STUDY ON THE INFLUENCE OF LIQUID FILM STIFFNESS ON SHAFTING VIBRATION UNDER MIXED LUBRICATION

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Bearing stiffness plays an important role in the study of the shafting vibration, and the structural stiffness is utilized to investigate the vibration characteristics. In the operation, actually, there is a layer of thin film which resists the exterior load between the journal and bushing, the real stiffness will be changed and has an impact on the vibration. In this paper, the investigation for the mixed lubrication of stern bearing will be presented based on the average flow model and the asperity contact theory. The film stiffness will be solved with finite differential method, and the coupled stiffness can be obtained by the stiffness series, ultimately, the effects of the film stiffness on the vibration characteristics of the shafting are analysed. Results show that the film stiffness has complicated influence on the natural frequency and the vibration response of shafting.

Keywords: mixed lubrication, film stiffness, stern bearing, shafting vibration

1. Introduction

Bearing stiffness is a key parameter in the analysis of propulsion shafting vibration, and the variations of stiffness will have effects on vibration characteristics. Some researchers [1-3] have investigated the influence of bearing stiffness on shafting vibration, including the natural frequency, vibration response, and the effects of bearing location, etc. However, a layer of oil film exits between the shaft and bearing shell, the oil film stiffness is like a spring element connected with the structural stiffness, which will result in the changes of the vibration characteristics, and it’s not been analysed.

The calculation of oil film stiffness is based on the solution of film pressure. In the traditional analysis, the sliding surface is assumed to be perfectly smooth as the clearance is larger or the load is smaller, based on this, many scholars [4-7] have studied the lubrication characteristics of radial journal bearing, including the film thickness and pressure distribution, flow-rate, friction coefficient, attitude angle, leakage flow-rate, eccentricity ratio、shaft deformation, cavitation, oil groove, etc. And the results are in good agreement with the experimental results. In engineering practice, however, machined surface has complex surface morphology, when the load is heavy, surface roughness heights are typically of the same order as or one order of magnitude greater than the film thickness in contact zone, which will result in direct asperity contacts when lubricant film thickness is below a certain limit, and the applied load is then shared by both hydrodynamic lubricant film and asperity contacts. It will be generally called mixed lubrication [8]. Some researchers have done some enormous work on this: Guha[9] studied the effects of isotropic roughness on steady-state characteristics. Cem Sinanoglu[10] investigated the influence of shaft surface texture on the film pressure of journal bearing, He Tao[11] researched the mixed-lubrication of the stern bearing under with the consideration of deformation of stern shaft and cavitation, etc.

Stern bearing is the main component of the propulsion shafting, it’ll bear a larger end load as it’s close to the propeller, and the minimum film thickness will decrease greatly, and lubrication state will change from the full film lubrication to the mixed lubrication. In this paper, thus, the oil film pressure and stiffness will be calculated under the mixed lubrication state, and the effects of oil film pressure on shafting vibration will be studied.

2. Theoretical analysis

The basic theory and method of mixed lubrication will be given firstly, the theory of asperity contact force and the numerical method of film stiffness will also be introduced in this section.

2.1 Lubrication theory of journal bearing

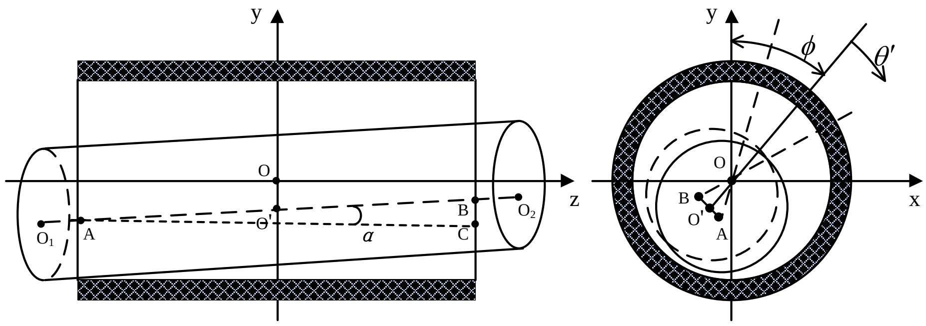


Fig1 Journal bearing

Fig1 shows the model of radial journal bearing, θ' denotes the circumferential angle, ϕ denotes attitude angle, α is the tilt angle, O is the centre of bearing shell and O' indicates the centre of shaft, A and B is the intersection of journal centre line and end face.

2.1.1 Film thickness equation

The film thickness formula is the following equation, and it's the 1st step to calculate the Reynolds equation.

The film thickness formula is equation(1), and it's the 1^{st} step to calculate the Reynolds equation.

$$h = c + e\cos(\theta - \phi) \tag{1}$$

Where c denotes clearance, e indicates eccentricity [12].

$$h_T = E(h) = \int_{-h}^{\infty} (h+\delta) f(\delta) d\delta \tag{2}$$

$f(\delta)$ is the probability density function of combined roughness, for a Gaussian distribution:

$$f(\delta) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{\delta^2}{2\sigma^2}\right) \tag{3}$$

Substituting equation(3) into equation(2) gives:

$$h_T = \frac{1}{\sigma\sqrt{2\pi}} \int_{-h}^{\infty} (h+\delta) e^{-\delta^2/2\sigma^2} d\delta \tag{4}$$

And after performing integration [13]:

$$h_T = \frac{h}{2}\left[1 + erf\left(\frac{h}{\sqrt{2}\sigma}\right)\right] + \frac{\sigma}{\sqrt{2\pi}} e^{-h^2/2\sigma^2} \tag{5}$$

Where erf(・) is error function.

2.1.2 Average Reynolds equation

In order to determine the effects of surface roughness on partially lubricated contacts, Patir and Cheng [12] proposed the average Reynolds equation in 1978, this equation has been widely used in the field of mixed lubrication.

$$\frac{\partial}{\partial x}\left(\phi_x \frac{h^3}{12\eta}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial y}\left(\phi_y \frac{h^3}{12\eta}\frac{\partial p}{\partial y}\right)=\frac{U_1+U_2}{2}\frac{\partial h_T}{\partial x}+\frac{U_1-U_2}{2}\sigma\frac{\partial \phi_s}{\partial x}+\frac{\partial h_T}{\partial t} \tag{6}$$

Where, p = film pressure, η = lubricant viscosity, x = the radical direction , y = length direction of bearing, U_1, U_2 = the velocity of two surfaces, ϕ_x, ϕ_y =pressure flow factors, ϕ_s =shear flow factor, σ_1 σ_2 are the standard deviations of journal and bearing surfaces roughness, $\sigma=\sqrt{\sigma_1^2+\sigma_2^2}$ = the standard deviations of combined roughness.

As the bearing shell is stationary while the shafting is rotating, thus $U_2=0, U_1=U$.

Substituting equation(5) and (7) into equation(6),

$$\left.\begin{aligned}&\theta=\frac{x}{R},\ \bar{y}=\frac{2y}{B},\ \bar{h}=\frac{h}{c}=\varepsilon\cos(\theta-\varphi),\\&\bar{h}_T=\frac{h_T}{c},\ \bar{p}=\frac{pc^2}{\eta\omega R^2},\ \omega=\frac{U}{R},\ \Lambda=\frac{c}{\sigma}\end{aligned}\right\} \tag{7}$$

then we can get：

$$\frac{\partial}{\partial\theta}\left(\phi_x\bar{h}^3\frac{\partial\bar{p}}{\partial\theta}\right)+\left(\frac{2R}{B}\right)^2\frac{\partial}{\partial\bar{y}}\left(\phi_y\bar{h}^3\frac{\partial\bar{p}}{\partial\bar{y}}\right)=-3\varepsilon\sin\left(\theta-\varphi\right)\left[1+erf\left(\frac{\Lambda\bar{h}}{\sqrt{2}}\right)\right]+\frac{6}{\Lambda}\frac{\partial\phi_s}{\partial\theta}+\frac{6}{\omega}\left[\dot{\varepsilon}\cos\left(\theta-\varphi\right)+\varepsilon\dot{\varphi}\sin\left(\theta-\varphi\right)\right]\left[1+erf\left(\frac{\Lambda\bar{h}}{\sqrt{2}}\right)\right] \tag{8}$$

The time item (or squeeze item) can be ignored if it’s the steady state, and so the steady state average Reynolds equation is given as follows:

$$\frac{\partial}{\partial\theta}\left(\phi_x\bar{h}^3\frac{\partial\bar{p}}{\partial\theta}\right)+\left(\frac{2R}{B}\right)^2\frac{\partial}{\partial\bar{y}}\left(\phi_y\bar{h}^3\frac{\partial\bar{p}}{\partial\bar{y}}\right)=-3\varepsilon\sin\left(\theta-\varphi\right)\left[1+erf\left(\frac{\Lambda\bar{h}}{\sqrt{2}}\right)\right]+\frac{6}{\Lambda}\frac{\partial\phi_s}{\partial\theta} \tag{9}$$

Equation(8) and (9) can be solved by finite difference method and successive over relaxation method.

2.2 Asperity contact model

The asperity contact load can be calculated with the contact theory proposed by Greenwood and Trip [14].

Film thickness ratio：

$$H=\frac{h}{\sigma}=\frac{h}{\sqrt{\sigma_1^2+\sigma_2^2}} \tag{10}$$

If two surfaces are both rough, the nominal contact pressure can be gained by the following equation:

$$p_c(h)=(\frac{16\sqrt{2}}{15})\pi(\eta\beta\sigma)^2E'\sqrt{\frac{\sigma}{\beta}}F_{5/2}(\frac{h}{\sigma}) \tag{11}$$

Contact area：

$$A_c(h)=\pi^2(\eta\beta\sigma)^2F_2(\frac{h}{\sigma}) \tag{12}$$

Where η is the surface density of asperity peaks on either surface, β is the radius of curvature at the peak, E' is the composite elastic modulus.

The function $F_{5/2}$ and F_2 are shown in equation (13)~(14).

$$F_{5/2}(H) = \begin{cases} 2.134\times10^{-4}\left\{3.804\ln(4-H)+1.34\left[\ln(4-H)\right]^2\right\} & (H \le 3.5) \\ 1.12\times10^{-4}(4-H)^{1.9447} & (4 \ge H > 3.5) \\ 0 & (H > 4) \end{cases} \tag{13}$$

$$F_2(H) = \begin{cases} 1.705\times10^{-4}\left\{4.504\ln(4-H)+1.37\left[\ln(4-H)\right]^2\right\} & (H \le 3.5) \\ 8.8123\times10^{-5}(4-H)^{2.15} & (4 \ge H > 3.5) \\ 0 & (H > 4) \end{cases} \tag{14}$$

2.3 Oil film stiffness coefficients

The oil film stiffness of radical journal bearing represents the response caused by the perturbation of displacement and velocity nearby the position of steady state. As the film pressure distribution is nonlinear, the film stiffness is nonlinear as well. The general numeric method [15] are the finite difference method, partial derivative method, etc. In this paper, the oil film stiffness will be calculated with FDM.

$$K_{xx}=\frac{\partial F_x}{\partial x},\ K_{xy}=\frac{\partial F_x}{\partial y},\ K_{yx}=\frac{\partial F_y}{\partial x},\ K_{yy}=\frac{\partial F_y}{\partial y} \tag{15}$$

The differential forms are as follow:

$$\begin{aligned} K_{xx}&=\left|\frac{F_{x1}-F_{x2}}{2\Delta x}\right|, K_{xy}=\left|\frac{F_{x3}-F_{x4}}{2\Delta y}\right| \\ K_{yx}&=\left|\frac{F_{y1}-F_{y2}}{2\Delta x}\right|, K_{xy}=\left|\frac{F_{y3}-F_{y4}}{2\Delta y}\right| \end{aligned} \tag{16}$$

Applying the displacement perturbation $\pm\Delta x$ and $\pm\Delta y$ into the journal centre point, which is in the steady state, the new eccentricity and attitude angle, can be gained according to geometric position, and the new oil film force of horizontal and vertical direction can be calculated finally. The flow chart of solution procedure is as follows:

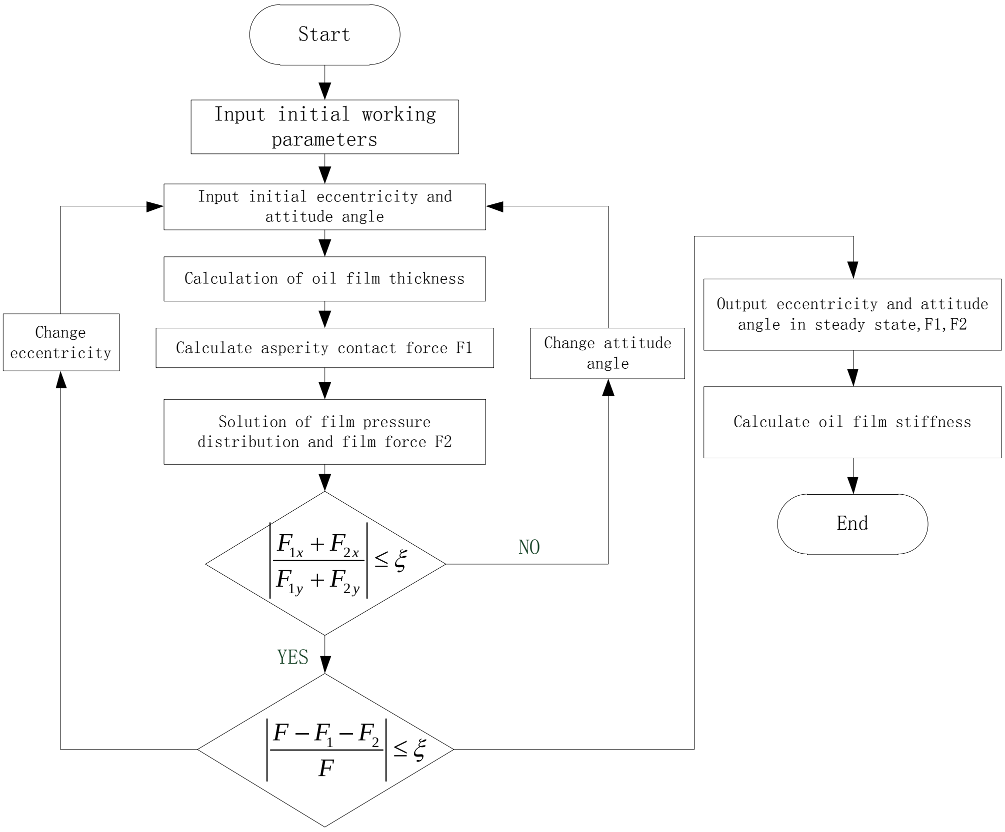
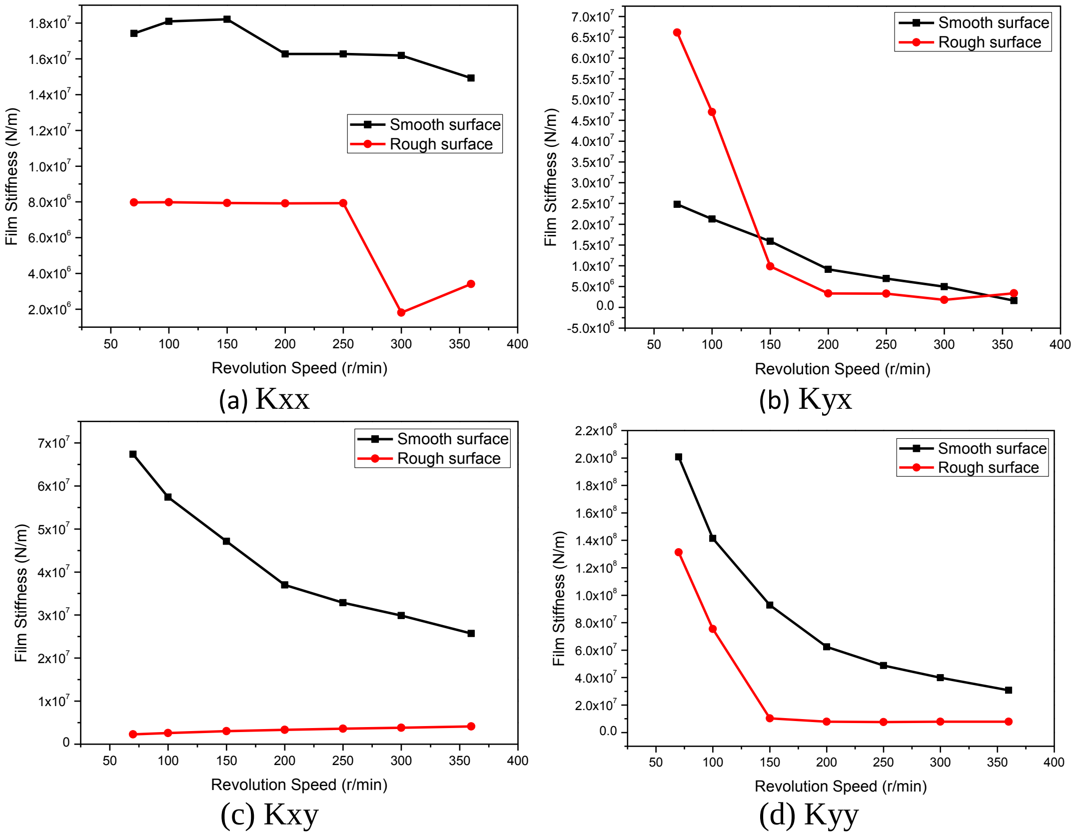


Fig2 Flow chart of solution procedure

2.4 Comparison of smooth and rough surfaces

In order to investigate the influence of rough surface on fluid film stiffness, the relevant comparative analysis has been presented as follows.



(a) Kxx (b) Kyx

(c) Kxy (d) Kyy

Fig3 Comparison analysis

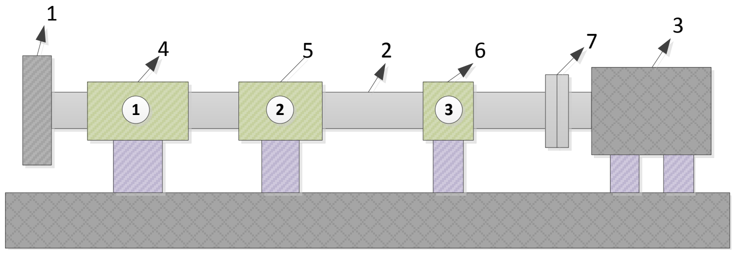
Fig3 shows that, with the increase of revolution speed, film stiffness decreases between smooth surfaces. For the rough surfaces, the film stiffness Kxx hasn’t obvious tendency, Kyx and Kyy decrease while Kxy increases. Overall, the stiffness Kxx, Kxy and Kyy between smooth surfaces is larger than rough surfaces, the Kyx in two types of surface topography alternate variation. The results show that the roughness has apparent influence on the oil film stiffness.

3. Results and discussion

In this section, the oil film stiffness of stern bearing will be included in the analysis of shafting vibration. And based on this, the natural vibration characteristics and response characteristics of the ship propulsion shafting will be studied.

3.1 Shafting model

The shafting model is built with ANSYS. BEAM188 element is used to simulate the shafting, the Spring-Damper14 element is selected to substitute the bearings, and the propeller is simulated by the MASS21 element. The engine is simulated by the element of SOLID185.



1-Propeller, 2-Ship shafting, 3-Main engine, 4-Stern bearing, 5-Intermediate bearing，6-Thrust bearing, 7-Coupler

Fig4 Shafting model

Fig4 shows the sketch map on the left and the FEM model on the right. Among the three bearings, the stern and intermediate bearings are water lubricated, the thrust bearings is oil lubricated. The signs ①~③ are the reference points of vibration response.

3.2 Calculation of Film stiffness

The revolution speed is set as 80r/min and the parameters of bearing are as follows:

Table1 The basic parameters of stern bearing

Length/m	Diameter/m	Clearance/mm	Viscosity/(Pa s)	Load/N
0.3	0.105	0.4	0.025	4000

Table2 Stiffness of stern bearing

Structural stiffness	Film Stiffness	Coupling Stiffness
1.00E+09	1.50E+09	5.9955e+08

The relationship of the two stiffness is like spring’s series, thus the coupling stiffness is less than the either one.

3.3 Analysis of shafting vibration

Table 3 Natural frequency (Hz)

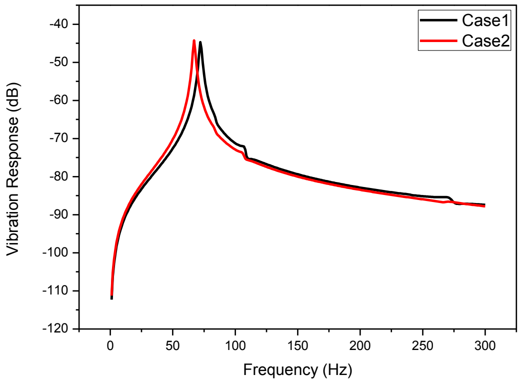
Order	Case1	Case2	$\triangle f$
1	72.19	67.05	5.14
2	84.25	83.82	0.43
3	108.18	106.92	1.26

4	240. 22	239. 99	0. 23
5	273. 065	268. 68	4. 385

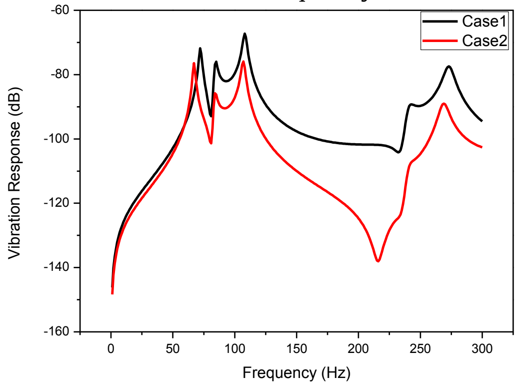
Case1: The initial condition

Case2: The oil film stiffness of stern bearing is considered.

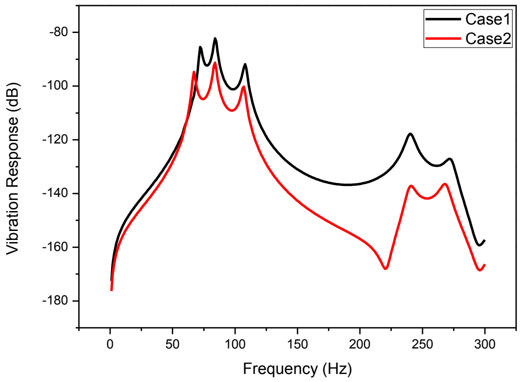
Table3 shows that Case2's frequency is less than Case1, and the 1st and 5th order frequency has larger change. The results indicate that oil film stiffness has an effects on the natural frequency.



（a）Point1



（b）Point2



（c）Point3

Fig5 Comparison of vibration response

Fig5 shows that the contrast results between Case1 and Case2. As a whole, Case2's peaks are lower than Case1. At point1, the curves are basic coincidence while the 1st peak of Case2 is on the left of Case1, which corresponds the natural frequency. At point2, Case2's curve is below Case1, the variation of 4th and 5th order peaks is larger. At point3, the tendency of the curve is consistent, the changes of the first three peaks are less than the 4th and 5th order peaks. The results show that the oil film stiffness has a greater influence on the vibration response of the higher natural frequency.

4. Conclusion

1. With the increase of speed, the oil film stiffness will decrease in both smooth and rough surfaces.
2. The oil film stiffness between smooth surfaces is larger than the rough surfaces.
3. The natural frequency will decrease under the consideration of the oil film stiffness.
4. Under the coupling stiffness, the vibration response has a downward trend, and it's more apparent at high frequency.

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