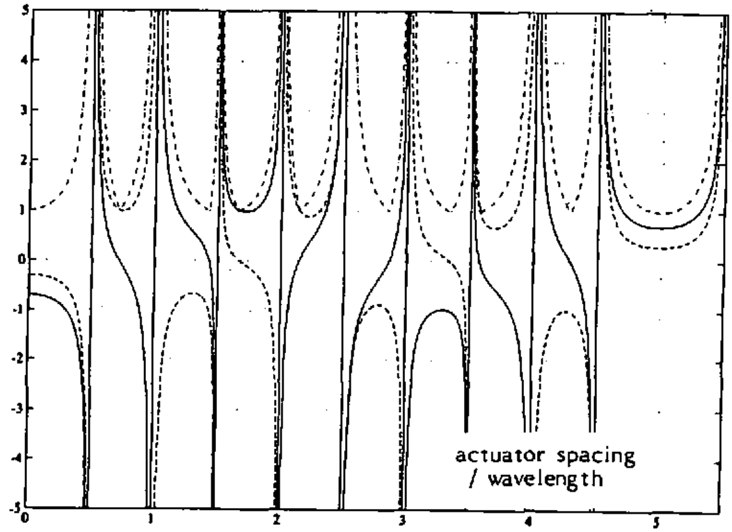


Fig. 1 : Optimum control forces acting on a beam : full line - control force 1 (-30 %) ; dotted line - control force 2 (+70 %) ; dash-dot line - total control force required

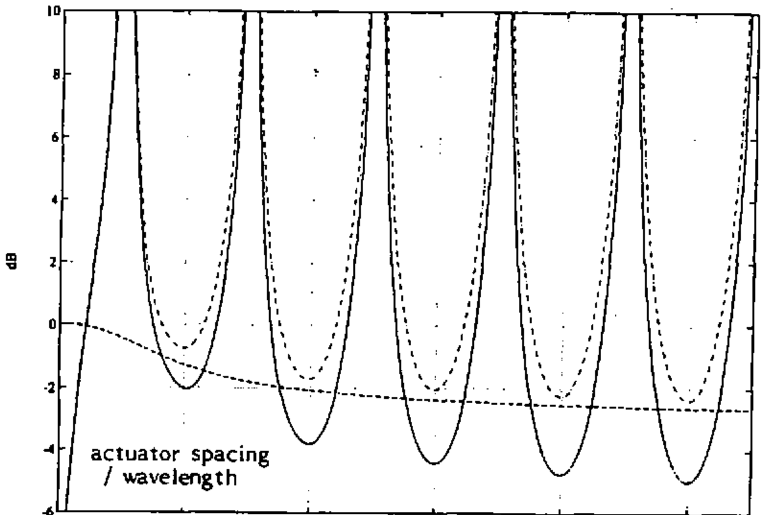


Fig. 2 : Average vibration level in the excitation zone : full line - control on ; dashed line - control off ; dash-dot line - ratio control on / control off

INPUT POWER SUPPRESSION OF VIBRATING INFINITE BEAMS AND PLATES BY SECONDARY SOURCES

The presence of damping may lead to demand for too high control forces if both actuators are located at the same side of the excitation zone many wavelength away from it.

In the case of a semi-infinite beam with a conservative termination, one actuator suffices to completely cancel the total power input, regardless of the excitation conditions. At certain discrete frequencies however, where the vibration wavelength unfavourably matches the excitation positions with respect to the boundary, no finite control force can produce enough vibration for a complete power cancellation.

Although optimally tuned control forces globally reduce vibration level, the local levels may increase. E.g. the vibration level averaged over the excitation zone, i.e. over the length of the beam between the excitation and control forces, will be, as a rule, higher than without the control forces operating. Fig. 2 shows the vibration levels averaged through the excitation zone in the case of two actuators symmetrically spaced around the primary excitation. Unless the actuators are very close to each other, less than 0,21 wavelength apart, the local vibration level with the control applied increases on average.

3. CONTROL OF PLATES

The power input by a normal force F_0, acting on an infinite plate, is given by (2) where in this case C is a constant. An actuator placed at a distance l away from the excitation, when adjusted to its optimum value (1), can be found to reduce the total power input by a factor $R^2_{01}(kl)$, where R is the real part of plate transfer function :

$$P = P_0 + P_1 = P_1 \; [1 - R^2_{01}(kl)] \tag{8}$$

The function R, a combination of Hankel functions [4], is at maximum for $l = 0$ where it reaches unity. For $l > 0$, R fluctuates between positive and negative values which permanently decrease with kl increasing. Unlike in the beam case, the actuator here becomes thus more and more inefficient as the distance from the primary excitation increases. At some frequencies where the function R has its roots, i.e. for $kl = 2.405, 5.520, 8.654$ ..., the actuator has no effect at all to the power input.

To reduce the total power input, more than one actuator has to be applied. The simplest arrangement in this case consists in placing the actuators symmetrically around the source. Depending on the number of actuators, a complete cancellation of the total power can be achieved up to a certain value of the distance source-actuator to wavelength ratio. This value increases non-linearly with the number of actuators. Figure 3 shows the total power input in the case of one, two and five actuators as a function of the normalised distance source-actuator. A careful analysis shows that the cancellation is never complete in a strict mathematical sense except for $l = 0$. However the values of reduced power which are few orders of magnitude below the value of the initial power can be considered as being zero. Fig. 3 clearly shows how the zone of (virtually) zero power widens with increase in the number of actuators.

The control forces minimizing the power input are exactly either in phase or out of phase with the primary ecitation. Fig. 4 shows the variation of the optimum control force with the distance-to-wavelength ratio for the same cases as in the previous example.

The next table displays the limiting value of distance / wavelength as a function of the number of actuators for a range of differences in power levels with / without the actuators.

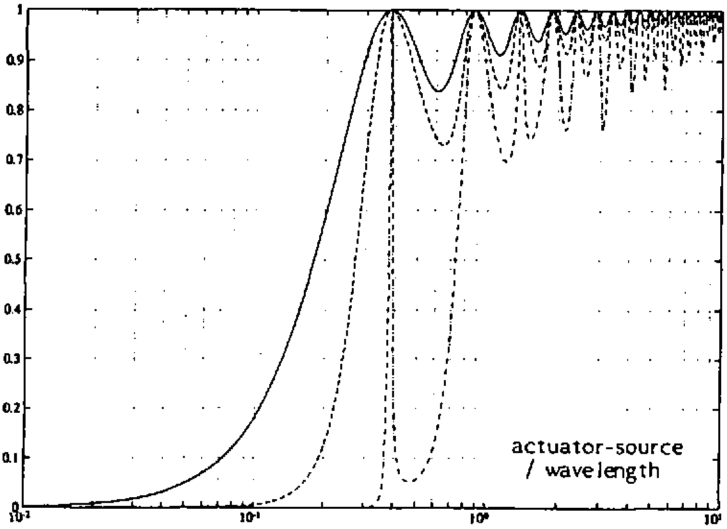


Fig. 3 : Minimum total power input to plate : full line - one actuator ; dotted line - two actuators ; dash-dot line - five actuators

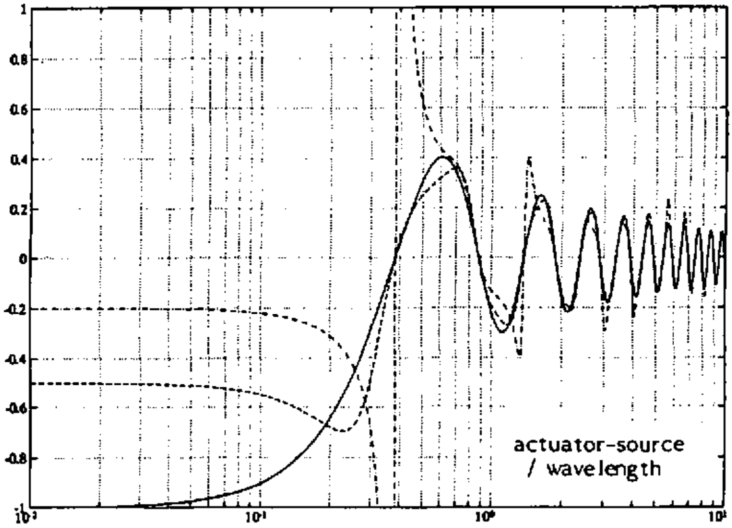


Fig. 4 : Optimum control force to plate : full line - one actuator ; dotted line - two actuators ; dash-dot line - five actuators

INPUT POWER SUPPRESSION OF VIBRATING INFINITE BEAMS AND PLATES BY SECONDARY SOURCES

Table 1 : Limiting distance / wavelength value for flexurally vibrating plate x 10^{-3}

Power w/wo actuators [dB]	- 30	- 25	- 20	- 15	- 10
number of actuators					
1	7	12	23	40	73
2	66	862	114	132	190
3	152	177	209	241	277
4	234	260	289	312	335
5	307	324	341	354	366
6	353	363	371	374	377
7	581	638	694	739	783
8	699	736	776	804	831
9	780	807	831	846	860
10	836	848	860	868	873

The increase in zero-power distance with the number of actuators is not monotonous ; a jump occurs for the case of 7 actuators due to specific features of the plate transfer function.

Fig. 5 shows three cases of optimally controlled power input produced by a single source and a single actuator in dependance of the source-actuator distance. The first case corresponds to an infinite plate, the second case to a semi-infinite plate having one straight simply-supported edge, while the thrid case corresponds to a simi-infinite plate with two perpendicular simply-supported edges. The distance source-edge(s) in the two cases is one quarter of a wavelength. The actuator is placed at the line of symmetry source-boundary. The figure shows that the effective zero-power distance increases from.01 to .04 and .3 wavelengths for the three cases respectively.

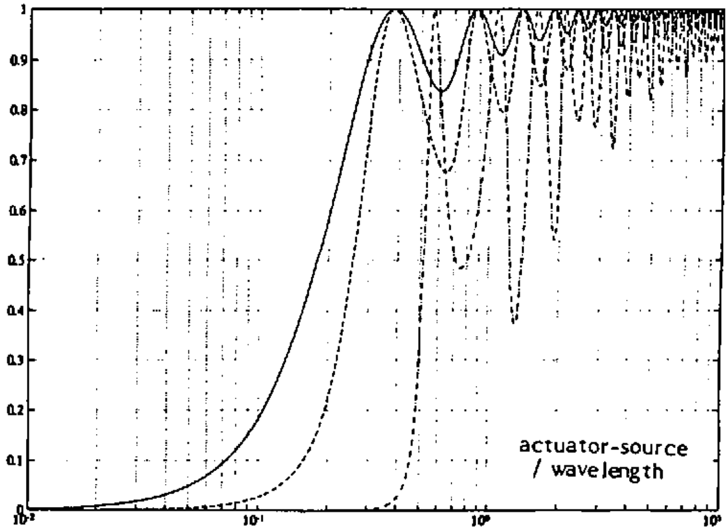
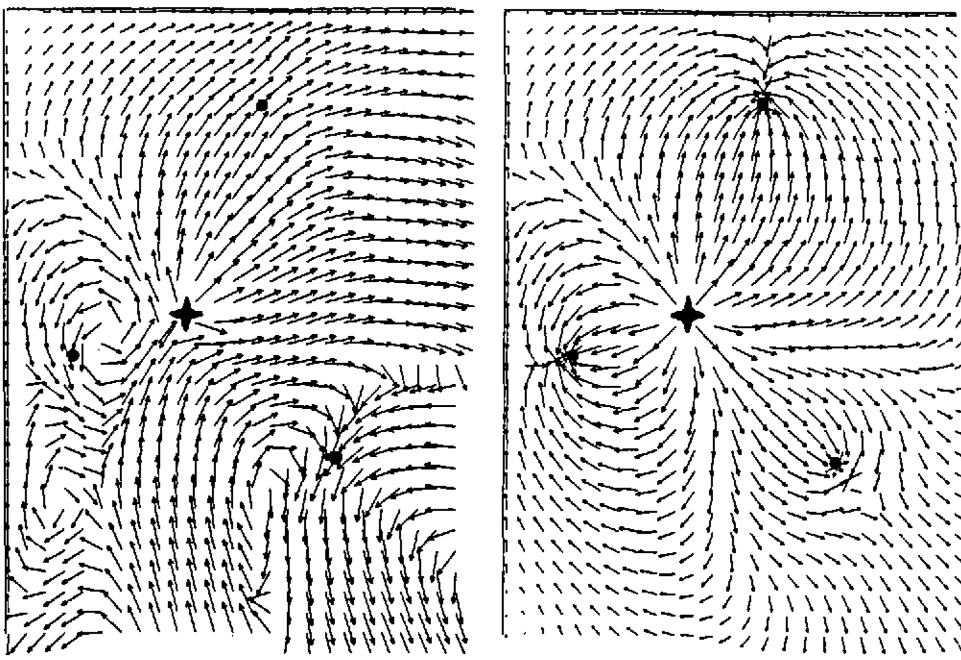


Fig. 5 : Power in plate : full line - no boundary ; dotted line - one edge ; dash-dot line - two edges

INPUT POWER SUPPRESSION OF VIBRATING INFINITE BEAMS AND PLATES BY SECONDARY SOURCES

When there is only one primary excitation, or many excitations being mutually in phase or out of phase, the power supplied by the optimum control force(s) is exactly zero. A slight de-tuning of control results in an absorption or emission of power by the control forces, as seen on Fig. 6.



re $2 \cdot 10^{-13}$ re $5 \cdot 10^{-7}$

Fig. 6 : Energy flow in semi-infinite plate near corner : source (middle) with three actuators ; left-optimum tuning ; right-de-tuning by 1°. Log scale, 30 dB / division.

4. CONCLUSIONS

In the case of infinite or semi-infinite beams and plates, corresponding to areas of damped real systems either far away from boundaries or close to one boundary, conditions of zero input power can be achieved with a limited number of actuators.

Two tuned actuators cancel the power in an infinite beam ; for a semi-infinite beam one actuator suffices. The control force vary considerably with frequency (wavelength) ; at some frequencies it increases to infinity. In the case of infinite plates, the limiting zero-power distance depends on the number of actuators. Close to boundaries (semi-infinite plate) the limiting distance may increase.

INPUT POWER SUPPRESSION OF VIBRATING INFINITE BEAMS AND PLATES BY SECONDARY SOURCES

5. REFERENCES

[1] P.A. Nelson, A.R.D. Curtis, S.J. Elliott and A.J. Bullmore, 'The minimum power output of free field point sources and the active control of sound', J. Sound Vib. , Vol. 116 (1987) pp. 397-414.

[2] S.J. Elliott, P. Joseph, P.A. Nelson and M.E. Johnson, 'Power output minimisation and power absorption in the active control of sound', J. Acoust. Soc. America, Vol. 50 (1991), pp. 2501-2512.

[3] D.U. Noiseux, 'Measurement of power flow in uniform beams and plates', J. Acoust. Soc. America, Vol. 47 (1970), pp. 238-247.

[4] L. Cremer, M. Heckl, E.E. Ungar, 'Structure-borne sound', Springer Verlag, Berlin (1972).