24th INTERNATIONAL CONGRESS ON SOUND AND VIBRATION
23–27 July 2017, London

THE INFLUENCE OF SLAB FOUNDATIONS ON GROUND-BORNE VIBRATION FROM RAILWAYS

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The ever increasing urban population is leading to the exploitation of building sites, close to surface or underground railways, that were once deemed unsuitable because of the level of ground-borne vibration. An important design consideration is whether or not levels of perceptible vibration and/or re-radiated noise will be acceptable in the completed buildings. A fundamental question concerns to what extent the mass and stiffness of the building foundation influences these levels. This paper explores this question in relation to the commonly encountered, concrete slab foundation. Previous research has explored the influence of the coupling between a flexural plate and an elastic half-space on the free-surface displacements arising from surface Rayleigh waves. Here, a numerical wave-based approach is followed using the ElastoDynamics Toolbox in MATLAB. The case of an infinite, horizontal layer on top of an elastic half-space is considered, subject to incoming plane waves from a buried source. The paper highlights the significance of differences in the wave excitation when considering the construction of a "thick" slab foundation as a means of mitigating ground-borne vibration.

Keywords: foundation, ground-borne vibration, railway

1. Introduction

Vibration sources represented by railway traffic are of particular interest for their effect on nearby buildings. The ground-borne vibration generated at the source propagates through the soil and reaches the building foundation. Depending on the mass, stiffness and damping distribution within the building, the level of vibration and re-radiated noise may lead to annoyance of the occupants.
A number of strategies are available for mitigating ground-borne vibration in buildings, the goal of these being to act either at the source [1, 2], along the propagation path [3] or at the receiver (building) [4]. A particular example of mitigation at the source is the implementation of a thick slab foundation at the base of the building. This represents a particular case of soil-structure interaction in which the displacement at the soil-foundation interface is constrained by the slab at relatively high frequency. Significant literature exists on the soil-structure interaction of both flexible and rigid foundation systems [5] with the main focus being earthquake-related problems, which involve low frequencies and long wavelengths. With reference to the ground-borne vibration induced by railway traffic, the frequency range of interest lies between approximately 20 Hz and 200 Hz, because of both the frequency content of the source and the range of human perception of vibration and re-radiated noise. Therefore, problems related to ground-borne vibration involve relatively short wavelengths that may be comparable with the dimensions of a typical concrete slab foundation. In this case, the flexibility of the slab must be considered.
Previous work by Auersch [6] has considered the response of finite and infinite plates resting on a

half-space to Rayleigh wave excitation. The work described herein first considers the results of Auersch, thereby validating the use of the Elastodynamics Toolbox [7] in MATLAB for the analysis of the slab foundation as an elastic layer on a half-space subjected to Rayleigh wave excitation. The variation of the free-field displacement, as a result of the soil-foundation interaction, is then investigated for the case of buried sources generating either incoming P or SV waves at several incidence angles.

2. Overview of the problem

As a first approximation, the concrete slab may be considered of infinite extent in both x an y (see Fig. 1). In this case, the slab can be treated as an additional layer, with shear modulus G_c, Poisson's ratio ν_c and mass density ρ_c, on top of the elastic half-space, and the problem can be studied in a plain strain formulation. The soil is assumed to be homogeneous and elastic with the shear modulus G, Poisson's ratio ν_s and mass density ρ_s. Both surface and body wave excitation are considered: a Rayleigh wave travelling along the x-axis with velocity V_R, and an incoming P or SV-wave travelling respectively with velocity V_P or V_S at an incidence angle θ_P or θ_S (see Fig. 1). The former is representative of a surface while the latter may represent an underground railway.

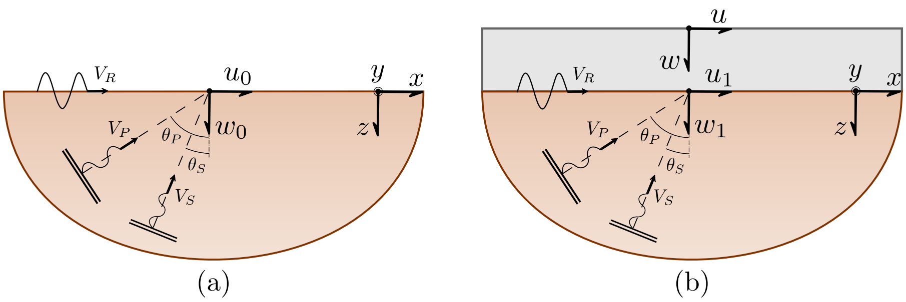

Figure 1: a) An elastic half-space subjected to surface and body wave excitation: a Rayleigh wave travelling along the x-axis with velocity V_R and a plane P or SV-wave travelling with velocity V_P or V_S at an incidence angle θ_P or θ_S; b) the same wave excitation is considered for the case of a concrete slab resting on the elastic half-space, with the resulting free-surface displacement $\mathbf{u}$ and the displacement $\mathbf{u_1}$ at the soil-foundation interface.

The wave excitation is assumed to be plane, and thus invariant along the y-axis. The problem can be then examined in the xz plane with reference to the vertical and horizontal displacements. The free-field displacement $\mathbf{u_0}$ of the elastic half-space, the interface displacement $\mathbf{u_1}$ and the free-surface displacement $\mathbf{u}$ of the slab can be expressed in vector form as:

$$
\begin{aligned}
\mathbf{u_0} &= \mathbf{A_0} \exp(ik_x x - i\omega t) \\
\mathbf{u_1} &= \mathbf{A_1} \exp(ik_x x - i\omega t) \\
\mathbf{u} &= \mathbf{A} \exp(ik_x x - i\omega t)
\end{aligned} \tag{1}
$$

The comparison of the amplitude vectors $\mathbf{A_0} = [u_0, w_0]^{\mathrm{T}}$, $\mathbf{A_1} = [u_1, w_1]^{\mathrm{T}}$, $\mathbf{A} = [u, w]^{\mathrm{T}}$, independent of the x coordinate, is of interest for investigating the influence of the slab foundation on the free-surface displacement. With reference to the type of wave excitation considered and its angular frequency ω of excitation, the horizontal wavenumber k_x can be written as:

$$
k_x = k_R\,; \quad k_x = k_P \sin\theta_P\,; \quad k_x = k_S \sin\theta_S \tag{2}
$$

where k_R, k_P and k_S are the wavenumbers of the Rayleigh, P and SV waves respectively.
The influence of a slab foundation on the displacement field $\mathbf{u_1}$ at the soil-foundation interface makes reference to a basic result in the literature of soil-structure-interaction (SSI) [5]. By ensuring equilibrium and compatibility at the soil-foundation interface, the displacement $\mathbf{u_1}$ can be expressed as:

$$
\mathbf{u_1} = [\mathbf{K_f} + \mathbf{K_s}]^{-1}\, \mathbf{K_s}\, \mathbf{u_0} \tag{3}
$$

where $\mathbf{K_f}$ and $\mathbf{K_s}$ represent the dynamic stiffness matrices of the foundation and the elastic half-space respectively, often found in the literature in terms of their respective frequency response function (FRF) matrices [4]. The dynamic stiffness matrices can be expressed in a frequency-wavenumber formulation for both the foundation and the elastic half-space. This approach was used by Auersch [6], assuming a relaxed boundary (RB) condition between the soil and the strip foundation. With the latter assumption, the horizontal and vertical displacements are effectively uncoupled and the problem of soil-foundation interaction can be studied with reference to the vertical displacement w_1 and w_0. Eq. (3) can be written in scalar form with reference to the vertical displacement as:

$$\frac{w_1}{w_0} = \frac{1}{\left(\dfrac{K_f}{K_s} + 1\right)} \tag{4}$$

3. A slab foundation on an elastic half-space subjected to Rayleigh wave excitation

This section explores the influence of a slab foundation, subjected to Rayleigh wave excitation, on the vertical displacement w_1 at the soil-foundation interface. First, the case of an inifinitely long strip foundation is reviewed, following the approach of Auersch [6]. In this case, only w_1 can be calculated because of the plate-like modelling of the strip foundation. Alternatively, the slab is considered infinitely long and wide, so to be effectively considered as an elastic layer on a half-space, with the calculation of both w_1 and w.

3.1 The case of an infinitely long strip foundation

The vertical dynamic stiffness of an elastic plate of width b and thickness t, in the frequency-wavenumber domain reads:

$$K_f(\omega, k_x) = B\, b\, k_x^4 - \rho_c\, b\, t\, \omega^2 \tag{5}$$

where $B = E_c\, t^3/12(1-\nu_c^2)$ is the bending stiffness of the plate.

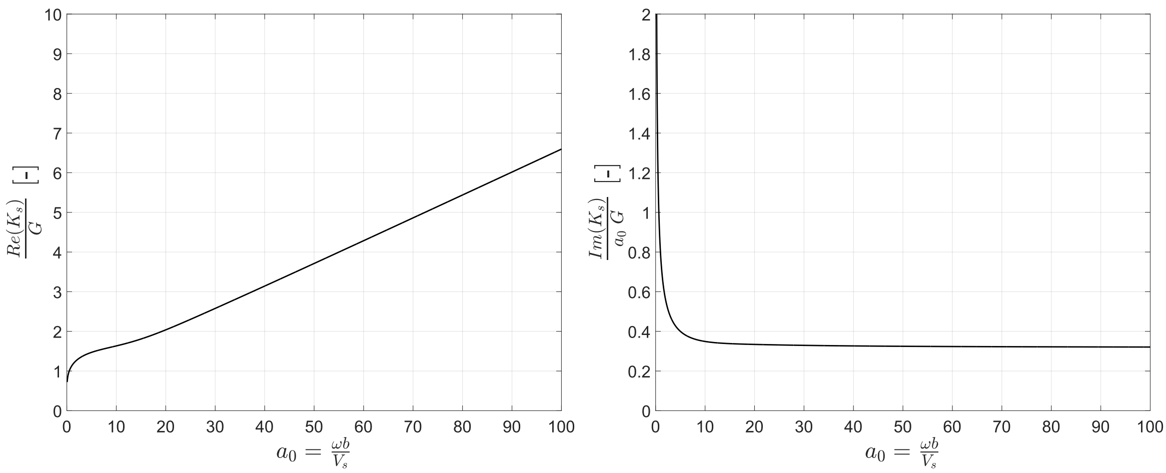

Figure 2: Normalised real (left) and imaginary (right) values of the soil dynamic stiffness beneath an infinitely long strip foundation of finite width b. Reproduced using the approach of Auersch [6].

The dynamic stiffness of the elastic half-space K_s effectively refers to a 21/2-D problem because of the finite width b of the plate along the y-axis. Auersch [6] reported the problem to a plain strain condition by an integration approach in the wavenumber k_y assuming the plate infinitely flexible along the y-axis.

The values of the real and imaginary part of the dynamic stiffness of the half-space are reproduced for

reference in Fig. 2, where the range of dimensionless frequency a_0 is consistent with the frequency range of interest and the data of the benchmark problem considered by Auersch (see Table 1 in the Appendix). According to Auersch, the real part of the soil dynamic stiffness may be approximated as constant up to a dimensionless frequency of 20, which corresponds to 30 Hz [6] for the current benchmark problem. However, assuming an hysteretic damping for the soil, the real part of K_s is proportional to frequency beyond a certain value of a_0 which depends on the damping loss factor η_s considered. The imaginary part of the soil dynamic stiffness is proportional to frequency and is representative of the damping within the soil.
It is instructive to consider several values of the foundation width b for the calculation of the ratio w_1/w_0, which can be obtained from Eq. (4). At a particular frequency, called "coincidence frequency" (f_{co}), the bending stiffness and inertia of the foundation plate are such that a unit value of w_1/w_0 is obtained. The coincidence point represents the scenario where the wavelength λ_f of the foundation plate coincides with the horizontal wavelength λ_x of the input motion. The corresponding frequency can be found as [8]:

$$f_{co} = \frac{V_R^2}{2\pi}\sqrt{\frac{\rho_c t}{B}} \tag{6}$$

The concept of coincidence frequency is of importance when assessing the mitigating effect of the foundation on the free-field displacement. It represents a turning point from a region where amplifications are to be expected and a region where considerable reduction can be achieved (Fig. 3). The influence of the bending stiffness and the mass of the foundation on the value of f_{co} can be then drawn from Eq. (6). Increasing the foundation thickness t will result in a reduction of both f_{co} and the ratio

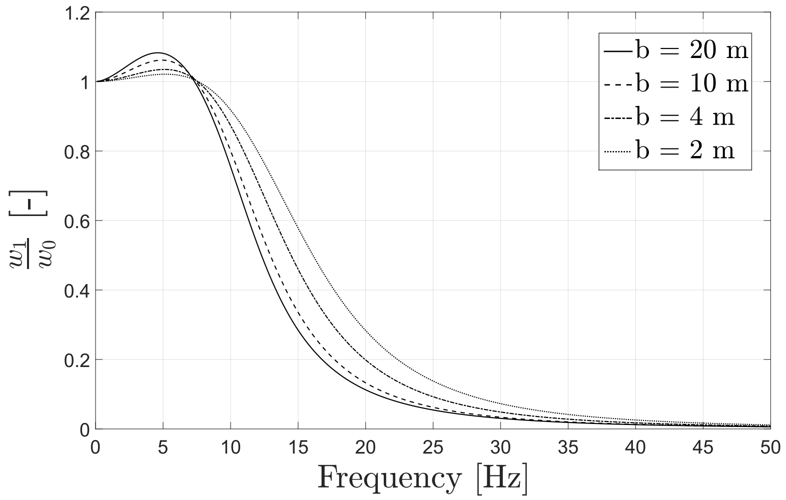

Figure 3: Influence of the width of the strip foundation b on the soil-foundation interaction with respect to the vertical displacement at the interface w_1 due to Rayleigh wave excitation.

w_1/w_0 at relatively high frequency ($f > f_{co}$). A similar effect may be obtained by decreasing the mass density ρ_s of the material comprising the foundation. A comprehensive discussion on the effect of the bending stiffness, mass and width of the foundation, and the shear wave velocity V_s of the soil can be found in Auersch [6], also with reference to a foundation of finite length along the x-axis. The results of Fig. 3 illustrate the significance of increasing the strip width b, making one think at the case of a very large slab foundation ($b \to \infty$).

3.2 An infinitely long and wide slab foundation

In order to consider a plain strain problem, let us assume the slab foundation to be infinitely long in both the x and y direction. With this assumption, the slab foundation can be regarded as an elastic layer of finite thickness on the elastic half-space. The case of a travelling Rayleigh wave can be investigated by means of the dynamic stiffness method (DSM) [9], with reference to the horizontal wavenumber k_R. This method makes use of the dynamic stiffness matrices of the elastic half-space and of the layer, obtained by means of the ElastoDynamics Toolbox (EDT [7]) in MATLAB [10], and being particularly convenient when evaluating the interface displacement $\mathbf{u_1}$ by means of the SSI Eq. (3). It is worth noticing that, with the assumption of relaxed boundary (RB), the actual retrogade elliptical motion of a particle at the soil-foundation interface, typical of a Rayliegh wave, is completely neglected, and the ratio w_1/w_0 can be retrieved from the scalar SSI Eq. (4), with dynamic stiffness values K_s and K_f in the vertical direction evaluated either from the DSM matrices or through Auersch's procedure outlined in Section 3.1. The fully coupled (FC) condition can be obtained when using the DSM matrices $\mathbf{K_s}$ and $\mathbf{K_f}$ in conjunction with Eq. (3), with reference to both the horizontal and vertical components of the Rayleigh wave [11] for the input motion $\mathbf{u_0}$. The results from Auersch [6], reported in Section 3.1, can be extended to an infinitely long and wide slab, and compared with those obtained using the DSM. It is clear from Fig. 4a that the two different methods agree well,

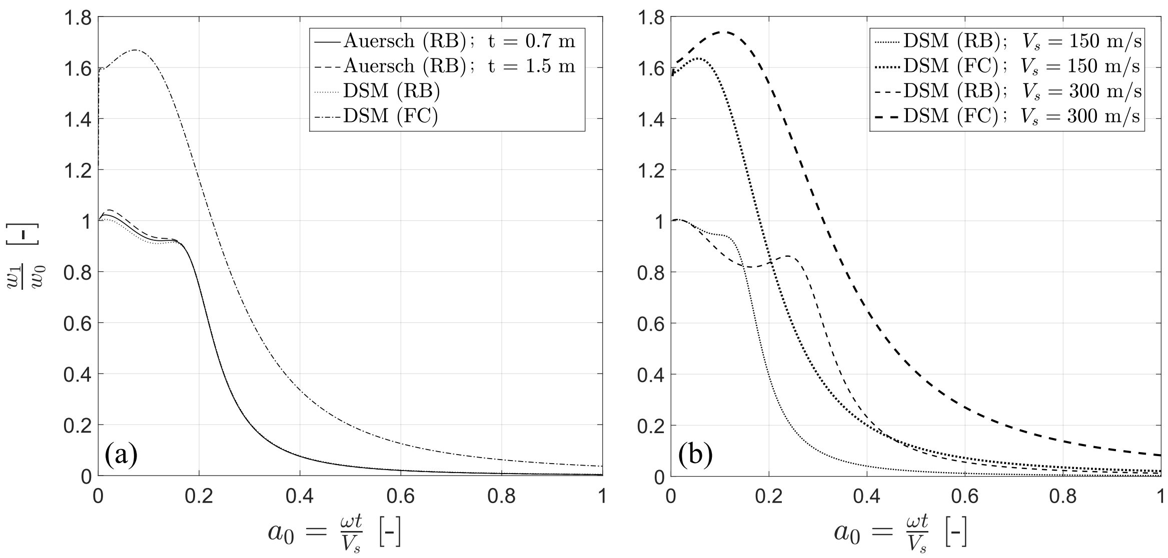

Figure 4: Values of the ratio w_1/w_0 obtained for an infinitely long and large slab foundation subjected to a Rayleigh wave excitation: (a) the values obtained by Auersch's procedure with $b = 1000$ m and $V_s = 200$ m/s are compared with ones obtained by the DSM method; (b) two values of $V_s = 150, 300$ m/s are considered with reference to both the fully coupled (FC) and the relaxed boundary (RB) conditions.

achieving convergent results for $b = 1000m$ with values for the other parameters reported in Table 1. Fig. 4 shows the results of w_1/w_0 in terms of the dimensionless frequency $a_0 = \omega t/V_s$. The integration approach of Auersch leads to slightly different results at relatively low frequencies, depending on the slab thickness t. On the other hand, the results are general, with respect to the slab thickness t, when the DSM approach is considered. Although the dimensionless frequency a_0 is convenient for investigating different values of the slab thickness, it does not lead to general results with respect to the shear wave velocity V_s of the soil. Higher values of V_s lead to a shift towards higher frequencies of the restraining effect of the slab foundation. The general trend reported by Auersch [6] is still valid for the relaxed boundary condition (RB) solution, although the region of amplification is considerably reduced and partly substituted by a region where a minor reduction is expected. The fully coupled condition (FC) highlights the importance of considering both the horizontal and vertical components of $\mathbf{u_0}$, and the cross-stiffness terms at the soil-foundation interface, that lead to a considerable amplification at relatively low a_0. It is worth noticing that, due to the small dynamic strains involved, and the likely influence of friction, the latter fully coupled condition may be more representative.

4. The slab foundation on the elastic half-space subjected to incoming P-SV waves

The case of incoming P or SV waves can be investigated by the use of a wave-based approach [11], using the dynamic stiffness method. Considering an incoming P or SV-wave at an incidence angle θ_P or θ_S (see Fig. 1), the displacement field $\tilde{\mathbf{u}}$ of the system in Fig. 1b can be calculated by superposition of several elastic states [7], assuming the material to be linear and elastic. The results in terms of

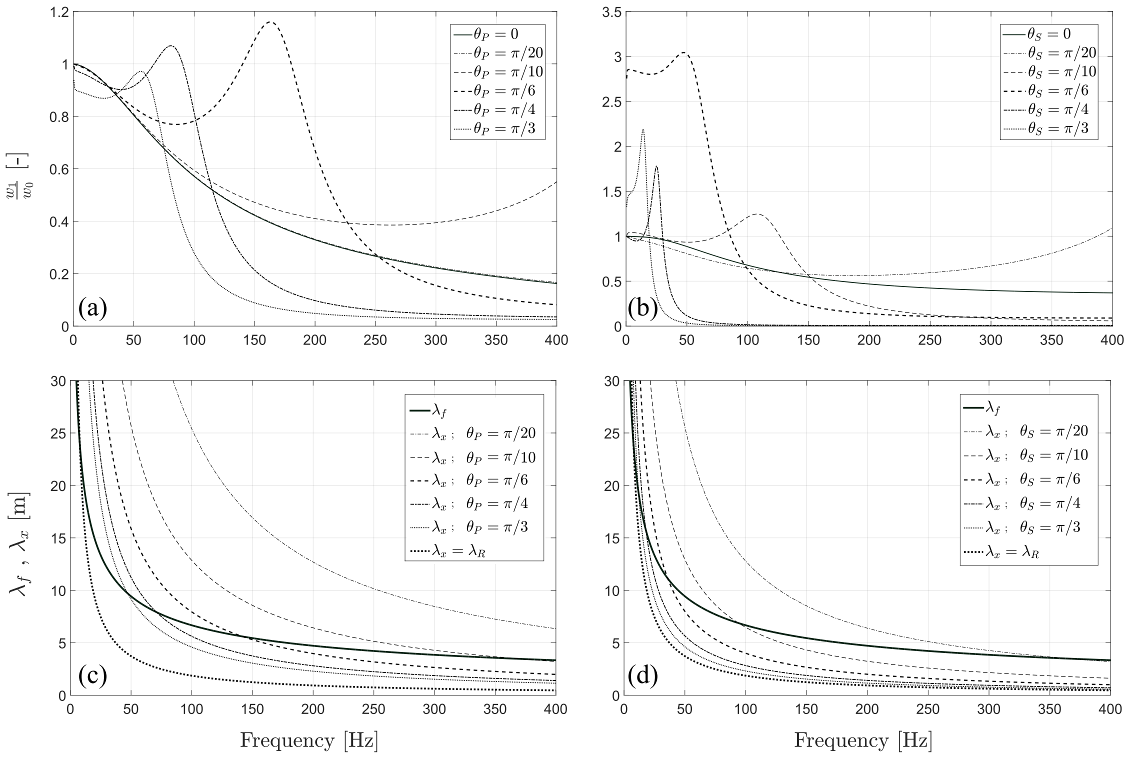

Figure 5: Values of the ratio w_1/w_0 obtained for an infinitely long and wide slab foundation subjected to either (a) an incident P-wave at an angle θ_P or (b) an incident SV-wave at an angle θ_S. The wavelength-frequency curves are plotted below for the two cases of incoming (c) P and (d) SV waves in order to visualise the coincidence frequency f_{co}.

w_1/w_0 are reported in Fig. 5. Figs. 5c and 5d show the wavelength-frequency curves of λ_f and λ_x for incoming P or SV waves respectively.

Considering an incoming P-wave, the normally incident wave ($\theta_P = 0$) represents a limit case for which $\lambda_x = \infty$ and the restraining effect of the slab foundation is maximum at the "natural frequencies" f_n of the layer (slab foundation):

$$f_n = \frac{V_{pc}}{4\,t}(2n-1) \tag{7}$$

Given the application and the large value of the P-wave velocity V_{pc} in the slab foundation, f_n assumes large values that lie well beyond the frequency range of interest ($f_{n=1} \approx 1300$ Hz for $t = 0.7$ m), leading to only moderate values of reduction. The Rayleigh wave excitation represents another limiting case for incoming P or SV waves. The horizontal wavelength λ_x for the incoming P and SV-wave is always larger than the Rayleigh wave case (see Figs. 5c and 5d). Consequently, the coincidence frequency f_{co} of incoming P and SV waves is greater than that of a travelling Rayleigh wave, and the region where strong reductions of the ratio w_1/w_0 are to be expected shifts to relatively higher frequencies.

The function $w_1/w_0(f)$ can be qualitatively divided into the following regions:

1. a first region of either a moderate reduction (for an incoming P-wave) or an amplification at a certain level (for an incoming SV-wave), up to a frequency $f_1 < f_{co}$;

2. a second region with increasing values of w_1/w_0 that are always greater than unity for incoming SV-waves, but not necessarily for an incoming P-wave. A maximum value is then obtained at a frequency $f_2 > f_{co}$;
3. a sharply sloped region where considerable reductions are to be expected up to a frequency f_3, beyond which the value w_1/w_0 tends to a limiting value.

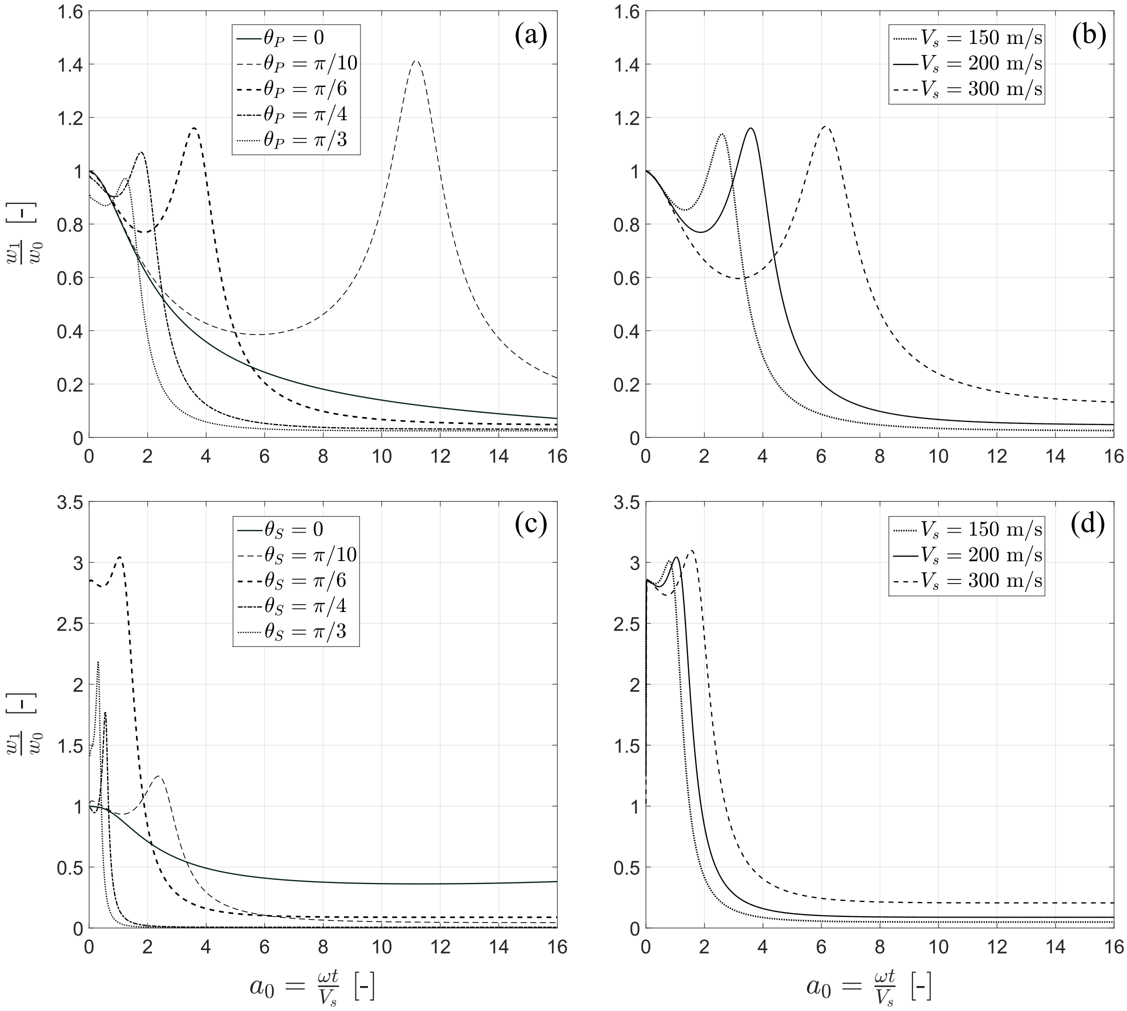

Figure 6: Values of the ratio w_1/w_0 obtained for an infinitely long and wide slab foundation subjected to either (a) an incident P-wave at an angle θ_P or (c) an incident SV-wave at an angle θ_S as a function of the dimensionless frequency a_0. The case of (b) $\theta_P = \pi/6$ and (d) $\theta_S = \pi/6$ is reported for several values of the soil shear wave velocity $V_s = 150,\ 200,\ 300$ m/s.

Referring to the values in Table 1, the slab foundation starts already to act as a plate, with the trend $w_1/w_0(f)$ as described above, at relatively low incidence angles (10°- 20°). The wavelength-frequency plots in Figs. 5c and 5d show the approximate correspondence between the “coincidence frequency” and the second region of local amplifications in Figs. 5a and 5b. The free-vibration wavelength λ_f of the slab foundation is calculated based on the bending plate assumption; the actual shear deformation may contribute to higher values of λ_f resulting in lower values of f_{co}.

It is convenient to plot the ratio w_1/w_0 for incoming P (Fig. 6a) and SV (Fig. 6c) waves in terms of the dimensionless frequency a_0. Given realistic values for the shear wave velocity in the soil $V_s = 150-300$ m/s, slab thickness $t = 0.7-1.5$ m and the frequency range of interest $f = 20-200$ Hz, the range of dimensionless frequency a_0 of interest for ground-borne vibration applications lies between 0.3 and 13. Unfortunately, the results in terms of w_1/w_0 cannot be generalised with reference to the shear wave velocity V_s; for soil deposits with higher V_s the coincidence frequency will shift to higher values of a_0 (Figs. 6b and 6d).

5. Conclusions

This review of the influence of a slab foundation on ground-borne vibration levels has brought to light the significance of differences in the wave excitation. While the slab foundation restrains considerably the level of vibration at relatively low frequencies in the case of an incoming Rayleigh wave, it may fail to do so when incoming P or SV waves are present. In this case, and depending on the values of the parameters involved, there may be amplifications associated with the construction of a slab foundation that is insufficiently thick for the frequency-range of concern. This leaves the matter of mitigation by means of a slab foundation still opened to questions, especially with regards to the actual incoming wave excitation.

ACKNOWLEDGEMENTS

The authors wish to thank Prof. Geert Degrande and colleagues in KU Leuven for providing the EDT Toolbox.

APPENDIX

Table 1: Data of reference

Soil	Shear modulus	$G = 8\,\text{x}\,10^7$ N/m^2
	Shear wave velocity	$V_s = 200$ m/s
	Poisson ratio	$\nu_s = 0.33$
	Mass density	$\rho_s = 2000$ kg/m^3
	Damping loss factor	$\eta_s = 0.1$
Concrete	Young's Modulus	$E_c = 3\,\text{x}\,10^{10}$ N/m^2
	Poisson ratio	$\nu_c = 0.15$
	Mass density	$\rho_c = 2500$ kg/m^3
	Damping loss factor	$\eta_c = 0.1$
Slab foundation	Width	$b = 2, 4, 10, 20, \infty$ m
	Thickness	$t = 0.7, 1.5$ m

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