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SEPARATION OF STRUCTURAL WAVE COMPONENTS

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INTRODUCTION

There exist parts of structures where unidirectional waves take place, i.e. where disturbances can propagate in two mutually opposite directions only. Transducers measure the resultant disturbance, and its opposite-travelling components cannot be detected separately from each other by direct measurements.

In this paper, two methods are formulated which enable separation of the components. This is achieved by a suitable processing of measurement signals. As a result, either the waveforms or the powers can be obtained for each component.

PRINCIPLE OF SEPARATION

For constant-velocity waves which propagate in one direction, a simple dependence relates the temporal and the spatial coordinate of motion. As a consequence, the temporal derivative is in a proportion with the gradient of the wave. The factor of proportionality is the same for each direction, except for its sign. The property that *sign grad* depends on the direction can be used for the purpose of separation of the components.

For disturbance waves which exibit dispersion, not only the propagation velocity depends on frequency, but also components of the motion exist which do not propagate as proper waves. It follows that the correspondence between the spatial and the temporal derivatives does not hold for dispersive waves. This property, however, does hold for each frequency component independently. The separation procedure, based on *sign grad* property, thus has to be performed in frequency domain.

REPRESENTATION OF MOTION

To make a spectral description of two opposite-travelling waves, an amplitude density function, $A(\omega)$, can be introduced, such that $A(\omega)\cdot d\omega$ is the amplitude of an elementary spectral component contained within the frequency band $\omega,\omega+d\omega$. $A(\omega)$ is made complex to include spectral phase distribution, $\underline{A}(\omega)=A(\omega)\cdot\exp\left(i\phi(\omega)\right)$. The spatial dependence is introduced via the wavenumber, $k=k(\omega)$. The total propagating disturbance ξ:

$$\xi(x,t) = \mathrm{Re}\int_0^\infty \left(\underline{A}_+\exp(-ikx)+\underline{A}_-\exp(ikx)\right)\exp(i\omega t)\cdot d\omega = \xi_+ + \xi_- \quad (1)$$

is made up of two components, $\xi_+(x,t)$ and $\xi_-(x,t)$, having the amplitude densities $\underline{A}_+$ and $\underline{A}_-$ respectively (x - direction, t -

time, ω - frequency, $i=\sqrt{-1}$, Re - real, _ - complex).

Most usually disturbances take the form of longitudinal, torsional or flexural waves. In not too high a frequency region, the first two types of waves are nondispersive ($k \propto \omega$), while flexural waves obey the $k \propto \sqrt{\omega}$ dispersion law. Dispersion in flexural waves gives rise to a reactive, exponentially changing part of the field. This part is for present purpose disregarded in the analysis.

GENERAL SEPARATION PROCEDURES

The most common way of transition from the time into the frequency domain is by use of the Fourier transform (FT). It turns out that a unique relationship connects the amplitude densities $\underline{A}$ and the FT[†] of the disturbance ξ^1:

$$\underline{\Xi}(x,\omega) = F\{\xi(x,t)\} = \P\left(\underline{A}_+\exp(-ikx)+\underline{A}_-\exp(ikx)\right) \qquad (2)$$

In normal circumstances, a field variable ξ would be available for measurement. If, however, the variable $\xi'=\partial\xi/\partial x$ can be measured at the same position, the separation of ξ_+ and ξ_- is achieved by applying the following algorithm:

$$\xi_{\pm} = \xi/2 \pm F^{-1}(i\underline{\Xi}'/2k), \qquad \underline{\Xi}'=F(\xi') \qquad (3)$$

which is derived from (1) and (2).

Once the FT's of ξ_+ and ξ_- are determined, the corresponding power spectral densities (PSD) are easily calculated[††]

$$S_{\xi\pm\xi\pm} = 1/4(S_{\xi\xi}+S_{\xi'\xi'}/k^2\pm\mathrm{Im}\{\underline{S}_{\xi\xi'}\})/2k), \quad \text{Im - imaginary} \qquad (4)$$

The total powers (mean square values) are then obtained by the PSD frquency integration.

If the gradient-sensitive transducers are not available, the separation can still be achieved by use of standard transducers and a slightly complicated processing procedure. Two identical transducers, refered to as 1 and 2, have to be employed for this purpose. The waveforms of the components are:

$$\xi_{\pm} = F^{-1}\{\underline{\Xi}_{\frac{1}{2}}\cdot\underline{z}^*+\underline{\Xi}_{\frac{2}{1}}\cdot\underline{z}\}, \qquad {}^* \text{ - complex conjugate} \qquad (5)$$

Here Ξ_1 and Ξ_2 are the FT's of the disturbance at 1 and 2 (positive direction is from 1 towards 2), and $\underline{z}$ is a complex variable:

$$\underline{z} = i\cdot\exp(-ikd/2)/2\sin(kd)$$

which depends on the frequency and on the distance between the transducers, d.

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† F denotes the direct FT and F^{-1} its inverse

†† $\underline{S}_{\eta\zeta}$ denotes the cross spectral density function of $\eta(t)$ and $\zeta(t)$

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The corresponding expression for the PSD functions:

$$S_{\xi\pm F\pm} = \left(S_{\xi_1\xi_1}+S_{\xi_2\xi_2}-2\mathrm{Re}\{\underline{S}_{\xi_1\xi_2}\exp(\pm ikd)\}\right)/4\sin^2(kd) \quad (6)$$

contains again both the direct and the cross spectral densities of the two measurement signals.

If the nonpropagating part of a dispersion field is significant, more than two transducers have to be used for separation of the components. Four transducers are needed for the separation of the flexural components. These can be either of different type (i.e. measuring ξ, ξ', ξ'', ξ''') placed at the same location, or of the same type but placed at different locations (or any combination of these possibilities). Algorithms for signal processing become more complex in such a case, but can be readily evaluated once the appropriate dependence F-x , such as given in Eq. 2, is known.

SIMPLIFIED SEPARATION PROCEDURES

Expressions obtained so far indicate that digital signal processing is required for the calculation of the waveforms and the power densities from measurements. However, in the case of nondispersive waves, processing procedures can be substantially simplified. It turns out that no transforms in frequency domain are necessary to get the waveforms and the total powers of the components:

$$\xi_\pm = \xi/2\mp\tilde{\xi}'\cdot c/2, \qquad \tilde{\ } \text{ - time integration, c - wave velocity} \quad (7)$$

$$\overline{\xi_\pm^2} = 1/4\,(\overline{\xi^2}+c^2\cdot\overline{\tilde{\xi}'^2}\pm 2c\cdot\overline{\xi\tilde{\xi}'}), \qquad \overline{} \text{ - time averaging} \quad (8)$$

and no spectrum weighting is necessary to get the PSD functions:

$$S_{\xi\pm\xi\pm} = 1/4\,(S_{\xi\xi}+c^2\cdot S_{\tilde{\xi}'\tilde{\xi}'}\pm 2c\cdot\mathrm{Re}\{\underline{S}_{\xi\tilde{\xi}'}\}) \quad (9)$$

Simplified processing procedures are applicable also when measuring with ordinary transducers, if these are placed close to each other (the spacing should be much smaller than the shortest wavelength). In such a case approximations give:

$$\xi_\pm = (\xi_1+\xi_2)/4\pm(\tilde{\xi}_1-\tilde{\xi}_2)\cdot c/2d \quad (10)$$

$$\overline{\xi_\pm^{\,2}} = \left(\overline{(\tilde{\xi}_1-\tilde{\xi}_2)^2}\cdot c/2d\pm\overline{\xi_2\tilde{\xi}_1}\right)\cdot c/2d \quad (11)$$

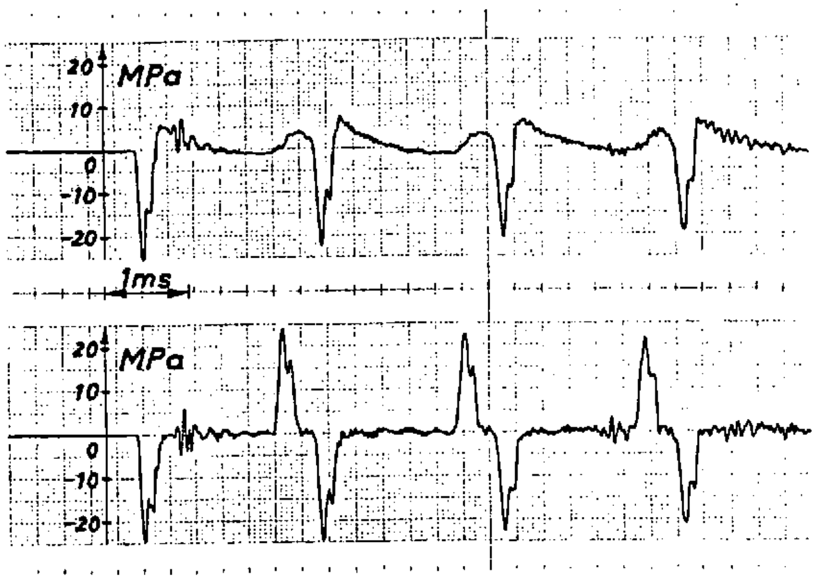
$$S_{\xi\pm\xi\pm} = \left(S_{\tilde{\xi}_1\tilde{\xi}_1}+S_{\tilde{\xi}_2\tilde{\xi}_2}-2\mathrm{Re}\{\underline{S}_{\tilde{\xi}_1\tilde{\xi}_2}\}\pm\mathrm{Re}\{\underline{S}_{\xi_2\tilde{\xi}_1}\}\cdot 2d/c\right)\cdot c^2/4d^2 \quad (12)$$

The last results are in accordance with those obtained previously for sound waves[2].

A TEST EXAMPLE

A steel pipe, 5,31 m long, with the edges unrestrained in axial direction was used for experiments. At the measurement location, 1,18 m from one of the edges, two pairs of precision foil strain gauges were bonded on the pipe (d=20 cm). The positions of the gauges were chosen in such a way that each pair, connected in the half-bridge configuration, was measuring only the axial stresses. Since in the measurement frequency region the wave motion had to be nondispersive[3], Eq. 10 was employed as a basis for signal processing.

A stress wave was produced by a hammer strike at the nearer edge. The forward propagated stress was detected by a conveniently made electronic circuit, and was recorded simultaneously with the total stress on a fast memory.



Propagation of stress wave along pipe: above - forward moving component, below - total stress (positive sign applies to tension)

The effect of separation is clearly seen from the two plots: the influence of backward propagating component is negligible.

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