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THE REFRACTION OF SOUND BY ROTATING FLOW

by

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INTRODUCTION

Sound propagation through rotating flows has been considered theoretically by several authors[1,2,3,4]. Lindsay[1] examines sound ray propagation through a potential vortex, while, for a cylindrically-stratified velocity field, both Cooke[2] and Salant[3] derive a simple differential equation for the path of a sound ray through a vortex and go on to consider the effects of particular velocity fields. Georges[4], on the other hand, applies a general ray-tracing computer program to calculate the ray-paths through a potential vortex with a viscous core. When the sound wavelength is much smaller than the length scale of the flow, so that ray theory is applicable, all conclude that, in general, sound rays are refracted in the direction of the flow velocity. Only Cooke, however, investigates the important implication of ray bending, which is that sound energy will be redistributed to give regions of amplification (ray focusing) and attenuation (ray spreading). While it is difficult to trace individual sound rays experimentally, changes of sound intensity can be measured, so that the prediction of the redistribution of energy provides a test for the validity of the ray theory approach in a given situation.

EXPERIMENTAL WORK

Such regions of amplification and attenuation have been observed in experimental investigations of the propagation of sound through the leading-edge vortex produced by a delta wing at incidence[5], carried out in the 24' wind-tunnel at RAE, Farnborough, which has been treated to make it suitable for acoustic experiments. A delta wing at incidence was used, since a stable, strongly rotating flow field is produced, whose properties are reasonably well-defined. In the first series of tests, the sound source was mounted above the wing and detailed interpretation of the experimental results was made difficult by acoustic interference effects due to reflection from the wing surface. To overcome this difficulty, further tests have now been performed with the source mounted behind the wing, so that the sound travelled through the rotating flow downstream of the trailing-edge. There the flow structure is complicated by bound vorticity being shed from the wing, which gives rise to a trailing-edge vortex rotating in the opposite sense. The leading-edge vortex is still, however, the dominant feature of the flow.

THEORETICAL MODEL

The flow-field of a leading-edge vortex consists of an inner viscous core, an outer core formed by a rolled-up vortex sheet and an outer inviscid flow. Measurements made in such a vortex[6], have indicated that within the outer core region, the azimuthal velocity V_θ, shows only small variations and that the diameter of the inner viscous core is less than 10% of the diameter of the outer core. This suggests that a flow model with V_θ = constant will be suitable for calculating the ray paths, except for rays which pass very close to the centre of the vortex. For such a velocity field, the ray equation of Cooke[2] and Salant[3] may be integrated exactly to yield the following expression for the ray paths (with reference to Fig 1):

$$\theta = \pm \left\{ V \alpha \operatorname{sgn} \Psi + \frac{2}{(1 - V^2)^{\frac{1}{2}}} \tan^{-1} \left[\frac{e^{\alpha} + \operatorname{sgn} \Psi \, V}{(1 - V^2)^{\frac{1}{2}}} \right] \right\} + K \qquad (1)$$

where $V = V_\theta / c$ is the non-dimensional azimuthal velocity,

c = the velocity of sound,

$\Psi = r A - V$

$\alpha = \cosh^{-1}(|\Psi|)$,

r, θ are defined in Fig 1,

and A, K are constants for a particular ray.

A is given in terms of the initial slope of the ray by

$$A = \frac{-V}{1 - V^2} \pm \left\{ (1 - V^2) \left[1 + (1 - V^2) \left(\frac{d_r}{d_\theta} \right)_o^2 \right]^{\frac{1}{2}} \right\}^{-1}$$

and K is determined from the condition that $\theta = 0$, $r = 1$ at the source.

Equation (1) is valid up to the point of closest approach of the ray to the vortex centre. After this point, the path is the mirror image of that before the turning-point.

Ray paths given by equation (1) are shown in Fig 1, for $V = 0.05$ over the region $r < 1$. Regions of amplification (ray focusing) and attenuation (ray spreading) are clearly indicated.

COMPARISON WITH EXPERIMENT

Analysis of the experimental results for propagation through the rotating flow downstream of a delta wing has not yet been completed, but typical measured sound intensity distributions with and without flow are shown in Fig 2a. The frequency was 12.5 kHz and the outer core radius was about 0.15 m, giving a non-dimensional wave-number ka of 36. The outer core occupied the region $r \lesssim 0.3$ and the azimuthal velocity within this region was estimated as $V = 0.05$. Fig 2b shows a theoretical calculation of intensity for $V = 0.05$ corresponding to the ray diagram shown in Fig 1, where ϕ is the angle at the source. It can be seen that there is some qualitative agreement between theory and experiment with regard to the overall position of the regions of amplification and attenuation, but quantitatively, the agreement is disappointing.

Several possible reasons for the difference are being investigated and preliminary indications are that taking account of the trailing-edge vortex will improve the agreement between theory and experiment.

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