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THE COUPLING OF BEM AND TLM METHOD TO SOLVE TRANSIENT ACOUSTIC PROBLEMS IN LARGE AREA - TWO DIMENSIONAL CASE

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1. INTRODUCTION

Airborne ultrasound based sensor systems have been applied to a variety of problems in robotics and advanced manufacturing. The mathematics describing the reflection of sound waves by objects are highly complex, and there are very few analytical solutions available. Any theoretical prediction has to be based on some form of numerical model. The Transmission Line Matrix (TLM) method can in principle provide this model.

However, like others domain-type modelling methods, such as finite element and finite difference methods, the TLM method need to discretize the domain of problems under consideration into a number of transmission line elements or nodes. There is a limitation of TLM which states that there must be at least 10, ideally 20 nodes per wavelength to give accurate results. These limit the model size that can be calculated within a reasonable time, even on a large computer.

To get around this problem, it is proposed to employ a hybrid technique. TLM will be used to give the near field solution for the space immediately around the target. The Boundary Element Method (BEM) will then be used to provide far field results at the transducer positions.

In the paper, the feasibility of combination of TLM method with BEM to model large two dimensional area has been presented. Some test examples have been demonstrated the remarkably good accuracy of the results and validate the application of this hybrid model to solve radiation and scattering problems.

2. BEM FOR TWO DIMENSIONAL TRANSIENT SCALAR WAVE EQUATION

2.1 Governing Equations

The governing equation of transient sound wave is the scalar wave equation and boundary conditions,

$$\nabla^2 p - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0 \quad in\,\Omega\,(closed\ field), \qquad \left.\begin{aligned} p &= \bar{p} \quad on\,\Gamma_1 \\ q &= \frac{\partial p}{\partial n} = \bar{q} \quad on\,\Gamma_2 \end{aligned}\right\} (boundary), \tag{1}$$

where p is the pressure, t the time and c the velocity of wave propagation. ∇^2 is the Laplace operator. q is the normal derivative of p, $\bar{q}$ and $\bar{p}$ indicate respectively known values of q and p on Γ (= $\Gamma_1 + \Gamma_2$).

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2.2 Boundary Integral Representation

The boundary integral solution to the exterior acoustic problem corresponding to the above equations is well known as

$$4\pi p(\xi,t) = \int_0^{t^+}\int_\Gamma p^* \, q \; d\Gamma d\tau + \int_0^{t^+}\int_\Gamma \frac{\partial R}{\partial n}\left(W p + \frac{p^*}{c}\,\frac{\partial p}{\partial t}\right) d\Gamma \; d\tau \;, \tag{2}$$

where

$$p^* = p^*(r,t/\xi,\tau) = \frac{2c}{\sqrt{c^2(t-\tau)^2 - R^2}}\; H[c(t-\tau) - R] \;, \tag{3a}$$

$$W = W(r,t/\xi,\tau) = \frac{2c[c(t-\tau) - R]}{\sqrt{[c^2(t-\tau)^2 - R^2]^3}}\; H[c(t-\tau) - R] \;. \tag{3b}$$

In the expressions above H(·) stands for the Heaviside (step) function. Note that R is the distance between source point x and the field (observation) point ξ, i.e., $R = |x - \xi|$.

2.3 Numerical Formulations

To discretize equation (2), the boundary Γ is divided into a series of segments or boundary elements. The equation (2) can be expressed in following algebraic equation [1]

$$4\pi p(\xi, t_N) = \sum_{n=1}^{N} \left\{ (A^n)^T \cdot \hat{U}^n + (D^n)^T \cdot P^n \right\} , \tag{4}$$

here the coefficient matrices $\{A^n\}$ and $\{D^n\}$ are determined from the geometrical data, the medium property, and the interpolation functions for variables q and p in the elements. In this paper, linear variations are assumed for both of q and p in an element. $\hat{U}^n$ and P^n are the known nodal values at a set of discrete points Γ_j (nodes) on the Γ boundary, $j=1,\cdots,J$ and a set of values of time t_n, $n=1,\cdots,N$. The symbols · and T indicate the vector multiplication and transformation in matrix calculations respectively.

3. TRANSMISSION LINE MATRIX (TLM) MODELLING

3.1 The Basis of TLM

TLM is a method which physically models wave propagation. The medium through which the waves propagate is represented as a cartesian mesh of electrical transmission lines. The transmission lines are joined where they cross, the junctions being termed nodes. It can be demonstrated [2] that as long as the spacing between adjacent nodes is less than on tenth of a wavelength of the ultrasound frequency being studied, waves will propagate on the mesh.

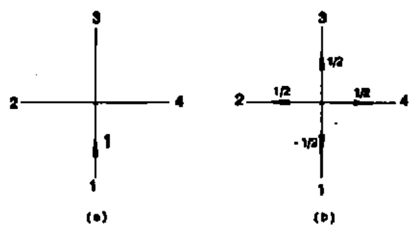

Figure 1 Two processes in the TLM modelling

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3.2 Numerical Treatment

Consider a sound-impulse delta function of unit magnitude introduced into terminal 1 (${}_0V_1^i = 1$) of the basic matrix element shown in figure 1(a). The magnitude of the pressure impulse at the junction is 1/2. Pressure impulse(${}_1V_n^r$, n=1,2,3,4) of 1/2 will be launched into lines 2, 3 and 4, while a reflected pulse of -1/2 appears on line 1, as shown below in figure 1(b).

Briefly, there are main two parts of the process, one is known as scattering given in the equation below,

$$ {}_{k+1}V_n^r = \frac{1}{2}\left[\sum_{m=1}^{4} {}_kV_m^i\right] - {}_kV_n^r , \qquad (5) $$

here k is the number of iterations.

In the next iteration the scattered pulses will be incident on four new nodes,as known as connection process. At each node the process will be repeated and the pulses being injected will disperse through the network.

Boundaries in the propagating medium are achieved by placing resistive loads at nodes. When sound travelling in air interacts with a solid the result is a total reflection, so object boundaries are modelled as open circuits. The edges of the mesh should behave as closely as possible to a free space boundary, and in this paper are modelled with a matching impedance to give a zero reflection coefficient.

4. HYBRID MODEL OF BEM AND TLM

As noted, the TLM method is especially suitable for a finite region because it has to divide the whole region which includes both the receiving points and sources into a fine grid. The larger this region is, the more data on nodes of grid need to be stored. When the volume of data exceeds a threshold for a given computer, the so-called swapping problem which causes a frequent exchange between the physical and virtual ROM will happen. It will occupy the main part of CPU time so that the running time in the modelling become unacceptable. However, in practice for a given source or target, the acoustic behaviour of near and far fields are generally of more interest than those in the between. It is obvious that the sole TLM strategy does much extra work and is therefore very inefficient.

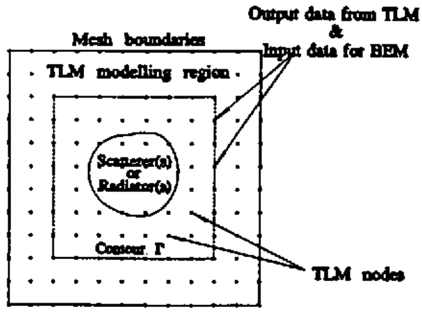

Figure 2 TLM mesh in hybrid TLM+BEM method

An improvement in the modelling efficiencies will be obtained by using the TLM method to model the acoustic behaviour in the near field and BEM to model the acoustic behaviour in the far field.

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Taking an acoustic scatterer of arbitrary shape as an example, the hybrid strategy is as follows. Firstly, in a TLM mesh as shown in Figure 2 a rectangular contour Γ is located at distance of about 20-30 nodes from the surface of the scatterer. In this mesh the TLM method is used to model the acoustic behaviour of the scatterer in the near field. A plane wave (in principle, any type of excitation [3]) can be generated to illuminate the scatterer. The mesh boundaries are terminated with a matching impedance to give a zero reflection coefficient. This type of mesh configuration has been developed in FD-TD method, originated by Umashankar and Taflove [4] in terms of a Huygen's source formulation. It was first applied to the TLM modelling in terms of the TLM formulation by Simons and Bridges [3] recently.

Secondly, the time series of pressure and normal velocity at each node on the rectangular contour Γ are derived and stored at each iteration processing. The relationships of sound pressure p and particle velocities u_x and u_y at (x,y) with TLM node vector are as follows,

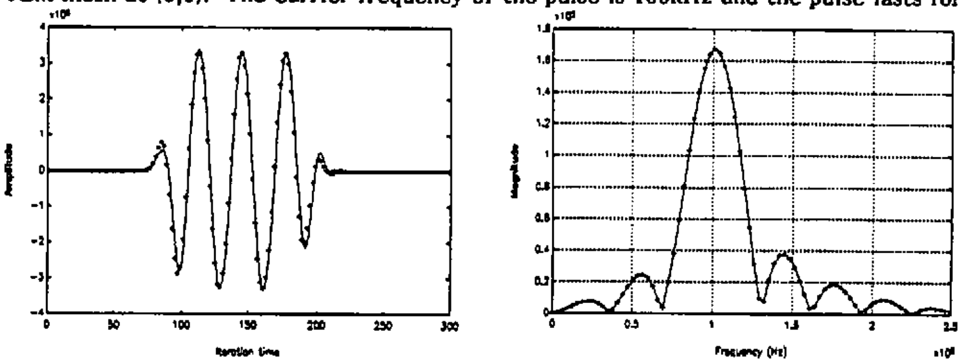
$$ {}_kp(x,y)=\frac{1}{2}\sum_{m=1}^{4}({}_kV_m^i), \qquad {}_ku_x(x,y)=({}_kV_1^i - {}_kV_3^i)/Z_0, \qquad {}_ku_y(x,y)=-({}_kV_2^i - {}_kV_4^i)/Z_0, \qquad (6) $$

where Z_0 is the characteristic impedance of the mesh lines.

Finally, these data are linked as equivalent pressure sources and velocity sources respectively with the BEM algorithm to predict the far field pressure we are interested in.
Substituting acoustic scatterers with acoustic radiators in Figure 2, the hybrid method can be applied to radiation problems.

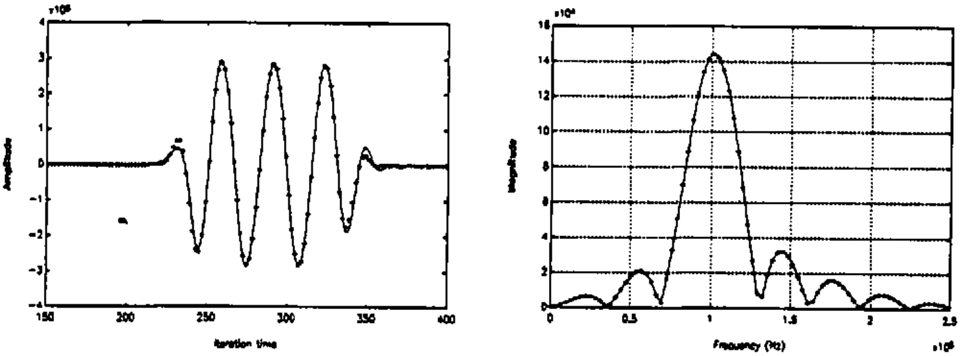
5. TEST PROBLEMS

Case 1: A line source with a continuous wave (CW) pulse excitation is placed in a 300×300 TLM mesh at (0,0). The carrier frequency of the pulse is 100kHz and the pulse lasts for



(a) Time domain waveform (b) Frequency domain waveform

Figure 3 The output from a line source with CW impulse excitation at receiving point (0, 300) four cycles. The radio of mesh size Δl and carried wavelength λ_c is chosen to be $\Delta l/\lambda_c$=0.05. In order to reduce high frequency components of the pulse a Tukey window is used to modulate the amplitude of pulse.



(a) Time domain waveform

(b) Frequency domain waveform

Figure 4 The output from a line source with CW impulse excitation at receiving point (45, 400)

The waveforms and their Fourier transformations at receiving points (0, 300) and (45, 400) are shown in Figure 3 and 4, respectively. The results from sole TLM modelling are represented as solid lines and those from TLM + BEM are represented as '+' symbols.

Case 2: A cube target whose dimensionless acoustic parameter is 16.5×16.9 (ka × kb) is located in the middle of 300×300 TLM mesh at coordinates (0,0), as shown in Figure 5. The same CW pulse plane wave as defined in case 1 illuminates the scatterer. The parameter of $\Delta l/\lambda_c$=0.05 is used. The results from selected receiving points are given in Figure 6 and Figure 7.

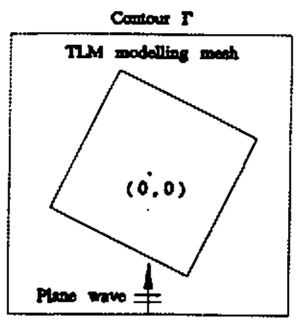
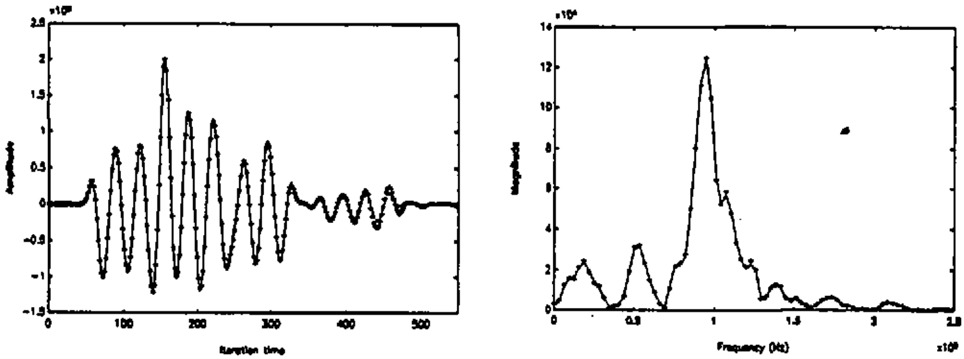


Figure 5 A cube target illuminated by a plane wave

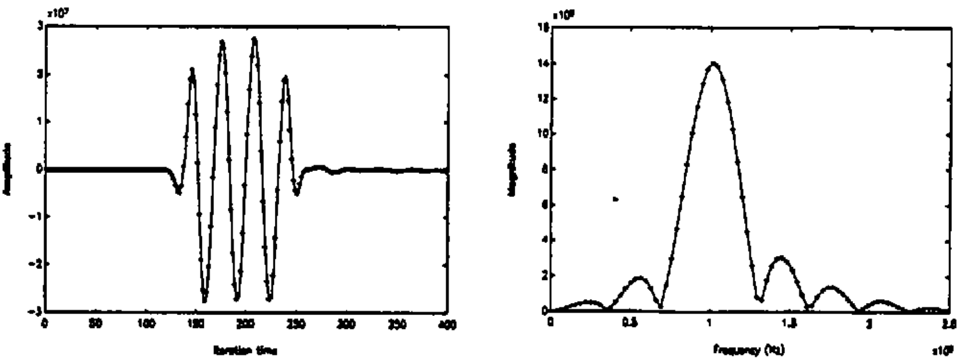
In the above TLM + BEM simulations, the parameter $\Delta l/\lambda$ in BEM modelling is chosen to be the same as that in TLM modelling. In these cases, 200 iterations took about 1.2 hours for transforming near field data from the TLM modelling to one receiving point of interest by BEM modelling. But BEM does not have the advantage of the TLM method which is able to extract information from all receiving points of interest with a single simulation. The computation time for BEM is directly proportional to the numbers of receiving points. Therefore, it is essential to reduce the computation time of single receiving point without losing accuracy in the BEM system.

It is noticed that BEM modelling is actually independent of the TLM modelling apart from the input data which comes from the TLM modelling. This gives us freedom to chose the parameters in the BEM modelling. Because the running time and accuracy of our BEM program mainly relate to the parameter $\Delta l/\lambda$, the method of choosing $\Delta l/\lambda$ is very important. In principle, an arbitrary value of $\Delta l/\lambda$ can be chosen in the BEM system by using an interpolation algorithm to the input data from the TLM modelling.



(a) Time domain waveform (b) Frequency domain response

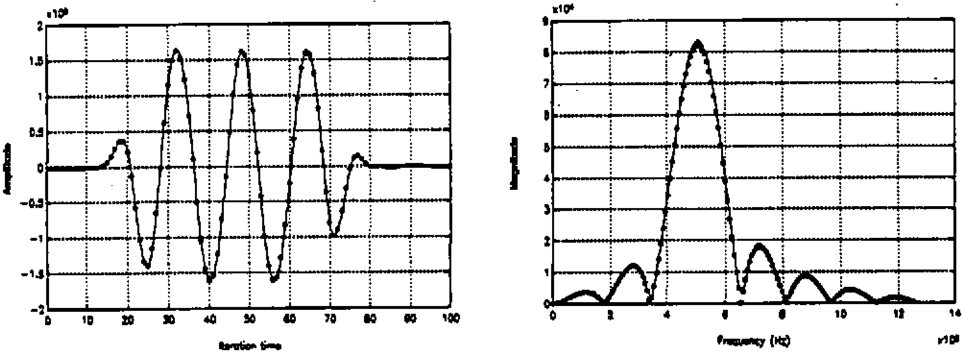
Figure 6 The scattering output from a cube target at receiving point (0, -350)



(a) Time domain waveform (b) Frequency domain response

Figure 7 The scattering output from a cube target at receiving point (0, 200)

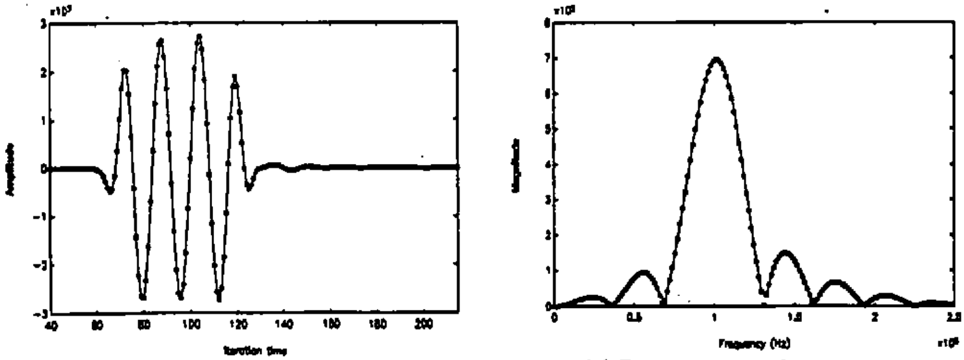
For instance, the results above are recalculated by only using those input data from the TLM modelling which have odd numbers both in space and time. It simply means that the BEM modelling use a mesh with a mesh size and time step which is double than those used in the TLM modelling. Therefore, the value of $\Delta l/\lambda$ is chosen to be $\Delta l/\lambda_c = 0.1$. The rerun results (indicated as '*' symbols) are shown in Figure 8 and Figure 9, together with the former results (indicated as solid line). 200 iterations cost about 0.14 hours for transforming near field data from the TLM modelling to the received point of interest by BEM modelling. Compared with the running time for same iterations using the fine mesh (1.2 hours), the running time is reduced by a factor of 7.5 when a coarse mesh is used.



(a) Time domain waveform

(b) Frequency domain response

Figure 8 The rerun results for a line source with the CW impulse excitation at received point (0, 300)



(a) Time domain waveform

(b) Frequency domain response

Figure 9 The rerun scattering output from a cube target at received point (0, 200)

6. CONCLUSIONS

In this paper, the feasibility of combining the TLM method with the BEM to model large area has been presented. Some test examples have demonstrated the remarkably good accuracy of the results and validates the application of this hybrid model to solve radiation and scattering problems. Some efforts have been made to run BEM modelling with a coarse mesh size approaching the upper limit of TLM modelling. The results present a reasonable accuracy and significant saving of computation time.

This hybrid model has the advantages as follows,

1. It offers a highly effective and flexible tool for most practical engineering problems.

2. It can be run on widely used computers such as the SUNs or PCs without causing swapping problem.

3. It is a decoupled hybrid strategy, i.e., the TLM and BEM are only related by a data pipeline in the algorithms. This means that the computer software can be maintained and further modified very easily.

4. It can be implemented by parallel programming. Both TLM and BEM are parallel algorithms. The special value of this method will be shown with the development of parallel processing techniques.

7. REFERENCES

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