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# The Measurement of the Bulk Modulus Loss Factor of Small Solid Specimens

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## Introduction

The elastic constants usually used to describe the properties of a linearly elastic homogeneous and isotropic medium are the Lame constants $\lambda$ (dilatational) and $\mu$ (shear). From these all other constants of interest can be derived. By permitting them to assume complex values the effect of internal losses can be taken into account. The shear modulus can be obtained directly from torsional measurements, but there is no motion that is controlled exclusively by the dilatational modulus, and its value must therefore be obtained from a measurement of one of the derived constants. In most of these the shear is the dominant factor, and the effects of inaccuracies in its determination can swamp the dilatational term. The constant with the least component of shear, and thus likely to lead to the most accurate determination of the dilatational modulus, is the bulk modulus $\lambda + 2\mu/3$. Since the imaginary component is usually relatively small it is convenient to write this as $\lambda_k(1+j\eta_k)$, where $\lambda_k$ is the real component and $\eta_k$ is the bulk modulus loss factor.

A knowledge of this loss factor is useful in the evaluation of materials such as acoustic rubbers for use as "windows" in sonar transducers, when a low value is desirable, and for materials to be used in vibration and shock mountings, for which a relatively high value may be perferred. However its measurement is difficult, particularly on low-loss samples, because it is necessary to excite a purely volumetric vibration in the material in such a way that the inherent material losses are not exceeded by the losses of the driving system.

In one method the volume changes have been measured directly, the loss factor being derived from the phase angle between the stress and the strain with forced vibrations at low frequencies (1). In another the attenuation of a traveling acoustic wave as a function of distance has been measured in a liquid containing the solid material in suspension (2). However, this method is of limited application, and useful only at relatively high frequencies (above 200 KHz). In a third method the sample was placed at a velocity node in a standing acoustic wave in a liquid filled tube, and the decay time of the resonance compared with the time with the specimen removed (3,4). A variation of this method, employing a U-tube with an open liquid surface at both ends of the column, was unsuccessful because of the losses at the walls and in the material of the tube (5).

By using a spherical shell container resonances of a body of liquid have been obtained with a Q as high as 80,000, enabling materials of a relatively low loss factor to be measured by the decay time method (5). However, there were a number of practical difficulties. It was necessary to use the same transducer for driving and for decay measurement, placed at the centre of the sphere so as to excite only radially symmetric modes, and the container had to be suspended in vacuum in order to reduce damping due to acoustic radiation. An attempt to dispense with the vacuum jacket by using a thick-walled container which would not itself vibrate was unsuccessful, because asymmetries introduced by the neck of the container and by the weld at its equator introduced a number of resonances which masked the radial resonance in the liquid that was being sought (6). Accordingly it was decided to try the use of cylindrical vessels.

## Losses to be Expected in the Measuring System

(a) Frictional loss at the walls of the vessel due to the relative tangential velocity between the walls and the neighbouring liquid: It can clearly be minimized by the use of a vessel with very thin, resilient walls which move with the liquid, and by choosing a predominantly radial mode of vibration.

(b) Losses in the material of the vessel: These should be small for a thick-walled, nearly rigid vessel, though some coupling can be expected. They may be significant for the thin-walled case.

(c) Acoustic radiation from the outside of the vessel and from the free surface: This can be reduced by the use of a rigid vessel or by suspending the vessel in a close-fitting rigid outer container (ideally spaced $\lambda/4$ away) or in a vacuum jacket (though it may be difficult to reduce the pressure below the vapour pressure of the liquid without introducing further losses from a sealing system).

(d) Losses in the driving and measuring transducers and mounts: These may be minimized by using loose coupling, since efficiency is not a problem, and by choosing a low-loss material such as quartz.

(e) Losses in the suspension of the vessel and of the sample: These can be minimized by careful design.

(f) Losses in the liquid due to its viscosity, and to thermal relaxation: These will usually be negligible compared with other unavoidable losses.

(g) Losses due to gas bubbles in the liquid or adhering to the surfaces of the vessel or sample: These are most pernicious, since a single bubble can cause significant losses and erroneous results. Care is required to degas the liquid and to ensure proper wetting of the surfaces so as to restrict adhesion of the bubble nuclei.

## Normal Modes and Energy in the Fluid contained by a Cylindrical Vessel

In terms of cylindrical coordinates r, $\varphi$ and z the contribution to the acoustic pressure of a mode of angular frequency $\omega$ may be written

$$P = P_o J_m(k_r r)\cos(k_z z)\cos(m\varphi)e^{j\omega t}$$

where $\omega^2/c^2 = k^2 = k_z^2 + k_r^2$ and m is zero or a positive integer. For an open vessel of radius a containing liquid of height h we have the boundary conditions $k_z h = (2n-1)\pi/2$ or $k_z h = n\pi$ for the situation where the bottom is thick (rigid) or thin (resilient) respectively, and $J_m'(k_r a) = 0$ or $J_m(k_r a) = 0$ for the case of thick and thin walls respectively. Using these equations the normal modes may easily be tabulated. Some deviation from the calculated values must be expected in a practical case, since no container can be asbolutely rigid or completely resilient. Additional resonances may also be developed in the vessel itself.

The kinetic energy contained in the mode may be obtained by integration over the volume, and is

$$(\rho\pi h V_o^2/\epsilon_m) \int_o^a \left[ k_z^2 r J_m^2(k_r r) + (m^2/r) Jm^2(k_r r) + k_r^2 r J_m'^2(k_r r) \right] dr$$

where $V_o = P_o/k\rho c$

In the radially symmetric case with which we will usually be concerned m = 0, and the integral becomes

$$\rho\pi h a^2 k^2 V_o^2 J_o^2(\gamma)/2 \qquad \text{(thick wall)}$$

or

$$\rho\pi h a^2 k^2 V_o^2 J_1^2(\gamma)/2 \qquad \text{(thin wall)}$$

where

$\gamma = k_r a$ is the appropriate root of

$J_1(\gamma) = 0$ or $J_o(\gamma) = 0$ respectively.

Frictional Loss at the Walls of a Thick Vessel

The loss per unit area can be shown to be $\sqrt{\omega\rho\mu/2}\, V_o^2$, where $\rho$ is the density and $\mu$ the viscosity of the liquid, and $V_o$ the relative velocity between the walls and the neighbouring liquid (3,5). From this we obtain the expression

$$\sqrt{\omega\rho\mu/2}\ (\pi h a/\epsilon_m)\ (k_z^2 + m^2/a^2)\ V_o^2 J_m^2(k_r a)$$

for the loss at a thick cylinder wall, and

$$\sqrt{\omega\rho\mu/2}\ \ (2\pi/\epsilon_m) \int_o^a \left[ (m^2/r) J_m^2(k_r r) + k_r^2 r J_m'^2(k_r r) \right] dr$$

for the loss at a thick bottom.

For the radially symmetric (m = 0) mode with which we are primarily concerned the total wall loss reduces to

$$\omega\rho\mu/2\ \pi V_o^2 (hak_z^2 + a^2 k_r^2) J_o^2(\gamma)$$

for the case of the thick walled vessel, or

$$\omega\rho\mu/2\ \pi a^2 k_r^2 V_o^2 J_1^2(\gamma)$$

for the case of a vessel with a thin side wall and a thick bottom.

Thus the component of damping factor due to wall frictional losses is $\sqrt{\omega\mu/2\rho}\ \left[(h/a)\ k_z^{\ 2} + k_r^{\ 2}\right] / k^2$ for the vessel with thick sides and bottom, or $\sqrt{\omega\mu/2\rho}\ k_r^{\ 2}/\ k^2$ for the thin-walled vessel with a thick bottom. These values can readily be tabulated along with the normal modes, and give a lower bound to the value to be expected from any particular apparatus.

## Experimental Arrangement

A can about 12.5 cm diameter and 15 cm high is suspended by long waxed nylon cords within a thick-walled container to reduce acoustic radiation. In the air space above the can was supended either a microphone (for receiving) or a loudspeaker (for transmitting); a second transducer, a lead-titanate-zirconate disc, was cemented to the bottom of the can. The transmitter was connected to a beat frequency oscillator through a transmit realy, and the receiver through a preamplifier to an oscilloscope with an afterglow tube and to a level recorder. The frequency was swept by hand until a resonance was found, and the decay time measured on the recorder when the transmitting transducer was disconnected.

Two thin-walled cans were tried with this arrangement. One was fabricated from a tube of stainless steel, originally .9 mm thick but reduced for most of its length to .4 mm, the bottom being a brass disc 9.5 cm thick affixed by epoxy cement. The other was a commercial stainless steel 1800 ml beaker of .8 mm wall thickness. With neither of these could a resonance of sufficiently high Q be found. At first this was attributed to the possible presence of minute air bubbles. Accordingly the cans were carefully cleaned and freshly boiled distilled water was siphoned in through a spray nozzle under vacuum to ensure that it was well below saturation. Since no change in Q was observed, it was assumed that the losses were in the material of the cans. To corroborate this hypothesis the Q of the longitudinal resonance in a stainless steel bar was measured and found to be of the same order of magnitude.

Thick-walled vessels are now being tried. It has been found possible to support these on springs without introducing undue damping, and this enables a small excess pressure to be applied immediately before or even during the measurement so as to reduce further the possibility of air bubbles by forcing them into solution. Preliminary results are promising.

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