Proceedings of the Institute of Acoustics

ATTITUDE SENSITIVITY OF THE FREQUENCY OF HELMHOLTZ RESONATORS

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1. INTRODUCTION

Lord Rayleigh writes that Liscovius and Sonhauss "found that the pitch of a flask partly filled with water was not altered when the flask was inclined [1]." This observation is confirmed in Stephens and Bate[2]. However, in the development of a device to measure the volume and flow of fluids[3], the resolution of a Helmholtz resonator was developed to a sensitivity of ± 1 part in 35,000 or ± 0.0000286 (±0.00286%). This device consistently showed deviations in the Helmholtz resonance frequency as a function of the attitude of the vessel. However, these changes were first ascribed to thermodynamic differences, as the Helmholtz resonance changes about 0.108% per degree F in the vicinity of room temperature. As the development work continued, it became clear that factors other than changes in temperature were the cause of the frequency differences.

2. EXPERIMENTAL PROCEDURE

A 500 ml, round bottom, long-neck flask and a 1000 ml, round bottom, short-neck flask were obtained for experiments. A test configuration was devised as shown in Figure 1. The loudspeaker and flasks were fixed to a platform that was hinged at its base. The base was then clamped to a laboratory bench. This arrangement allowed the flasks and loudspeaker to remain in a fixed position relative to each other while the platform to which they were attached was rotated through 90° from a vertical to a horizontal position. The distance from the loudspeaker to the mouth of the flasks was 7cm to 10cm, depending on which flask was used. The output of the loudspeaker was 80dB to 85dB at the mouth of the flask.

Tests were performed by determining the frequency of the peak sound pressure inside the flasks as a function of the angle of tilt of the platform. The test sequence consisted of first measuring the Helmholtz frequency at the vertical position (0°tilt). Then the angle of tilt of the axis of the flask was increased until a 90° angle was reached or until the water was in danger of spilling from the flask. After this test sequence, the flask was again oriented to the vertical and the original measurement was repeated. A small piezoelectric microphone was placed inside each flask to sense the sound pressure. A frequency synthesizer of a very stable design was used to provide the frequency input to excite the Helmholtz resonances. The frequency synthesizer also permitted the frequency to be controlled to 0.1Hz.

3. TEST RESULTS

Figure 2 shows the results of tilting the empty 500 ml flask. The Helmholtz frequency remains relatively unchanged, within ± 0.2Hz, as one would hope.

Figure 3 shows the results of tilting the 500 ml flask when it contained 140 ml of water. The total change in the Helmholtz frequency was 2.7Hz. The initial Helmholtz frequency of 206.7Hz in the vertical position was exactly repeated after the test sequence.

Figure 4 shows the results of tilting the 500 ml flask when it contained 250 ml of water. The flask could only be tilted to 60° without spilling the water. The total change in the Helmholtz frequency was 2.0Hz. Again, the initial Helmholtz frequency of 249.2Hz at the vertical position was exactly repeated after the test sequence.

Figure 5 shows the results of tilting the empty 1000 ml flask. The change in the Helmholtz frequency throughout the test sequence was ± 0.25Hz, so it also remained essentially unchanged.

Figure 6 shows the results of tilting the 1000 ml flask containing 280 ml of water. The change in the Helmholtz frequency was 1.4Hz. The initial Helmholtz frequency of 141.4Hz was repeated after the test sequence to within 0.3Hz.

Figure 7 shows the results of tilting the 1000 ml flask containing 500 ml of water. The flask could not be tilted more than 60° without spilling the water. The change in the Helmholtz frequency was 1.1Hz. The initial Helmholtz frequency of 166.3Hz was exactly repeated after the test.

4. DISCUSSION

The data indicates that the Helmholtz resonance of the flasks has a sensitivity to its angle of inclination. This seems to be well substantiated by the fact that the Helmholtz frequency at the vertical position, as measured at the beginning of the test sequence, can immediately be duplicated afterwards by quickly reorienting the vessel. Furthermore, the change in the Helmholtz frequency shows a smooth and steady downward shift as the vessels are tilted to progressively larger angles. The empty flasks, on the other hand, show no substantial change. The small amount of change that was measured with the empty flasks, and in one case with a partially filled flask, could result from two obvious causes. First, temperature fluctuations can cause changes in the Helmholtz frequency. To minimize this problem, the apparatus was placed in an electrically heated and controlled room. The temperature was monitored during the tests. Each individual test was also performed as quickly as possible to minimize problems associated with temperature changes. Another source of error could result in determining the frequency of the Helmholtz resonance which was always taken as the frequency, to the nearest 0.1Hz, where the largest output of the microphone occurred. As the data was not interpolated, this could have resulted in an error of ± 0.1Hz.

Acoustic pressure patterns were also measured inside the flasks. Patterns similar to those found by Cummings [4], [5] were measured. It was noted that these patterns were disturbed by the asymmetrical. The patterns also shifted in such a way that they resembled those of vessels of slightly larger volume. This phenomenon is also consistent with the simultaneous decrease in the Helmholtz frequency as the angle of tilting increased. Figure 8 shows sound pressure patterns in a 500 ml flask. The figure shows that the sound pressure pattern in the flask containing 250 ml of water and inclined at 65°, shifts away from its pattern at 0° toward the pattern for an empty flask. The inset shows this more clearly.

It was also observed that regular vessels, such as spherical bottom flasks, had relatively small variations in their Helmholtz frequencies as a function of attitude. More irregularly shaped vessels had greater variations and were less well behaved. Figures 9 and 10 show the results of tilting a larger (60.6 liters) and very irregularly shaped vessel half full of water. The tests were performed in a manner similar to that described for the flasks. The results show greater frequency changes and erratic patterns rather than smooth trends.

Finally, it must be stated that previous observers [1], [2] did not have the use of the precise instrumentation available today. It is not surprising that they were unaware that the lumped parameter solution was not exactly correct.

5. ACKNOWLEDGEMENT

Applied Acoustic Research acknowledges The Ben Franklin Partnership Fund of Pennsylvania and Oxford Speaker Company for their support of this work.

7. REFERENCES

[1] John William Strutt, Baron Rayleigh, The Theory of Sound, Dover Publications, 1896, 1945, Vol. II, p. 187.
[2] R.W.B. Stephens and A.E. Bates, Acoustics and Vibration Physics, St. Martin's Press, 1966, p. 176.
[3] Mark E. Warneka, Joseph A. Kulikauskas, Glenn E. Warnaka, and Jiri Tichy "The Measurement of the Volume and Flow of Fluids by a Helmholtz Resonator System," Acoustics '90, Southampton, 28-30 March 1990.
[4] A. Cummings, "Acoustics of a Cider Bottle," *Appl. Acoustics*, 5 (1972) pp. 616-170.
[5] A. Cummings, "Acoustics of a Wine Bottle," *J. Sound and Vib.*, *31*, (3) (1973), pp. 331-343.

ATTITUDE SENSITIVITY OF THE FREQUENCY OF HELMHOLTZ RESONATORS

Figure 1. Test Configuration

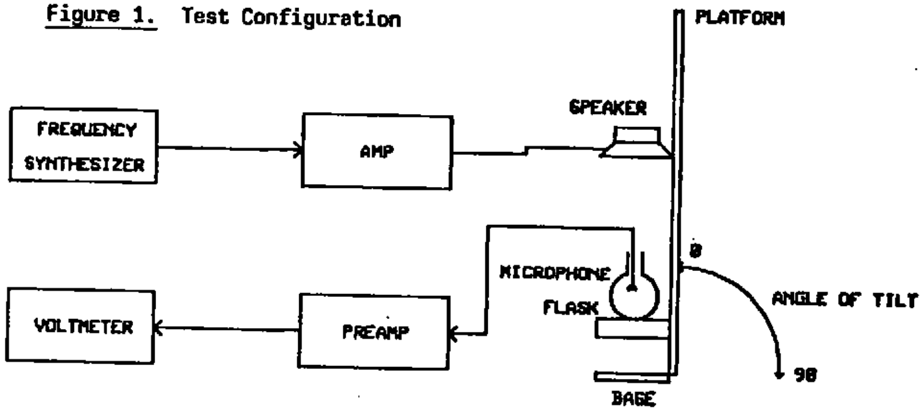


Figure 2. Helmholtz Frequency as a Function of Attitude for an Empty 500 ml Flask.

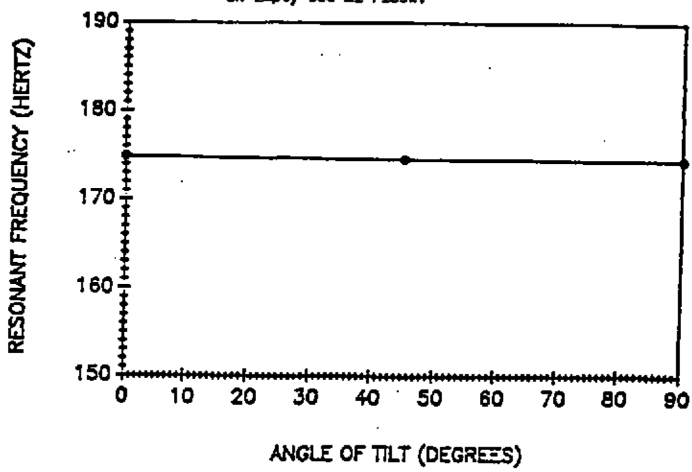


Figure 3. Helmholtz Frequency as a Function of Attitude for 500 ml Flask Containing 140 ml of Water.

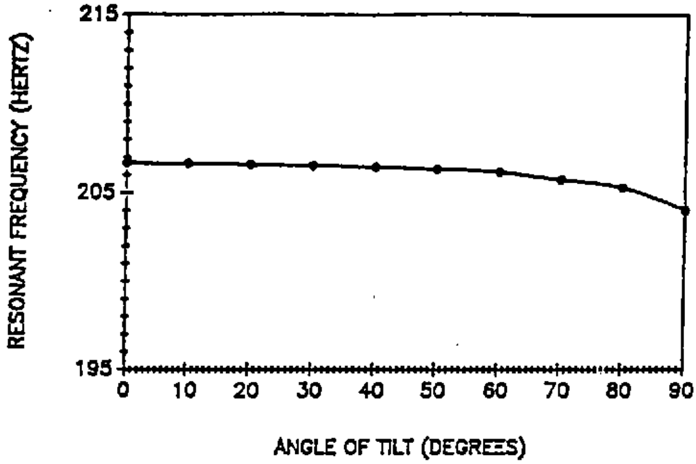


Figure 4. Helmholtz Frequency as a Function of Attitude for a 500 ml Flask Containing 250 ml of Water.

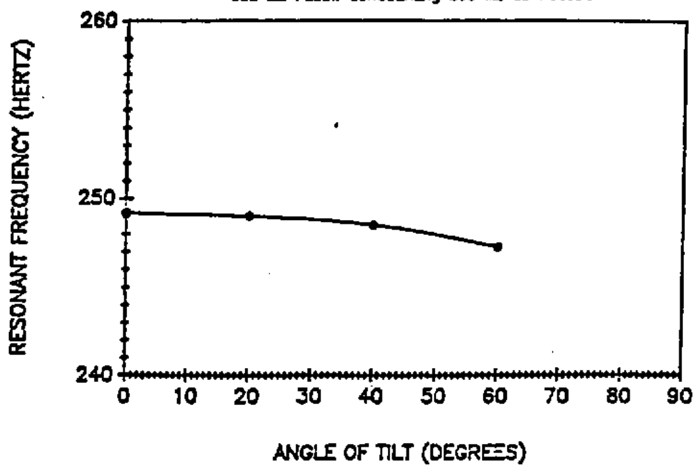


Figure 5. Helmholtz Frequency as a Function of Attitude for an Empty 1000 ml Flask.

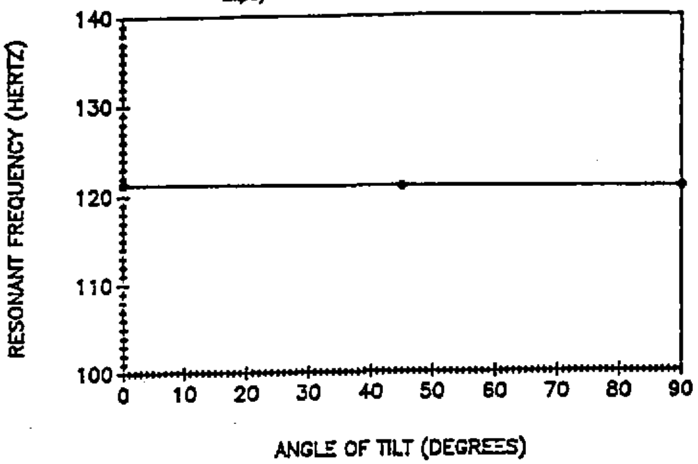


Figure 6. Helmholtz Frequency as a Function of Attitude for a 1000 ml Flask Containing 280 ml of Water.

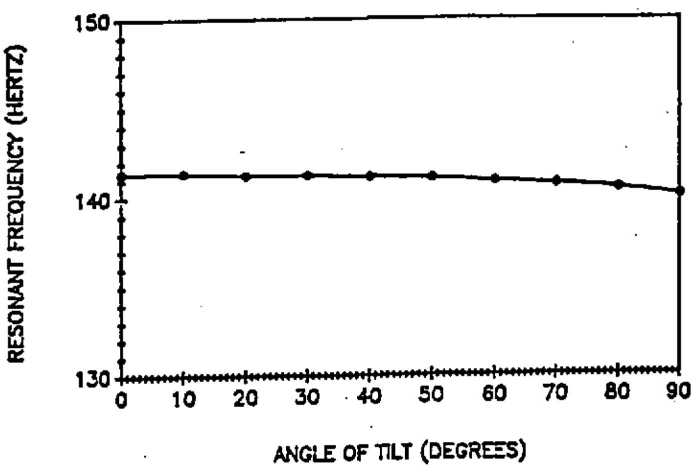


Figure 7. Helmholtz Frequency as a Function of Attitude for a 1000 ml Flask Containing 500 ml of Water.

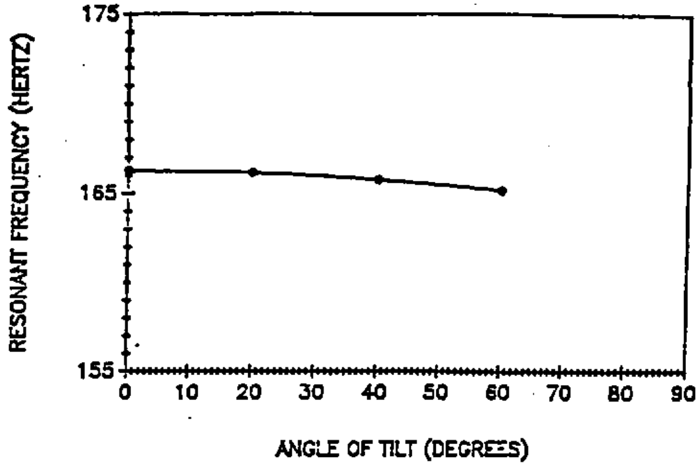


Figure 8. Sound Pressure Patterns in a 500 ml Flask.

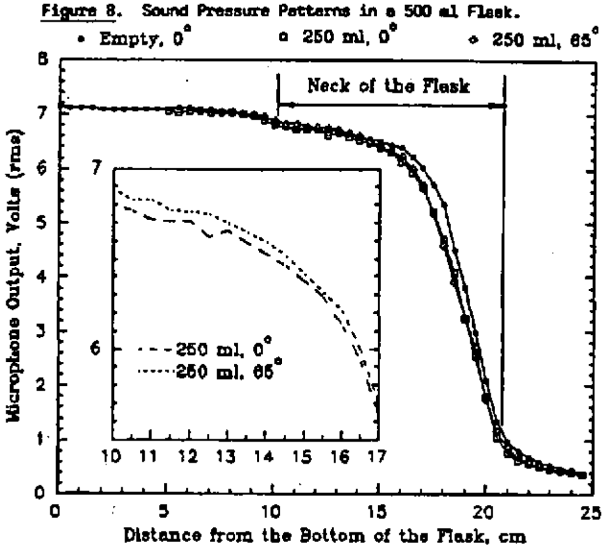


Figure 9. Helmholtz Frequency as a Function Attitude for an Irregularly Shaped 60.6ℓ Vessel Tilted Front to Back.

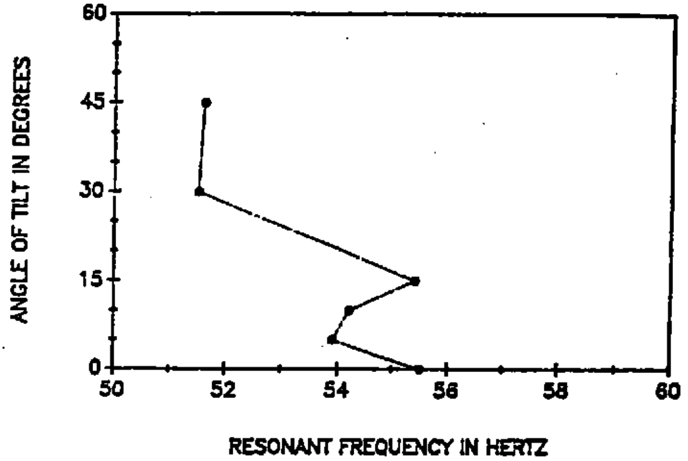
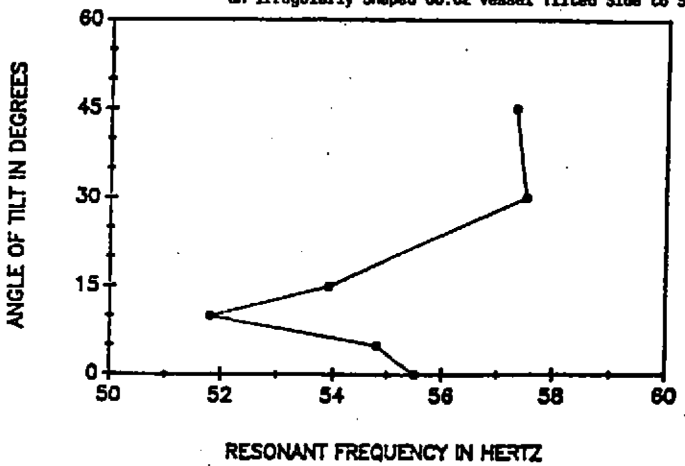


Figure 10. Helmholtz Frequency as a Function of Attitude for an Irregularly Shaped 60.6ℓ Vessel Tilted Side to Side.



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THE MEASUREMENT OF THE VOLUME AND FLOW OF FLUIDS BY A HELMHOLTZ RESONATOR SYSTEM

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1. INTRODUCTION AND TECHNICAL DESCRIPTION

There are a great many applications that require the determination of the amount of liquid or liquid-like materials in a vessel. There are a variety of devices that monitor the level of the liquid surface either continuously, intermittently, or occasionally. Since the liquid level is indicated directly, the amount of liquid in the vessel must be inferred. This is not difficult for vessels with constant cross-section. However, for irregularly shaped vessels, the relationship of the volume of liquid must be empirically determined from its level. Since liquid storage tanks are used in a variety of attitudes and/or in conjunction with moving vehicles, current measurement devices that sense the liquid level are inherently subject to two serious drawbacks: 1.) The tank must always be oriented in a given attitude relative to the sensor position. Otherwise, the sensors will not detect the fluid level properly. 2.) If the tank is on a moving vehicle, acceleration forces cause motion of the fluid within the tank so that the level of the fluid may vary considerably. During the period of agitation of the liquid, all of the state-of-the-art devices that measure the level of the liquid surface are unable to provide accurate measurements.

The volume/flow meter under development makes use of the principles of acoustic cavity resonance first set forth by Helmholtz and Rayleigh [1], [2]. The classic work has been developed into new methods for measuring the volume of liquid in a closed, or nearly closed, vessel.

2. PHYSICAL CONSIDERATIONS

The Helmholtz resonant frequency f_n of a simple cavity is given by

$$f_n = [c/2\pi][a/(\ell V)]^{\frac{1}{2}}$$

c = Speed of sound

a = Neck area

ℓ = Neck length

V = Empty Volume in the vessel.

The ratio a/ℓ is the "geometric factor" for each resonator. It is sometimes called the "Conductivity Factor" of the neck. It appears to be quite a simple relationship, but it has long been known that this factor is complicated by the requirement of an effective end-correction for ℓ. The geometric factor may be represented as $a/\ell = V[2\pi f_n]^2/c$. The ratio, a/ℓ may be replaced by k, and this factor can be found from experimental measurements. For a vessel of given geometry, the volume of the airspace above the liquid may be found from the relationship $V = kc/(2\pi f_n)^2$. Thus, the volume of the airspace is a function of its resonant frequency, and the resonant frequency is an inverse function of the airspace volume. Hence, as is well known, the vessel has a lower resonant frequency when it is empty and a higher resonant frequency when it is full. By observing that

$$V_c = V + V_\ell$$

where V_c = *Tank volume in cubic meters,*

V = *Airspace volume in cubic meters,*

V_ℓ = *Liquid volume in cubic meters,*

It is easy to determine the volume of liquid present from

$$V_\ell = V_c - V = B - V.$$

Since the capacity of any given vessel is a constant, it may be replaced by the term, B. The indicating or "read out" system for the vessel can be calibrated to read the volume of liquid directly.

The flow rate of liquid into or out of the vessel is the change in volume as a function of time, the expression for the volume of liquid present may be differentiated to find the flow rate: $dV_\ell/dt = dV/dt$. The acoustic volume/flow meter currently under development can measure either the volume or flow, or both, simultaneously.

3. TRACKING THE FREQUENCY

A method was devised for automatically tracking the changes in the Helmholtz resonant frequency as a function of the change in volume of a closed vessel. The test configuration is shown in Figure 1. The block diagram of the electronic control and detection system is shown in Figure 2. This system proved to be workable and its accuracy was quite good. However, the response to volume changes (flow) was quite slow.

4. TRACKING PHASE CHANGES

In an effort to simplify the system and improve both the response time and accuracy, a phase tracking method was devised. This method chooses a fixed frequency representing a Helmholtz resonance. Changes in volume are then measured from the phase changes between the acoustic input and acoustic output of the vessel. Figure 3 shows a diagram of the measuring system. The speaker provides acoustic input to the system, and the microphone measures the acoustic response (output) of the system. To accomplish the measurement, the electrical input to the speaker and the electrical output of the microphone are converted into TTL square waves and used as inputs to an exclusive OR gate. The output of the exclusive OR gate then converts the out-of-phase relationship of the two waves into a pulse. The time width of this pulse represents the phase angle relationship of the two original waves. With this method, a small change in volume will produce an easily measureable phase change. The change in phase can be taken as a measure of the volume change from the relationship

$$\phi = 360^\circ (\text{Phase Time})/(\text{Frequency Time})$$

where ϕ is the phase to be determined, Phase Time is defined as the time width of the pulse representing the difference between the input and output and Frequency Time is defined as one-half of the period of the fixed input frequency.

For larger volumes more than one frequency must be used, and the container is divided into frequency steps. Each of these steps is used for a specific range of volume within the vessel. Many frequency steps can be used to ensure that the container is always very close to resonance. One major advantage of choosing the set of frequencies to be used is that frequencies that may cause problems, such as column resonances or other resonances within the container, can be avoided.

Another advantage of using the phase tracking method is that the duty cycle out of the exclusive OR gate can be used to measure flow into and out of the container. By integrating the output of the exclusive OR gate, we get a DC voltage equivalent to the phase relationship. If we differentiate this voltage with respect to time, the output reflects any change of the volume, i.e. flow. A block diagram of the system is shown in Figure 4.

Actual measurement accuracy obtained with the phase tracking method is as great as 0.000286 (0.00286%) with 10 to 12 frequency steps when thermal equilibrium is achieved. The thermodynamic changes of air in the vicinity of room temperature alter the Helmholtz resonance about 0.108% per degree F.

The circuit for measuring flow directly has been able to measure a flow less than 0.00246 (0.246%) of the volume per hour. The accuracy has since been improved but has not been quantified. However, a flow of only one drop at a time can easily be measured in a 3.79 liter vessel.

The system can be used to make volume measurements on such technically difficult fluids as beer with foam. Figure 5 shows the results of such a test. The beer was placed in a mixing device which was then closed with an acoustic driver such as shown in Figure 3. The liquid was allowed to out-gas and come to an equilibrium state. It was then very briefly agitated by the mixer at time zero. As a high-foaming beer was used, this produced a great deal of foam. The decrease in the air volume is clearly shown by the increase in the resonant frequency. This indicated that the system can easily measure the volume of such a delicate fluid. However, the foam eventually collapses, accompanied by a thermal reaction. Ultimately, the system warms up, stabilizes, and returns to its original state. The amount of foam present and the mixture of gasses present affects the Q of the system as well as its frequency.

Flow in a pipe may also be measured with this device. This is accomplished by creating a chamber containing a weir, oriface, etc. within the pipe. Hence, the level of the liquid inside the chamber will be higher or lower, depending on the flow rate.

5. SOME OTHER DETAILS

Some additional "details" arose in the course of this work. These details are of considerable importance but lie outside the scope of this paper. The first issue is whether or not a Helmholtz resonance exists in a closed vessel. As it turns out, it does exist, but that is a story in itself. References [4],[5],[6] give some comfort.

A second issue involves the design of the neck. This is crucial in making the device operate properly. The neck must be designed to provide the largest possible frequency span between empty and full while maintaining a high system Q. Figures 6 and 7 show the frequency range and Q for a 3.79 liter vessel. Considerable development effort showed that a "distributed" neck geometry consisting of a plate with a number of holes of different diameters gave the best results. A third issue is whether or not the Helmholtz frequency is independent of attitude [2],[3]. Unexpectedly, it turns out not to be [7].

7. ACKNOWLEDGEMENT

Applied Acoustic Research acknowledges the support of The Ben Franklin Partnership Fund of Pennsylvania and Oxford Speaker Company in the development of this device.

8. REFERENCES

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[2] John William Strutt, Baron Rayleigh, The Theory of Sound, 2nd Ed., Vol. II, Dover Publications, 1878, 1945, pp. 187 ff.
[3] R.W.B. Stephens and A.E. Bate, Acoustics and Vibration Physics, St. Martin's Press, 1966, pp. 176 ff.
[4] A. Cummings, "Acoustics of a Cider Bottle," *Appl. Acoustics*, 5 (1972), pp. 161-170.
[5] A. Cummings, "Acoustics of a Wine Bottle," *J. Sound and Vib.*, *31* (3) (1973), pp. 331-343.

[6] Philip M. Morse, Vibration and Sound, 2nd Ed., McGraw-Hill, 1948, pp. 235 ff.
[7] Glenn E. Warnaka and Mark E. Warnaka, "Attitude Sensitivity of the Frequency of Helmholtz Resonators," Acoustics '90, Southampton, UK 28 - 30 March 1990.

Note: Because of space limitations, the figures are not in numerical sequence.

Figure 5. System Performance vs Time For Beer. Foaming Took Place At Time =0.

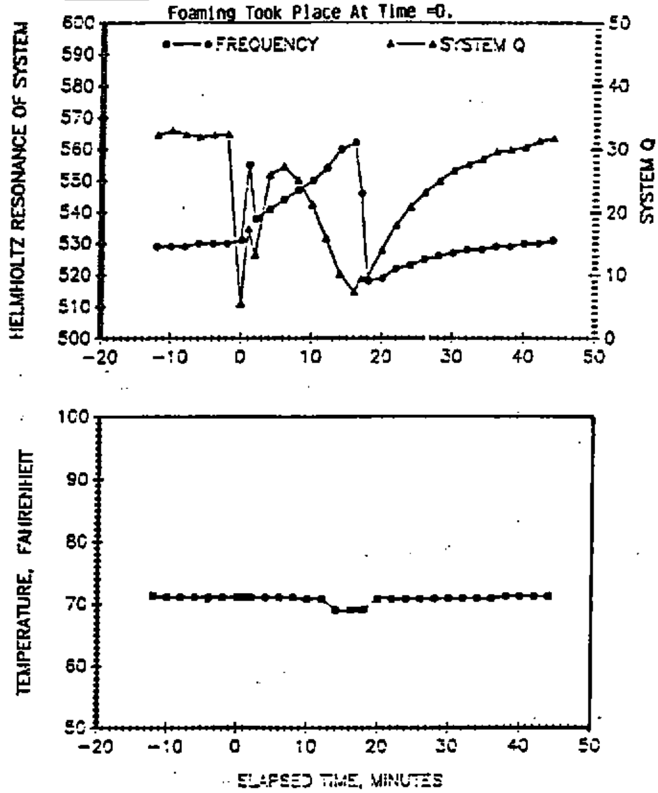
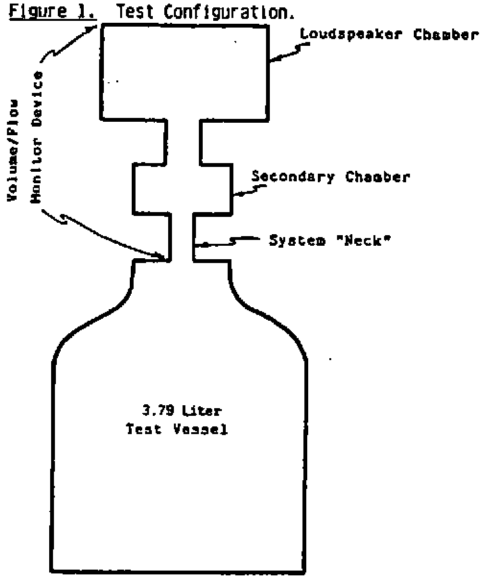


Figure 1. Test Configuration.



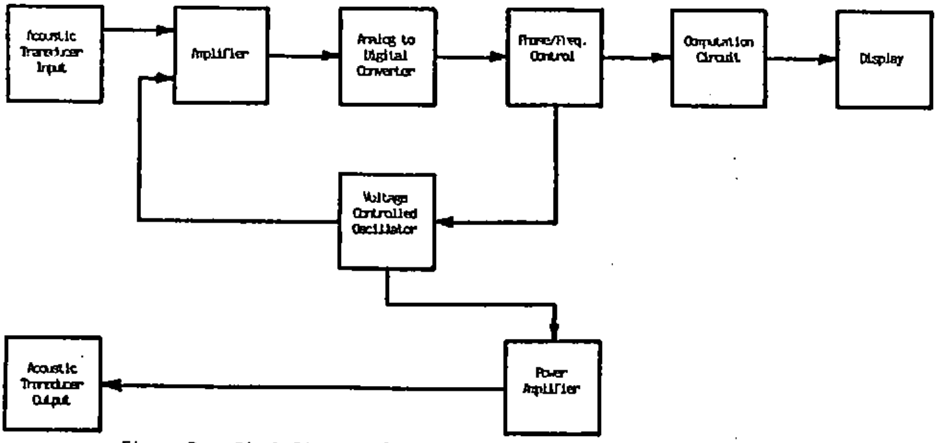


Figure 2. Block Diagram of the Frequency Tracking System.

THE MEASUREMENT OF THE VOLUME AND FLOW

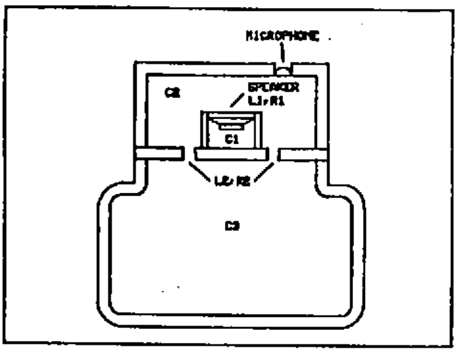
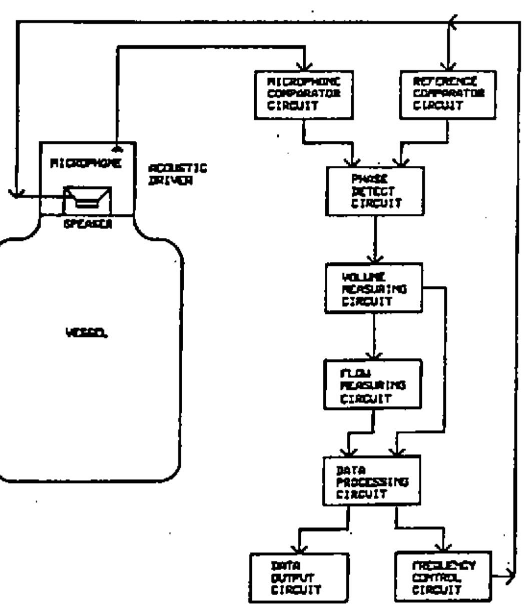


Figure 3, Schematic Representation of Volume/Flow Measuring System.

C1	Compliance of volume behind loudspeaker
L1	Loudspeaker mass
R1	Loudspeaker damping
C2	Compliance of driver chamber
L2	Neck mass
R2	Neck damping
C3	Compliance of measuring volume

Figure 4, Block Diagram of the Phase Tracking System



VOLUME/FLOW METER BLOCK DIAGRAM

Figure 6. Helmholtz Frequency of a 3.79 Liter Vessel as a Function of Fluid Volume

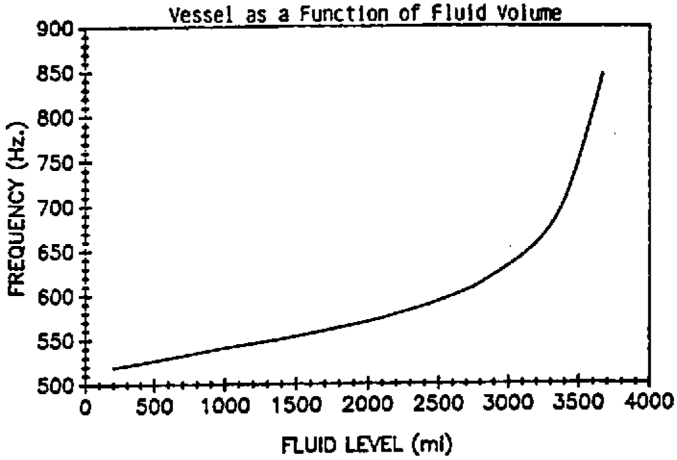


Figure 7. System Q of a 3.79 Liter Vessel as a Function of Fluid Volume