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A THEORETICAL ANALYSIS OF THE SUBDIVISION OF THE SURFACE OF A FINITE SOUND-ATTENUATING BARRIER INTO ELEMENTAL REGIONS

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Introduction

Given a configuration of a point, monotonic source of sound at S, an arbitrarily shaped plane, finite, non-transmitting barrier and a monitoring position P in the free-field, a method has been developed which predicts the sound attenuation at P due to the inclusion of the barrier using the Fresnel-Kirchhoff diffraction theory. The method used was to solve the Fresnel-Kirchhoff area integral over the surface of the barrier. This could be done however, after first subdividing the region occupied by the barrier into rectangularly shaped elemental regions and finding a solution to the integral separately on each element. The necessity of this construction arises because it is a requirement in the use of the integral that the linear dimensions of the area of integration are small compared with its distances from the source and monitoring position. It is at this stage that the question arises of the nature of the subdivision in a particular configuration of source, barrier and monitoring position whcih is most acceptable to the use of this technique. It is the aim of this paper to investigate this problem.

To expand further on what will be required in the investigation, it may first be recalled from (1) that certain approximations were required to the integrand of the Fresnel-Kirchhoff formula on each element in order to affect a solution. These approximations tend to give rise to possible errors in the final attenuation prediction. It will therefore be necessary to estimate the error incurred due to a given subdivision. Firther, the behaviour of the integrand over a typical element, and consequently the contricution of this element to the final prediction error, depends on the elements dimensions. It is therefore intended to produce a correlation between the size of the elements forming a subdivision and the estimated attenuated prediction error, from which it will be possible to propose an optimum subdivision of the barrier surface area which gives the most accurate attenuation prediction.

Quantitative Evaluation of the Approximations made to the Diffraction Equation

In a given subdivision of the barrier into elemental regions R_1, R_2, . . . R_n, it was necessary to solve the Fresnel-Kirchhoff diffraction integral

$$U_{\bar{R}j}(P) = -\frac{A_i}{2\lambda}\iint_{Rj}\frac{e^{ik(\ell+m)}}{\ell m}[\cos(n,\ell) - \cos(n,m)]dS \qquad (1)$$

on each element R_j. In order to effect a solution, it was necessary to make the following assumption on each element.

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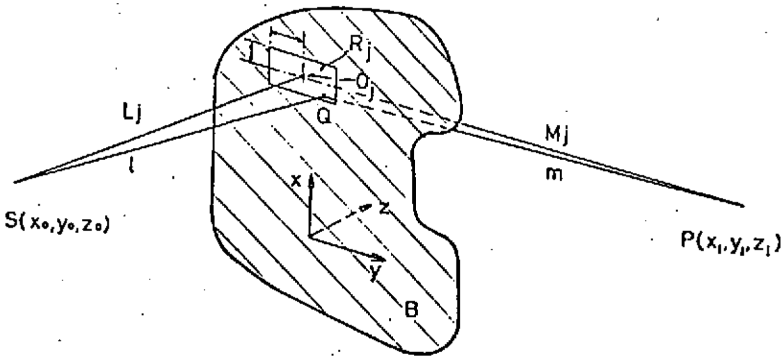

Fig 1 The Rectangular Element R_j of the Subdivision of the Region B

(a) Where R_j and the distances ℓ, m, L_j and H_j are shown in figure 1, it was assumed that R_j is small enough to write

$$\frac{\cos(n,\ell) - \cos(n,m)}{\ell m} = \frac{\cos(n,L_j) - \cos(n,M_j)}{L_j M_j} \qquad (2)$$

on R_j.

(b) It is assumed that R_j is large enough for the term $e^{ik(\ell+m)}$ to fluctuate appreciably over R_j, so that the term is retained as a variable, unlike $\frac{\cos(n,\ell) - \cos(n,m)}{\ell m}$ in (a) above.

(c) It is assumed that R_j is small enough to assume that the curtailed expansions of L and m in terms of

$$\frac{x - X_j}{L_j},\ \frac{y - Y_j}{L_j},\ \frac{x - X_j}{M_j} \text{ and } \frac{y - Y_j}{M_j} \quad \text{given by:}$$

$$\left.\begin{aligned} \ell &= L_j - \frac{(x_0 - X_j)(x - X_j) + (y_0 - Y_j)(y - Y_j)}{L_j} \\ m &= M_j - \frac{(x_1 - X_j)(x - X_j) + (y_1 - Y_j)(y - Y_j)}{M_j} \end{aligned}\right\} \qquad (3)$$

are sufficiently accurate approximations of the actual values of ℓ and m on R_j.

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The validity of these assumptions on the element R_j is investigated as follows. The percentage variations over R_j of the various functions are calculated by finding their maximum and minimum values and exposing their difference as a ratio of the function's value at O_j. For greater accuracy, it is therefore necessary to choose the R_j such that the percentage variations of the functions in (a) and (c) are minimised whilst the variation of the function in (b) is still of a reasonably large extent.

In a given configuration of source, barrier and monitoring position and for any subdivision of the barrier into rectangular elements R_1 etc, therefore, the validity of the assumptions above can be expressed quantitatively using the computer programs developed here. By examination of the variation of the function of various subdivisions of the barrier into elements and by noting where convergence of the predicted attenuation occurs when the number of elements is increased an optimum subdivision giving the required accuracy whilst minimising the number of elements can be determined in a given configuration.

Summary

A method of obtaining an optimum subdivision of a barrier into elemental regions in a given configuration of source barrier and monitoring position to give a satisfactory degree of attenuation prediction while minimising the computer run time has been achieved. The theory involved the use of the attenuation prediction method covered by Phillips (1) and a method of investigating the approximations incurred when any subdivision is assumed.

Reference

(1) S PHILLIPS, G J McNULTY and J L WEARING Proc Ins Acoustics Spring Conference 1978 University of Cambridge. Sound reduction by a finite barrier of arbitrary shape. (At Press)

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