OPTIMUM ABSORBER PARAMETERS FOR MINIMIZING VIBRATION RESPONSE

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In the classical problem a viscously damped single degree-of-freedom absorber system is attached to an undamped single degree-of-freedom main system. The optimum absorber parameters (i.e., tuning ratio, or ratio of absorber natural frequency to that of the main system, and absorber damping ratio), which will minimize the harmonic response of the main mass, are simple functions of the assumed mass ratio (1). The author (2, 3) has shown that these simple expressions for optimum absorber parameters and for the optimized response of the main system can be used when the main system is an elastic body, provided that an effective mass is used for the latter and that the natural frequencies of the body are well separated. This has been demonstrated for absorbers attached to various beams and plates. However, for cylindrical shells, where the frequency separation is smaller, optimum absorber parameters diverge from Den Hartog's values with this divergence increasing as the effective mass ratio (i.e., ratio of absorber mass to effective mass of the shell) increases.

In order to study the effect of natural frequency distribution, optimum parameters have been determined for absorbers, which minimize the maximum response of a 2DOF main system (4), for which the natural frequency ratio ω_2/ω_1 can be adjusted by varying the ratio of spring stiffnesses δ (Fig.1) Minimization is applied over an excitation frequency range, which includes only the first and second resonances of the combined system (narrow band optimization). As ω_2/ω_1 increases, the optimum parameters for the 2DOF main system are asymptotic to the relevant parameter from Den Hartog. For small and practical values of the effective mass ratio μ_{eff} (absorber mass/effective mass of 2DOF system), the equivalent 1DOF system can be used to predict optimized maximum response with acceptable accuracy if $\omega_2/\omega_1 \geq 2$; for large values of μ_{eff} this becomes $\omega_2/\omega_1 \geq 3$. This is in broad agreement with the results for absorbers attached to elastic bodies.

In the classical problem the main mass is subjected to a harmonic force and optimum absorber parameters are determined to minimize the displacement response of that mass. Consequently, there has been emphasis on this excitation and response in subsequent work. However, other types of excitation and response are of practical importance. The other excitation-time history which is amenable to simple analytical treatment is random with a white noise spectrum. For this the absorber parameters to minimise some mean square response quantity are determined. An alternative input is the acceleration of the frame of the main system. Response quantities of interest include the velocity and acceleration of the main mass. Some of these alternative problems have been considered for one DOF main systems (3,5) and by Snowdon for beams (6) and plates (7,8). In this paper the author obtains expressions for the optimum absorber parameters and minimum response quantity in terms of the mass ratio μ, which are of similar form to the classical expressions. For harmonic excitations there is a pattern of dependence of the absorber parameters and response on μ. For white noise excitations the

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mean square response is proportional to $\int_{-\infty}^{\infty} |H(\omega)|^2 d\omega$, where $H(\omega)$ is the appropriate complex transfer function or receptance. This can be evaluated only if $H(0)$ is finite and $H(\omega) \to 0$ as $\omega \to \infty$. Within this limit the dependence of parameters on μ is similar for harmonic and white noise excitations.

Real systems have an infinite number of degrees of freedom and are usually modelled by approximate multi DOF systems, for example, by the finite element method. Thus the collected expressions for absorber parameters for 1DOF main systems are of practical significance, only if the concept of replacing real systems by equivalent 1DOF systems, which has been established for harmonic force excitation and displacement response (4), applies to the other types of excitation and response. This has been demonstrated for all the cases considered except one : the relative displacement response to an acceleration of the frame of the main system. It is believed that it is the use of a relative, rather than an absolute, quantity for response that leads to the breakdown in the analogy; this is unfortunate as this case has considerable importance in structural dynamics.

If a real system is represented by an equivalent 1DOF system, contributions to response from higher modes are neglected; the numerical effect of this approximation may vary for different cases. For harmonic excitation, when the equivalent system is used, absorber tuning and damping ratios, f_{eq} and $\gamma_{A,eq}$ respectively, are given by simple expressions; the corresponding maximum response, R_{eq}, is based on the value at the invariant points. If an absorber defined by f_{eq} and $\gamma_{A,eq}$ is attached to the 2DOF main system of Fig.1, the maximum response of this system R_m is somewhat greater than R_{eq}. Optimum absorber parameters f_{opt} and $\gamma_{A,opt}$, which minimize the maximum response of this system can be found also; in general, $R_m > R_{opt} > R_{eq}$. If the difference between R_m and R_{eq} is small, the use of the equivalent 1DOF system is justified. For force excitation and displacement response of the 2DOF system with $\omega_2/\omega_1 = 2$ and $\mu_{eff} = 0.1$, R_m exceeds R_{eq} by 8.4 per cent. For other harmonic excitation and response parameters (with ω_2/ω_1 and μ_{eff} unchanged) this difference is slightly smaller. Thus the conclusion for the displacement case is generally true; i.e., for small practical values of μ_{eff} the equivalent 1DOF system predicts optimized maximum response with acceptable accuracy provided that $\omega_2/\omega_1 \geq 2$.

For harmonic excitation the equivalent 1DOF plus absorber system must give acceptable values for two maxima of the complex transfer function $H(\omega)$ for the real system plus absorber. In narrowband optimization other maxima of $H(\omega)$ are neglected. For random excitation by white noise all maxima of $H(\omega)$ contribute to the mean square response. Thus predictions from the equivalent 1DOF system will be reasonable only if contributions to response from the higher maxima are negligible. For random problems R_m and R_{eq} are the non-dimensional mean square responses using f_{eq} and $\gamma_{A,eq}$ to define the absorber parameters and $H(\omega)$ for the actual and equivalent systems respectively in the integrand. For the 2DOF main system the level of agreement between R_m and R_{eq} depends upon the case studied; for force excitation good agreement occurs if (a) $\omega_2/\omega_2 \geq 3$ and the main system is lightly damped for optimization of the displacement response; (b) $\omega_2/\omega_1 \geq 9$ and the main system is lightly damped for velocity response; and (c) $\omega_2/\omega_1 \geq 2$ for optimization of the force transmitted to the frame. The presence of the light damping reduces appreciably the third resonance in $H(\omega)$ for the system. The inapplicability of the equivalent system for some cases when the main system is undamped is not of practical significance, as undamped systems do not exist, but is theoretically inconvenient, as the analogy and simple expressions for optimized absorber parameters exist only for undamped systems. However, numerical results which show the effect of main system damping on the basic absorber problem exist for harmonic excitations (2, 9, 10, 11) and are included here for random excitations.

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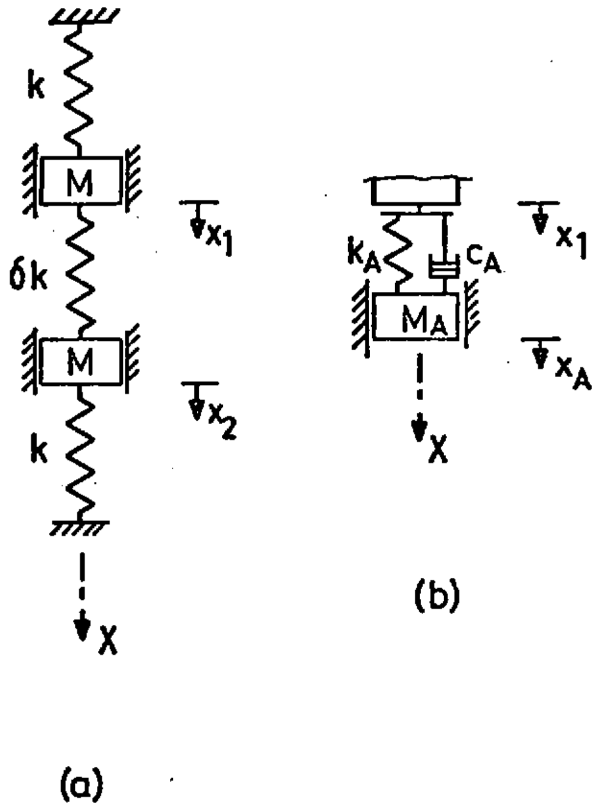

Fig.1 (a) Two degree-of-freedom main system.
(b) Absorber system, attached to upper mass of (a).