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OPERATIONAL TRANSFER PATH ANALYSIS OF A WASHING MACHINE

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Although the design process of complex mechanical systems have been improved significantly, the need for vibration and noise reduction has become even more important. In this sense, Transfer Path Analysis (TPA) can be used for the identification of significant vibration source(s) and critical transmission path(s) in a system to improve the vibro-acoustic design. The method utilized in this study is the Operational Transfer Path Analysis (OTPA) using Crosstalk Cancellation (CTC) and Singular Value Decomposition (SVD).

OTPA is implemented and used in two applications in this study. In the first application, a simple plate test case is used to carry out the basic steps of the OTPA method using both numerical simulations and experimental data in order to validate the MATLAB code developed for the OTPA based on Least Square Algorithm (LSA) and SVD techniques. Numerical simulations as well as experimental measurements on this plate structure are carried out in order to generate data for the OTPA analyses. In both cases, computed (synthesized) results are compared with the actual results in terms of responses and Transfer Functions (TFs). In the second application, the OTPA method is applied to a front-loading drum-type washing machine to determine and rank the critical vibration transmission paths. The source-transfer paths-receiver model, sensor positions and operational conditions for the analysis are explained in detail. The response locations to be used in the OTPA are determined using the results of the experimental modal analysis, which is carried out before the OTPA. Two different operational cases of a washing machine are considered and measurements at input and output channels are made for individual operational cases. The OTPA is then applied using the acquired data sets and the contributions of vibration transmission paths via individual springs and dampers of the washing machine are obtained and discussed.
Keywords: Operational Transfer Path Analysis, Least Square Algorithm, Singular Value Decomposition, Vibration Signal Analysis, Washing Machine

1. Introduction

Transfer Path Analysis (TPA) is utilized to investigate the contributions from various sources to the output and to optimise the transmission paths, in Noise, Vibration, and Harshness (NVH) studies [1]. It is a technique to determine and rank vibro-acoustic transfer paths in dynamic systems [2, 3]. This method is based upon indirectly measured operational input forces, needing the preliminary information of Transfer Functions (TFs) between input forces and responses [2]. More recently, the

Operational Transfer Path Analysis (OTPA) has been presented as an alternative to the Classical TPA [2, 4].

The concept of the OTPA technique carried out in the scope of this study is based on the Multiple Input-Multiple Output (MIMO) method introduced by Bendat [1, 5]. Noumura and Yoshida [4] presented the implementation of Singular Value Decomposition (SVD) into the OTPA to determine the linearized Transfer Functions (TFs) with an interior sound application. Riberio et al. [6] proposed the generalization of transmissibility approach, which has been widely used in OTPA applications for Multi-Degrees-of-Freedom (MDOF) systems. De Klerk and Ossipov [7] detailed the resemblances and dissimilarities between the MIMO and OTPA techniques with a tire noise application. Van der Seijs et al. [8] recently published a review paper about a general framework for TPA containing history, classification and theory of the various TPA methods including OTPA.

The main purpose of this work is to assess the performance of the OTPA in the case of a washing machine application. For this purpose, an OTPA code based on Least Square Algorithm (LSA) and SVD is developed and verified using a simple flat plate test case. OTPA is then applied to washing machine for the identification of critical vibration transfer paths. The outline of the manuscript is as follows. The theoretical background is summarised in the next section, followed by the verification of the developed computer program in the third section. The fourth section describes the application of this technique to a front-loading drum-type washing machine. Finally, in the last section, the paper is concluded with a brief summary of the main findings.

2. Theoretical background

The aim of the OTPA is to determine the contributions of various sources, propagated over different vibro-acoustic transmission ways, to the receivers under operational conditions [1, 7]. The input Degrees of Freedom (DOFs) for the OTPA are defined as the sources or excitations of the system and the output DOFs for the OTPA are defined as the receivers [1, 2, 7]. The fundamental objective of the OTPA is to find linear correlations between input (reference) and output (response) DOFs by simultaneous measurements of source/excitation and receiver/response signals of the system under operational conditions [1, 7, 8]. An arbitrary linear(ized) system model characterized by a group of input and output DOFs is indicated as:

$$\boldsymbol{H}(\boldsymbol{j\omega})\boldsymbol{x}(\boldsymbol{j\omega}) = \boldsymbol{y}(\boldsymbol{j\omega}) \tag{1}$$

where $\boldsymbol{H}(\boldsymbol{j\omega})$ is the TF matrix connecting the input vector $\boldsymbol{x}(\boldsymbol{j\omega})$ to the output vector $\boldsymbol{y}(\boldsymbol{j\omega})$ and $(\boldsymbol{j\omega})$ indicates the dependency on frequency. The OTPA method attempts to identify all the elements of the TF matrix from measured input and output data. To examine this estimation, firstly the transpose of Eq. (1) is taken and it is rewritten as:

$$\left[\boldsymbol{x}^{(\mathbf{1})}, \dots, \boldsymbol{x}^{(\boldsymbol{m})}\right] \begin{bmatrix} \boldsymbol{H}_{\mathbf{11}} & \cdots & \boldsymbol{H}_{\mathbf{1}\boldsymbol{n}} \\ \vdots & \ddots & \vdots \\ \boldsymbol{H}_{\boldsymbol{m}\mathbf{1}} & \cdots & \boldsymbol{H}_{\boldsymbol{mn}} \end{bmatrix} = \left[\boldsymbol{y}^{(\mathbf{1})}, \dots, \boldsymbol{y}^{(\boldsymbol{n})}\right] \tag{2}$$

where m and n indicate the number of reference (input) and response (output) DOFs. Here, the dependency on frequency $(\boldsymbol{j\omega})$ is dropped from Eq. (2) for the sake of brevity. Although Eq. (2) has to be computed at each frequency, a group of synchronized linearly independent operational measurement blocks is required for the determination of the TF matrix. Thus, the operational measurements are augmented r times, for which synchronous frequency spectra for each input (reference) and output (response) channel are computed. This leads to the expansion of Eq. (2) for all measurement blocks r as:

$$\begin{bmatrix} \boldsymbol{x}_{\mathbf{1}}^{(\mathbf{1})} & \cdots & \boldsymbol{x}_{\mathbf{1}}^{(\boldsymbol{m})} \\ \vdots & \ddots & \vdots \\ \boldsymbol{x}_{\boldsymbol{r}}^{(\mathbf{1})} & \cdots & \boldsymbol{x}_{\boldsymbol{r}}^{(\boldsymbol{m})} \end{bmatrix} \begin{bmatrix} \boldsymbol{H}_{\mathbf{11}} & \cdots & \boldsymbol{H}_{\mathbf{1}\boldsymbol{n}} \\ \vdots & \ddots & \vdots \\ \boldsymbol{H}_{\boldsymbol{m}\mathbf{1}} & \cdots & \boldsymbol{H}_{\boldsymbol{mn}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{y}_{\mathbf{1}}^{(\mathbf{1})} & \cdots & \boldsymbol{y}_{\mathbf{1}}^{(\boldsymbol{n})} \\ \vdots & \ddots & \vdots \\ \boldsymbol{y}_{\boldsymbol{r}}^{(\mathbf{1})} & \cdots & \boldsymbol{y}_{\boldsymbol{r}}^{(\boldsymbol{n})} \end{bmatrix} \tag{3}$$

which can be expressed in more compact form as:

$$\boldsymbol{XH} = \boldsymbol{Y} \tag{4}$$

Crosstalk between the input DOFs can cause coherency between m measured input (reference) signals [1, 7]. It is presumed that the number of input DOFs is smaller than the number of measurement blocks (i.e., $m < r$). This allows the TF matrix to be obtained as:

$$\boldsymbol{H} = \left(\boldsymbol{X}^{\boldsymbol{T}}\boldsymbol{X}\right)^{-\boldsymbol{1}}\boldsymbol{X}^{\boldsymbol{T}}\boldsymbol{Y} = \boldsymbol{X}^{+}\boldsymbol{Y} \tag{5}$$

where matrix $\boldsymbol{X}^{+}$ is the pseudo-inverse of matrix $\boldsymbol{X}$, which is given by:

$$\boldsymbol{X}^{+} = \left(\boldsymbol{X}^{\boldsymbol{T}}\boldsymbol{X}\right)^{-\boldsymbol{1}}\boldsymbol{X}^{\boldsymbol{T}} \tag{6}$$

Since the $\boldsymbol{X}$ matrix has less number of columns than rows (i.e., $m < r$), the pseudo-inverse computation becomes an overdetermined least-squares problem. At this stage, SVD can provide a general and more reliable solution [1, 7, 9]. Accordingly, the input matrix $\boldsymbol{X}$ is decomposed by a SVD as:

$$\boldsymbol{X} = \boldsymbol{U\Sigma V}^{\boldsymbol{T}} \tag{7}$$

where $\boldsymbol{U}$ is an $r \times r$ unitary column-orthogonal matrix, $\boldsymbol{V}^{\boldsymbol{T}}$ is an $m \times m$ unitary matrix that defines the conjugate transpose of an $m \times m$ unitary column-orthogonal matrix $\boldsymbol{V}$. $\boldsymbol{\Sigma}$ is $r \times m$ diagonal matrix with nonnegative numbers along the diagonal, representing singular values of the matrix $\boldsymbol{X}$. It is usually the case that very small singular values, mostly due to noisy data, are removed during this process [1, 4, 7] and the pseudo-inverse $\boldsymbol{X}^{+}$ via SVD is obtained as:

$$\boldsymbol{X}^{+} = \boldsymbol{V\Sigma}^{-\boldsymbol{1}}\boldsymbol{U}^{\boldsymbol{T}} \tag{8}$$

Finally, Eq. (8) is inserted into Eq. (5) to estimate the linearized TF matrix $\widetilde{\boldsymbol{H}}$ as:

$$\widetilde{\boldsymbol{H}} = \boldsymbol{V\Sigma}^{-\boldsymbol{1}}\boldsymbol{U}^{\boldsymbol{T}}\boldsymbol{Y} \tag{9}$$

The term $\widetilde{\boldsymbol{H}}$ in Eq. (9) indicates the synthesized TF matrix. The synthesized output (response) matrix $\widetilde{\boldsymbol{Y}}$ is computed using measured input (reference) matrix $\boldsymbol{X}$ and estimated TF matrix $\widetilde{\boldsymbol{H}}$ as:

$$\widetilde{\boldsymbol{Y}} = \boldsymbol{X}\widetilde{\boldsymbol{H}} \tag{10}$$

As can be noted from Eq. (10), the total response at the receiver location is determined by summing up the output contributions from individual inputs (sources) [1, 4]. The overall response synthesis $\widetilde{\boldsymbol{Y}}$ (i.e., the total synthesis contributions) is compared with the actual measured total response $\boldsymbol{Y}$ to represent the usefulness of the OTPA calculations [1, 4, 7].

3. Verification of the OTPA procedure: Application to a simple plate

In this section, numerical and experimental studies about the application of OTPA to a simple plate are presented in order to verify the OTPA procedure and the MATLAB [10] program developed for this investigation.

3.1 Numerical simulations

Numerical simulations of the simple plate are carried out using IDEAS [11] and ADAMS [12] environments. Boundary conditions (as illustrated in Fig. 1(a)), input and output locations are defined on the structure. Two types of analyses are used to generate desired data for comparison purposes. First of all, forced vibration analysis is carried out so as to obtain the Frequency Response Functions (FRFs) between selected input and output coordinates. Secondly, dynamic analysis is carried out by acting operational forces on input coordinates and by measuring the structural responses at the defined output coordinates. Due to the fact that the OTPA procedure is carried out in frequency domain, Fast Fourier Transform (FFT) is utilized to transform time domain data into frequency-domain in ADAMS. Three types of data (FRFs, applied operational forces and measured structural responses) are imported into MATLAB environment. A MATLAB code developed for OTPA is run to estimate the complex transfer matrix by multiplying the pseudo inverse of the input

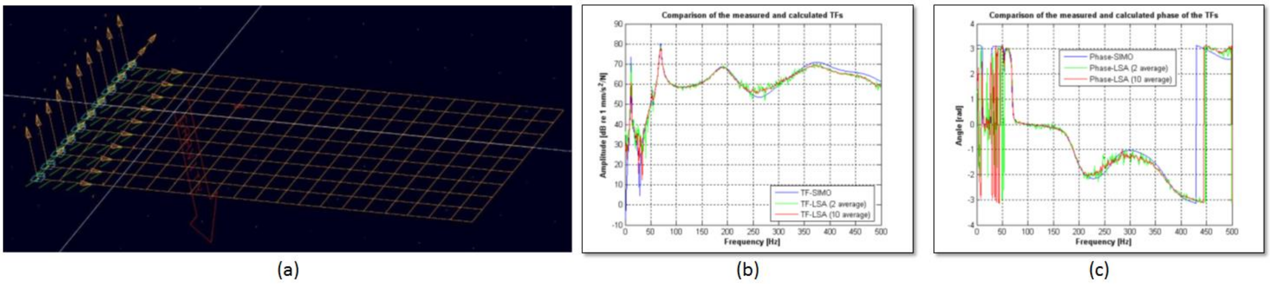

Figure 1: (a) Numerical model for the flat plate, (b) TF comparison, (c) phase angle comparison.

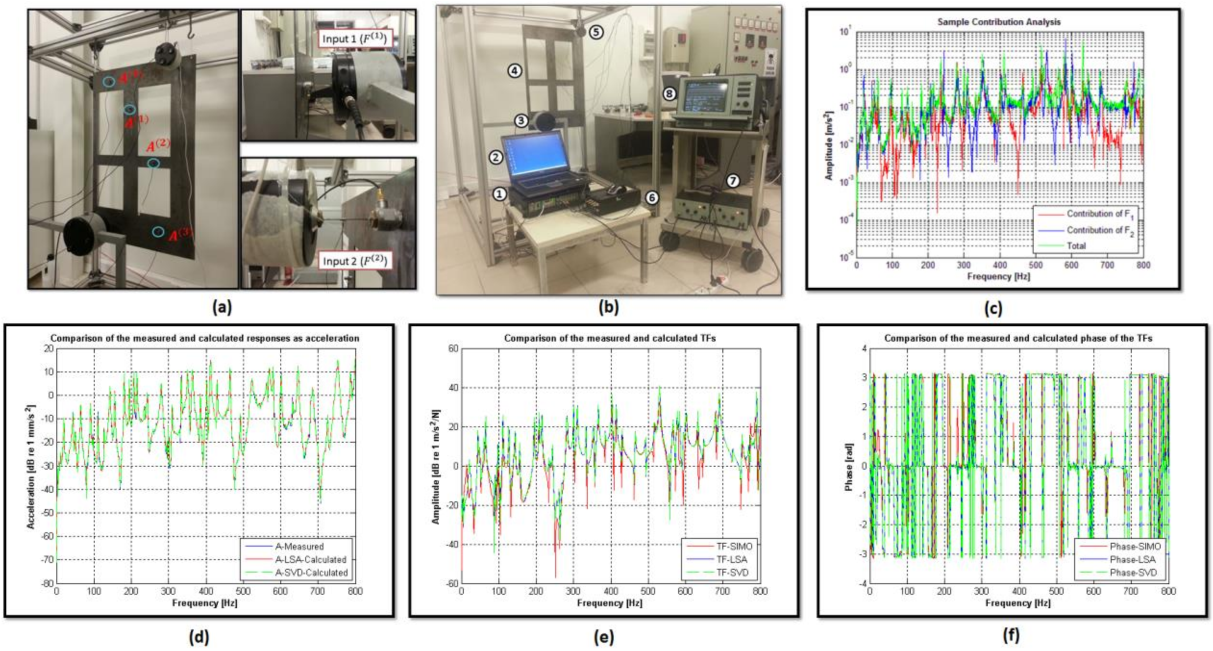

Figure 2: (a) Input and output locations, (b) test setup, (c) contribution analysis, (d) output comparison, (e) TF comparison, (f) phase angle comparison.

matrix with the measured output matrix. LSA and SVD algorithms are used for this purpose. Also, overall response (output) matrix is synthesized by using the estimated TF matrix and the input matrix. In the final step, the estimated FRFs and their phase angles are compared with their measured counterparts, as illustrated in Fig. 1(b) and Fig. 1(c), respectively. It is worth stating that measured FRFs are obtained via Single Input/Single Output (SISO) model. Although not presented here for brevity, the simulated responses at the defined output coordinates are also compared with the synthesized responses and expected results are obtained.

3.2 Experimental studies

Input locations where the operational loads are to be applied, and the output locations where the structural responses are to be measured are defined on the simple plate structure, as illustrated in Fig. 2(a) and 2(b). Considering Single Input/Multi Output (SIMO) model assumption, the single excitation is applied to the input location and structural responses are measured as accelerations at related output locations. This allows direct measurements of the FRFs between selected input and output coordinates. Then, operational forces at two excitation locations are applied simultaneously, corresponding structural responses are measured and MIMO model is used to estimate TFs. The acquired data (FRFs, applied operational input forces and the measured structural responses) are imported into MATLAB environment to process according to OTPA algorithm. At each frequency, input and output matrices are generated, LSA and SVD techniques are utilized to take the inverse of the input matrix in order to determine the operational TF (FRF) matrix. As a result, the FRFs and their phase angles obtained using SIMO and MIMO model are compared, as illustrated in Fig. 2(e) and 2(f) respectively, and the OTPA implementation in MATLAB is validated. Furthermore, synthesized output matrix computed according to Eq. (10) is allowing us to compare measured responses with synthesized responses. The results presented in Fig. 2(d) confirm the validity of the implemented OTPA procedure. A contribution analysis is also performed on this application as illustrated in Fig. 2(c).

4. Application of OTPA to a washing machine

4.1 Establishing the source-transfer path-receiver model

OTPA procedure requires an accurate establishment of source-transfer path-receiver model for reliable decomposition of critical vibro-acoustic transmission paths. The main vibration source for the front-loading washing machine under investigation here is the drive group that contains the tub, the rotary drum and the motor. The vibrations of the drive group are assumed to be transmitted to the cabinet through the suspension system that includes two damping units, two coil springs and the bellow. Vibration transmission from drive group to the cabinet through the bellow is ignored in this study. Furthermore, some preliminary vibration measurements of the washing machine tub in a steady-state at spinning cycle indicated that axial vibrations were small compared to vibrations along horizontal and vertical directions. As a result, the source is confined as the vibrations of the attachment points (along horizontal and vertical directions) of the 2 springs and 2 dampers to the tub, leading to 8 input variables for the OTPA model.

Another critical issue in the OTPA application is the selection of output variables including their number as well as their measurement locations. To decide the most significant outputs, experimental modal analysis of washing machine is performed. The main purpose of carrying out the experimental modal analysis here is to optimize the number and the location of the output variables. First of all, an experimental model of the washing machine cabinet excluding base, as given in Fig. 3(a), is created using ICATS [13] platform. The same measurement points are marked on the cabinet of the washing machine as also presented in Fig. 3(a). For the modal analysis of the washing machine, the shaker is attached to the right panel of the washing machine to excite the structure using random signal. The corresponding responses are simultaneously measured as accelerations at each of the defined measurement points. The measurements are made along the directions perpen-

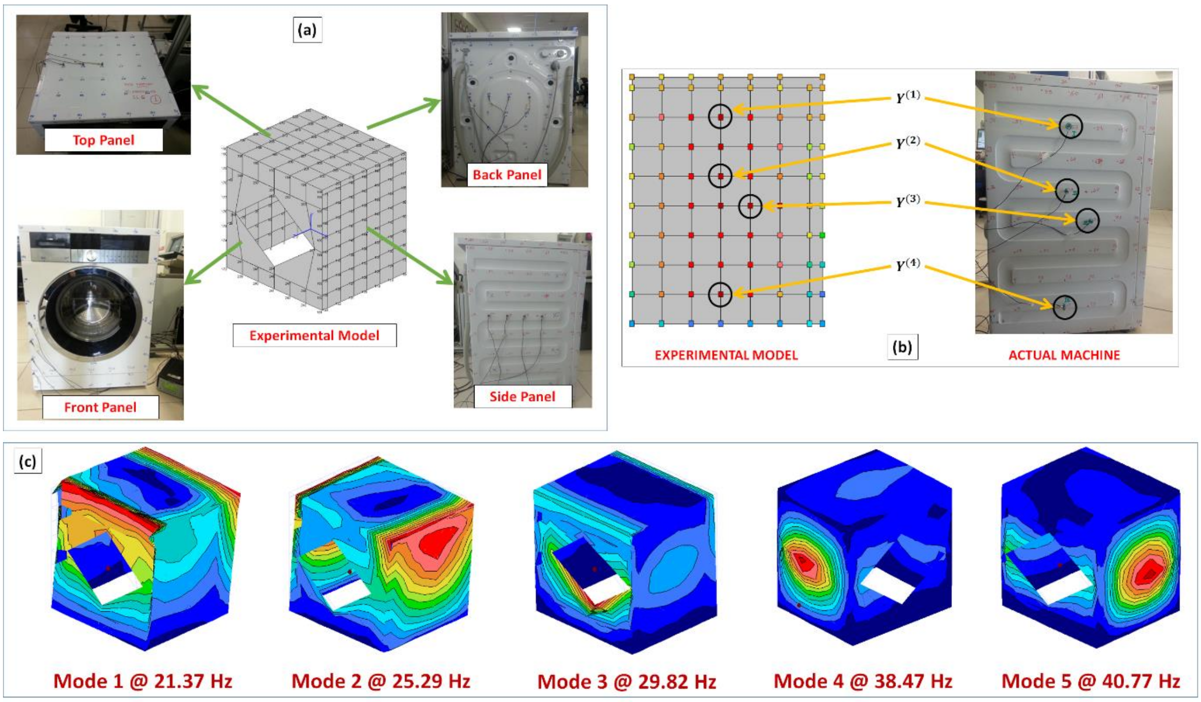

Figure 3: (a) Experimental of the washing machine, (b) selected output sensor positions on the washing machine side panel, (c) the first 5 mod shapes of the washing machine.

dicular to the surfaces of the top, front, back and side panels. In experimental modal analysis, the FRFs between measured forces and accelerations are estimated using spectrum analyser and FRFs are analysed to extract 17 natural frequencies and the corresponding mode shapes using ICATS. The first 5 mode shapes of the cabinet are presented in Fig. 3(c). These mode shapes are then used to determine the best response locations using MODPLAN module in ICATS. The selected output positions, shown in Fig. 3(b), are located on the left side panel of the washing machine.

4.2 Experimental setup, operating conditions and measurement details

A signal analyzer with 12-channel is used, allowing the measurements of 12 signals simultaneously. In order to predict the TFs, an important issue for OTPA is the selection of appropriate operational conditions for the measurements. The necessary operational conditions for the application of OTPA to a front-loading washing machine are provided by changing the influence of the centripetal force. For this washing machine application, the main source of excitation is altered by changing the unbalanced mass and the rotational frequency. In other words, the operational conditions are varied by changing the rotational speed of the washing machine drum and total unbalance mass inside it.

The washing machine is tested at different rotational speeds, from 400 to 1400 rpm, with a step of 100 rpm. Moreover, two critical spin cycles at 1200 rpm and 1400 rpm are retested for different unbalanced masses. Two variations are created for the placement of the unbalanced mass in the drum. In the first variation, a single unbalanced mass (a rubber plate) is attached to the periphery of the spin drum, the amount of individual unbalanced masses ranging from 100 gr to 500 gr with an increment of 50 gr and 475 gr. For the second variation, in addition to the individual unbalance masses in the first variation, two rubber plates, each with a mass of 800 gr, are positioned to the inner wall of the drum with equal angular distances. Furthermore, two types of damping units, so-called single effect and dual effect friction dampers, are utilized for the operational measurements. Under each operational condition, 8 input and 4 output signals are measured simultaneously. Measured signals are obtained as phase-assigned spectrums in 0-1600 Hz frequency range and the verified MATLAB code is used to process the measurement results using the OTPA procedure to obtain the required data for the identification of critical transfer paths.

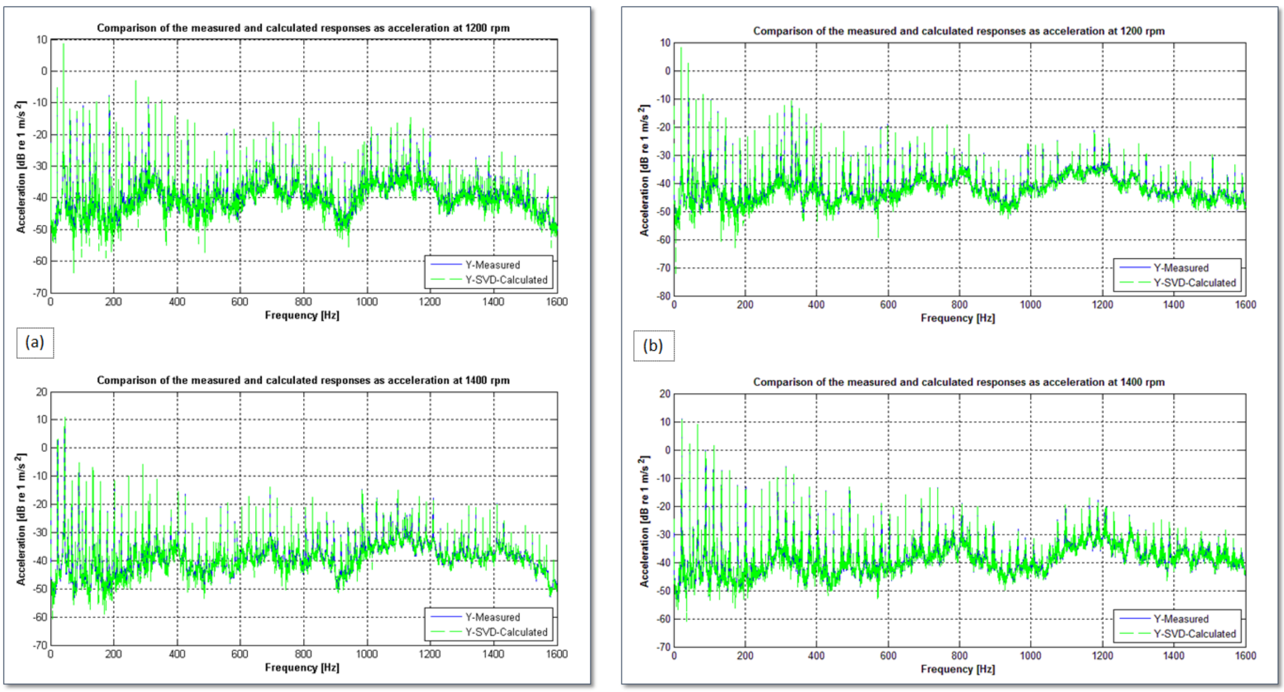

Figure 4: Comparison of measured (**blue**) and calculated (**green**) responses $\boldsymbol{Y}_r^{(n)}$ in dB scale at 1200 rpm (top) and 1400 rpm (bottom) (a) with single effect friction dampers set, (b) with dual effect friction dampers.

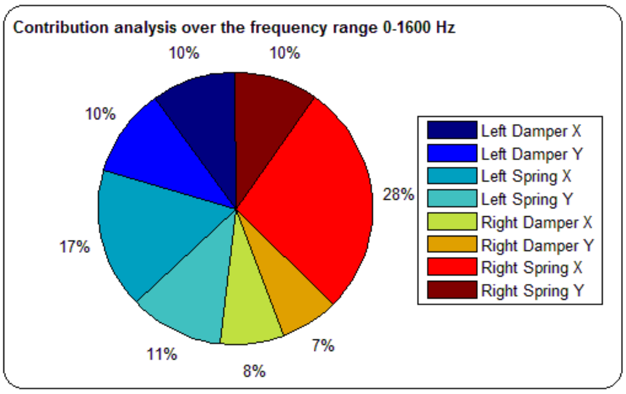

Figure 5: Results of contribution analysis, where the single effect friction dampers are used, obtained as percentage values for the 1400 rpm measurement condition over the complete frequency range 0-1600 Hz.

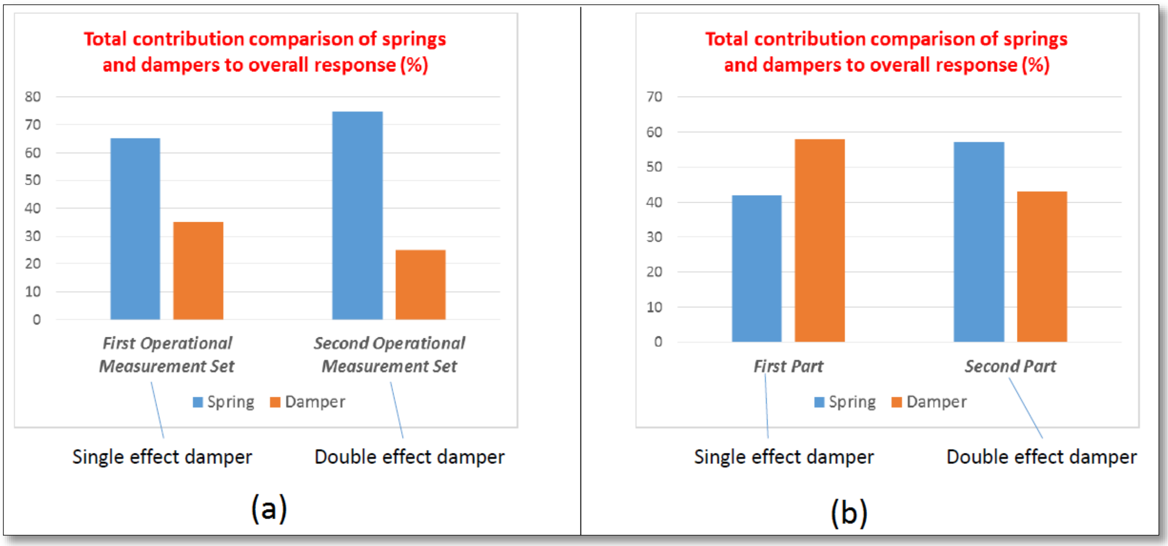

Figure 6: Comparison of total averaged contributions of springs and dampers. a) experimental case I (fixed unbalance, rpm: 400-1200 rpm), b) experimental case II (variable unbalance, 1200 rpm).

4.3 Results and discussion

After establishing the source-transfer paths-receiver model with the appropriate selection of sensor positions and operational conditions for the OTPA, TF estimation is made using the SVD method. Then, to verify the TF estimation, some comparisons of measured and calculated (synthesized) responses, shown in Fig. 4, are made for one of the response locations corresponding to using single and dual effect friction dampers. It is seen that there is a good accordance between measured and synthesized response signals.

After the validation process, the dominant transfer paths are identified by the contribution analysis where the overall response is separated into individual path contributions. A sample contribution analysis is shown in Fig. 5, which presents the result of contribution analysis as percentage values for the 1400 rpm measurement condition over the entire frequency band (0-1600 Hz). According to this contribution analysis, the most dominant transfer path is found to be the horizontal (X) component of the right spring that transmits the most vibration from the drive group (main source) to the left side panel of washing machine (receiver).

The contribution analyses are performed using single and dual operational friction dampers under two different operational cases. In case I, starting from 400 rpm, steady state input and output signals are measured. Then rpm is incremented by 100 rpm and new measurements are made. Results are then processed for OTPA and contribution analysis. For this case, i.e., case I, contribution of transmission paths through the springs and dampers are presented in Fig. 6(a). It is seen that the contributions through the springs are larger than those of the dampers. However, it is also seen that the use of the use of dual operational friction dampers reduces the contribution of the damper path. Similar tests and analyses are also performed for so-called case II where the rotational speed is kept either at 1200 or 1400 rpm and unbalance is changed. Contribution analyses this time yields the results presented in Fig. 6(b). This time, contributions through the dampers are larger than those of the springs. However, it is seen once again that the use of the use of dual operational friction dampers reduces the contribution of the transmission through the dampers.

5. Conclusions

In this study, it is aimed to apply the OTPA to a washing machine for contribution analysis. For this purpose, OTPA procedure is implemented and it is validated first using numerical and experimental results obtained from simple plate test case. It is shown that accurate synthesizing of overall response signal is not sufficient for satisfactory contribution analyses, correct estimations of MIMO FRFs and their phase angles are also required to get more reliable results. Furthermore, it is noticed that LSA and SVD techniques yield very similar result for the prediction of operational TF (or FRF) matrix if sufficient data is used. After the validation process, the OTPA method is applied to a washing machine. In the washing machine application, the sources to the transfer paths are assumed as input variables; hence transmissibility approach is used for the operational TF estimation. The operational conditions are varied by changing the rotational speed of the drum and generating various unbalanced conditions inside the spinning drum. Various contribution analyses are performed in order to rank the importance of the transmission paths along the springs and dampers of the washing machine using single and dual operational friction dampers. It is found that the use of dual operational friction dampers always reduces the contribution of the transmission through the dampers.

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