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USING ENERGY FLOW ANALYSIS TECHNIQUES FOR THE PREDICTION OF STRUCTURAL NOISE RADIATION

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1. INTRODUCTION

Energy Flow Analysis techniques, i.e. Statistical Energy Analysis, Energy Accountancy, etc., offer a very powerful method for modelling the noise radiation from many types of structures: machinery, automotive, aerospace, etc. The technique basically involves defining a model of the structure built up of a number of connected component parts (subsystems). A matrix formulation of power balance equations can then be obtained which in general takes the form:

$$[\text{Loss Factor Matrix}]\ [\text{Vibrational energy matrix}] = [\text{Input Power Matrix} + \omega] \qquad (1)$$

Noise radiation is calculated from the vibrational surface velocity obtained from the vibrational energy matrix [1]. In the normal analysis the vibration of energy matrix which defines the distribution of vibrational energy throughout the component parts of the structure is obtained by inverting the loss factor matrix and premultiplying the input power matrix. Thus, for the analysis it is necessary to obtain values for loss factors and input powers. The loss factor matrix is built up from both the internal loss factors of the individual components and coupling loss factors which relate to the transmission of vibrational energy between the component subsystems. For simple structural elements such as beams and plates these coupling factors can generally be defined with reasonable accuracy from theoretical relationships. For more complexly shaped components typical of many practical machinery structures these factors are more difficult to define theoretically and it is often necessary to obtain these from measurements on an actual structure. This type of analysis provides a very powerful and versatile tool for modelling the noise generating characteristics of a particular structure and can be used for predicting and optimising the effects of structural changes on noise radiation. The effects of changes to materials, component sizes, method of connection, internal damping and absorption can be assessed. The technique permits a valuable insight (as one overall model) into the important paths of vibrational power flow and noise radiation characteristics of a particular structure or connected structures in a manner easily understandable in an engineering sense. The technique can be used not only during development with an existing structure but also actually at the design stage.

2. ENERGY FLOW ANALYSIS

For N substructures under steady-state conditions, the power balance equations may be written as [2]:

$$P_i = \omega E_i \sum_{j=1}^{N} \eta_{ij} - \omega \sum_{\substack{j=1 \\ j\neq i}}^{N} E_j \eta_{ji} \qquad i = 1 - N \qquad (2)$$

with $\eta_{ij}n_i = \eta_{ji}n_j$ (3)

where n = modal density.

Consider the case of a structure divided into two subsystems. Equation (2) now becomes, when expanded:

$$\omega\left\{(\eta_1 + \eta_{12})\,\bar{E}_1 - \eta_{12}\,\frac{n_1}{n_2}\,\bar{E}_2\right\} = \bar{P}_1$$

$$\omega\left\{-\,\eta_{21}\,\frac{n_2}{n_1}\bar{E}_1 + (\eta_2 + \eta_{21})\,\bar{E}_2\right\} = \bar{P}_2 \tag{4}$$

and equation (3) is

$$\eta_{12}\,n_1 = \eta_{21}\,n_2 \tag{5}$$

Eliminating the modal density ratio, $\frac{n_1}{n_2}$, between equations (4) and (5) and restoring the equations to matrix form gives

$$\omega\begin{bmatrix}(\eta_1 + \eta_{12}) & -\eta_{21}\\ -\eta_{12} & (\eta_2+\eta_{21})\end{bmatrix}\begin{bmatrix}\bar{E}_1\\ \bar{E}_2\end{bmatrix} = \begin{bmatrix}\bar{P}_1\\ \bar{P}_2\end{bmatrix} \tag{6}$$

or $\omega[L]\,[\bar{E}] = [\bar{P}]$ (6a)

The power injection method can now be applied to equation (6) in the following manner.

Power is first injected only into subsystem 1. Hence, equation (6) becomes

$$\omega\begin{bmatrix}(\eta_1 + \eta_{12}) & -\eta_{21}\\ -\eta_{12} & (\eta_2+\eta_{21})\end{bmatrix}\begin{bmatrix}(\bar{E}_1)_1\\ (\bar{E}_2)_1\end{bmatrix} = \begin{bmatrix}\bar{P}_1\\ 0\end{bmatrix} \tag{7a}$$

where $(\bar{E}_i)_j$ = time averaged energy of subsystem i when only subsystem j is excited.

$\bar{P}_i$ = time averaged power injected into subsystem i

Similarly, if power is then injected only into subsystem 2,

$$\omega\begin{bmatrix}(\eta_1+\eta_{12}) & -\eta_{21}\\ -\eta_{12} & (\eta_2+\eta_{21})\end{bmatrix}\begin{bmatrix}(\bar{E}_1)_2\\ (\bar{E}_2)_2\end{bmatrix}=\begin{bmatrix}0\\ \bar{P}_2\end{bmatrix} \tag{7b}$$

Combining equations (7a) and (7b) and rearranging terms gives

$$\omega\begin{bmatrix}(\bar{E}_1)_1 & (\bar{E}_1)_1 & -(\bar{E}_2)_1 & 0\\ 0 & (\bar{E}_1)_1 & -(\bar{E}_2)_1 & -(\bar{E}_2)_1\\ -(\bar{E}_1)_2 & -(\bar{E}_1)_2 & (\bar{E}_2)_2 & 0\\ 0 & -(\bar{E}_1)_2 & (\bar{E}_2)_2 & (\bar{E}_2)_2\end{bmatrix}\begin{bmatrix}\eta_1\\ \eta_{12}\\ \eta_{21}\\ \eta_2\end{bmatrix}=\begin{bmatrix}\bar{P}_1\\ 0\\ 0\\ \bar{P}_2\end{bmatrix} \tag{8}$$

or $\omega[A]\,[\eta]=[P_*]$ (8a)

All the loss factors $[\eta]$ can therefore be obtained from equation (8) viz.

$$[\eta]=\frac{1}{\omega}[A]^{-1}\,[P_*] \tag{9}$$

It should be noted here that all the loss factors have been obtained without a knowledge of subsystem modal densities, the first major source of error. For a small number of subsystems equation (9) can be used with confidence. However, as the number of subsystems (N) gets larger the errors in calculating the loss factors increase since the dimension of [A] increases as N^2. [A] also tends to be rather ill-conditioned (note the large off-diagonal terms in rows 1 and 4 of equation (8)). In order to overcome this latter problem it has been shown (8) that the internal loss factors can be eliminated from equation (8) giving N sets of generally well conditioned matrix equations, each of dimension (N-1). The internal loss factors are then obtained by back substitution into equation (8). Thus, for N subsystems, the coupling loss factors are given by N sets of equations of the form:

$$\begin{bmatrix}\eta_{1i}\\ \vdots\\ \eta_{ri}\\ \vdots\\ \eta_{Ni}\end{bmatrix}_{r\neq i}=\frac{P_i}{\omega(E_i)_i}\begin{bmatrix}\left(\left(\frac{E_1}{E_i}\right)_1-\left(\frac{E_1}{E_i}\right)_i\right) & \left(\left(\frac{E_r}{E_i}\right)_1-\left(\frac{E_r}{E_i}\right)_i\right) & \left(\left(\frac{E_N}{E_i}\right)_1-\left(\frac{E_N}{E_i}\right)_i\right)\\ & \left(\left(\frac{E_r}{E_i}\right)_r-\left(\frac{E_r}{E_i}\right)_i\right) & \\ \left(\left(\frac{E_1}{E_i}\right)_N-\left(\frac{E_1}{E_i}\right)_i\right) & \left(\left(\frac{E_r}{E_i}\right)_N-\left(\frac{E_r}{E_i}\right)_i\right) & \left(\left(\frac{E_N}{E_i}\right)_N-\left(\frac{E_N}{E_i}\right)_i\right)\end{bmatrix}^{-1}\begin{bmatrix}1\\ \vdots\\ 1\end{bmatrix} \tag{10}$$

The internal loss factors are then calculated from

$$\eta_i = \frac{\frac{P_i}{\omega} - \left\{\sum_{j=1}^{N} (E_i)_i \, \eta_{ij}\right\}_{j \neq i} + \left\{\sum_{j=1}^{N} (E_j)_i \, \eta_{ji}\right\}_{j \neq i}}{(E_i)_i} \tag{11}$$

N.B. The time averaged bars have been omitted from the E's and the P's for clarity.

The matrix containing the E's in equation (10) is generally much better conditioned than [A] in equation (8). This is because in the former case terms involving $\left(\frac{E_r}{E_i}\right)_r$, which are usually the largest, only occur on the leading diagonal.

Equations (10) and (11) will therefore give accurate values of the internal loss and coupling loss factors provided the measured input powers and subsystem energies are measured with equal accuracy. As far as the input powers are concerned, there is no fundamental reason why they should be in error. This is not true, however, of the subsystem energies. As was indicated in the discussion following equation (1), the total subsystem mass can only be used in the calculation of total energy for uniform structures of constant thickness (plate) or cross-section (beam). This is hardly ever the case for an actual vibrating structure. The difficulty can be overcome by making use of the concept of Equivalent Mass, M_{eq}. This is defined as:

$$M_{eq} = \frac{\bar{E}_{true}}{\langle \bar{V}^2 \rangle} \tag{12}$$

where $\bar{E}_{true}$ = time averaged true total subsystem energy.

It can be shown [3] that

$$(M_{eq})_i = \frac{\bar{P}_i}{0.23\gamma_i \langle \bar{V}_i^2 \rangle} \tag{13}$$

where γ_i = initial energy decay rate of subsystem i when the power to it ($\bar{P}_i$) is suddenly switched off.

3. APPLICATION EXAMPLE - AUTOMOTIVE ENGINE/TRANSMISSION

A basic model of an engine and transmission assembly can be considered as being built up of five component subsystems, one for each important sound radiating region - see fig. 3. The model could, of course, be made more detailed if required by splitting down the more complex components such as the block further into separate subsystems. The limiting requirement as to how small the individual subsystems can be is the condition that relatively weak coupling exists between the subsystems. This generally means that the power dissipated within a subsystem be greater than the net power transmitted out of that subsystem and equipartition of vibrational energy does not exist, i.e. it would not be valid to split a single flat plate into two subsystems.

As components such as the engine block and transmission casing are very complex accurate values of coupling loss factors can only be obtained with certainty from measurements. However, theoretical values of coupling loss factors for the bolted flange joints typical of engine components have been developed [4] which can give a good approximation of typical values as is shown in fig. 1.

Also needed to form the model are values of internal loss factors and the effective masses of the components (these can be measured on a particular engine or estimated from data from similar structures), and also geometric and weight details of the components and the input power. Obtaining absolute values of input power for an engine presents some difficulties but the shape of the excitation easily predicted. Thus, a relative model which will predict the spectrum shape of the noise radiation and relative changes in level can be formed fairly simply. Input power is related to input forces and the structural response at the point of application [5]. For Diesel engines, for example, input forces have the typical spectra shown in fig. 2(a) which lead to input power spectra of the shape shown in fig. 2(b).

Fig. 3(a) shows a typical result of overall sound power from an engine structure compared with a measurement and fig. 3(b), the breakdown of sound radiation from the individual components. Fig. 4(a) shows an alternative form of output from the model in a graphical form showing net structural power flow between components and sound radiation related to the size and direction of the arrows and circles. Once formed the model can be used to predict and optimise the effects of changes to the structure, e.g. material type, component dimensions, connection changes and changes of internal damping. Fig. 4(b) shows a prediction of the effects of changing from a pressed steel to an aluminium sump and incorporating damped and isolated rocker and front covers on the engine shown as standard in fig. 4(a).

4. CONCLUDING COMMENTS

As can be seen this type of analysis can provide engineers with a valuable insight into the mechanisms, interactions and noise radiation characteristics of complex connected structures. The example given with only five subsystems and relatively few inter-connections is relatively simple and power flow paths and the effects of changes could possible have been envisaged intuitively. However, for a more complex situation this could not be done. Currently, a model is being developed for a car body structure with 32 subsystems having many interconnections and numerous power inputs. In this instance, the power flow paths and distributions of vibrational energy are not at all obvious and this type of analysis can be very enlightening.

5. REFERENCES

[1] G. Stimpson, J. Sun, E.J. Richards. Predicting sound power radiation from built-up structures using Statistical Energy Analysis. J. Sound Vib., 107(1), 107-120 (1986).

[2] D.A. Bies, S. Hamid. In situ determination of loss and coupling loss factors by power injection method. J. Sound Vib., 70(2), 187-204 (1980).

[3] N. Lalor. The experimental determination of vibrational energy balance in complex structures. Paper No. 1084 29, SIRA Conference "Stress and Vibration: Recent Developments in Industrial Measurement and Analysis", London, March 1989.

[4] R.S. Ming, G. Stimpson, N. Lalor. The effect of flanged joints on noise and vibrational transmission. Proc. IOA Spring Conference, Oxford, 619-626, April 1989.

[5] G.J. Stimpson. The use of energy techniques for the prediction of machinery noise - application to the Diesel engine. Proc. Internoise 88, Avignon, 315-318 (1988).

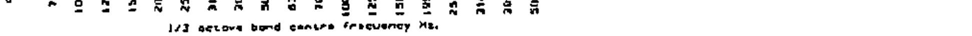


fig 1 coupling loss factors for two plates connected by a bolted flange

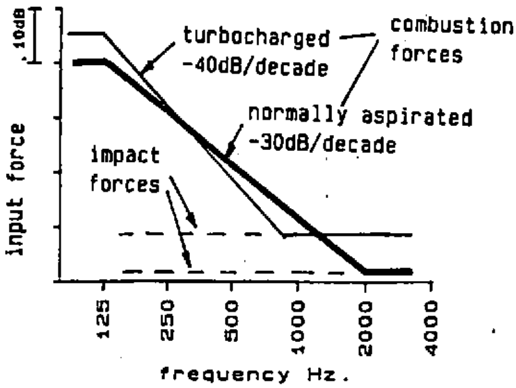


fig 2(a) input forcing spectra for a Diesel engine

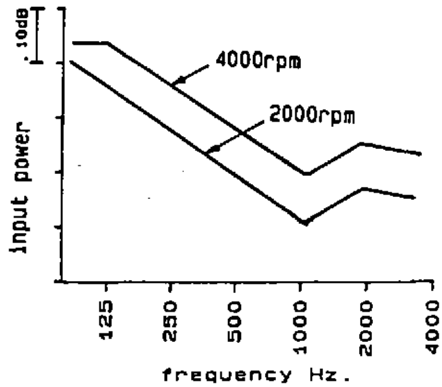


fig 2(b) input power weighting (turbocharged)

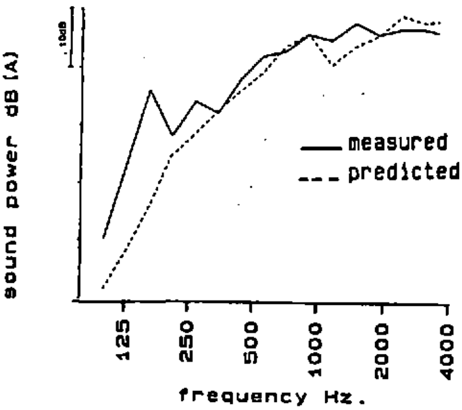


fig 3(a) total engine sound power measured and predicted

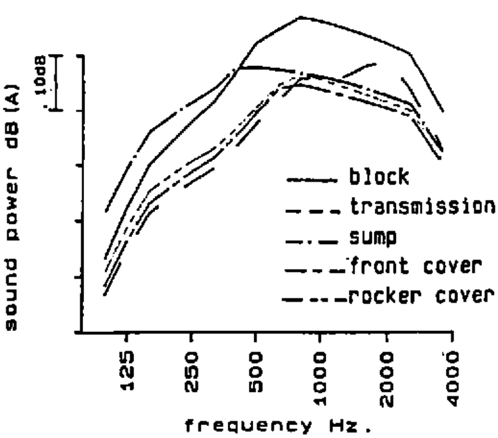
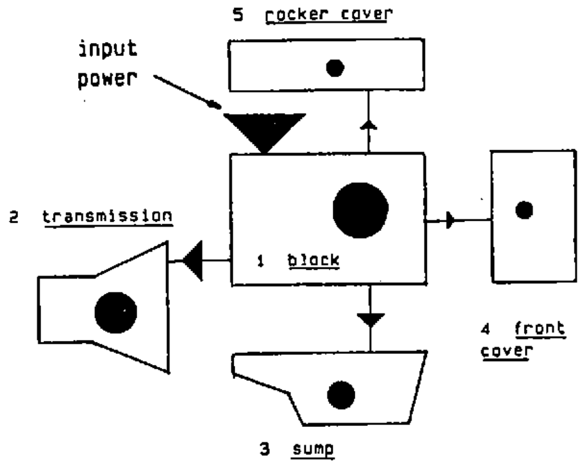


fig 3(b) predicted breakdown of sound power fig 4(a) power flow and sound radiation diagram std. engine



net structural power flow between components
-10dB

sound power radiation
-10dB

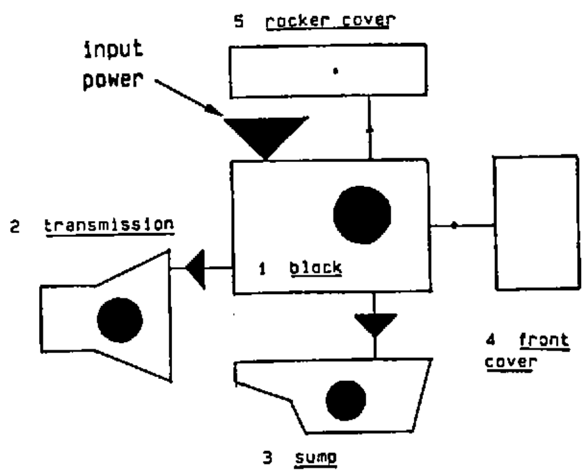


fig 4(b) power flow and sound radiation diagram modified engine