ANHARMONIC MUSICAL SCALES

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1. INTRODUCTION

The present authors recently completed a study of the normal modes of a spiral clock gong [1]. The sound of such a gong is pleasant and would be instantly recognised by most workers. However, unless they are in the habit of looking inside mantel clocks, they would probably be unaware that it emanates from such a geometrically unlikely source. Since it is essentially a very long thin cantilever wound into a flat constant pitch spiral one might expect that the frequencies would be proportional to $(2n-1)^2$ with the mode number n being simply the number of nodes present [2]. Such a sequence, on the basis of conventional ideas about musical intervals, would however not be expected to be particularly harmonious.

Experimentally we found that every mode was either purely in the plane of the spiral or purely out-of-plane. For each of these types there were two definite regimes with a changeover occurring at about $n = 11$ where the wavelength of the standing wave became comparable to one turn of the spiral. The frequencies for the in-plane cases are illustrated in figure 1 which makes the change of regime very clear and also shows the excellent agreement we were able to achieve with finite element calculations. The latter were invaluable in sorting out the nature of the various modes and allocating values of n to them. Despite the good straight line fit to the lower branch in the log plot the frequencies did not follow the expected cantilever law but rather $f_n \propto (2n-1)^{0.62}$ while the out-of-plane equivalent was $f_n \propto (2n-1)^{0.58}$. With the higher frequency regimes the best fits we could find were those shown in figure 2 which were $f_n \propto (2n-15)^{1.744}$ and $f_n \propto (2n-15)^{1.856}$ for the in-plane and out-of-plane cases respectively.

The outer part of the spiral ended in a short straight leg which was brazed into a small brass block which screwed firmly into a large steel block. This in turn was attached to the end of a brass rod which originally screwed into the wooden base of the clock's case, the latter acting as a very effective sounding board. The chime is produced by a padded hammer falling, in the plane of the gong, normally onto the centre of the top of the short straight leg. Using these conditions the out-of-plane modes were hardly excited but, because of the position of the impact, the relatively high n-value modes were excited strongly. Furthermore, all the modes in the lower regime were below 100 Hz and would contribute little to any harmonic effect even if they had been excited. Thus the frequencies of the acoustically important modes are given by

$$f_n = 3.090\,(2n - 15)^{1.744} \text{ where } n \geq 11$$

Other spiral gongs appear to obey similar frequency laws but with different values of the parameters so that in general it seems the acoustically important modes always obey

$$f_n = a(m + 2n)^k \qquad (1)$$

where m is evidently a negative integer. Clearly this equation can be used to fit all four sets of modes in our gong provided four different sets of parameters are used.

Horologists do not seem to go to any particular trouble to "tune" the gongs. No two sound quite the same yet all seem to be pleasant despite the fact that equation (1) can only be made to give a conventionally harmonious frequency sequence by the selection of very special values of the parameters, eg $k = 1$, $m = \bar{1}$. It therefore appears that equation (1) could be a recipe for producing a whole range of pleasant sounding chords most of which would have no connection whatsoever with conventional musical harmonies.

2. OTHER SYSTEMS

Rayleigh showed analytically that for a flat circular plate, with either free or clamped circumference, the frequency $f_{m,n}$ for a mode with m nodal diameters and n nodal circles takes the asymptotic form $f = c(m + 2n)^2$ for large $(m + 2n)$ where c is a constant for a given plate [see reference 2, section 218]. This is sometimes called Chladni's law because it was first discovered by him empirically [3]. It has been found that a wide variety of flat and non-flat circular plates can have their frequencies fitted by a modified form of the law

$$f_{m,n} = c_n(m + 2n)^{k_n} \qquad (2)$$

In the flat plate k_n is nearly 2 but in cymbals, bells and gongs it can vary between about 1.4 and 2.4 [4]. This equation also holds with $n = 0$ for thin ring vibrations which have $p = 2$ for inextensional and axial modes but $p = 1$ for extensionals and torsionals. As for the clock gong, only one quantum number is required to specify the order of the mode, i.e. the number of nodal "diameters" m in the case of the ring.

A particularly interesting case is the church bell where our own results on a D_5 Taylor bell [5] have been analysed using both equation (2) and an alternative modification of it

$$f_{m,n} = c(m + bn)^k \qquad (3)$$

Using the extra parameter b enabled us to fit all the various families of "shell driven" inextensional modes using parameters c and k which did not depend upon n. It was necessary to fit large and small m separately because $m \approx n$ gave the changeover point from a regime where the crown of the bell did not participate in the motion to one where it did. The results are shown in figure 3. The best fits were obtained with b = 0.81, k = 1.81 for large m and b = 1.41, k = 1.40 for small m. In the case of "RIR" modes, the only family of acoustical importance other than the shell driven, an excellent fit was obtained for all m with $f_m = c(m - 1)^{1.404}$ for $m \geq 3$ as shown in figure 4 [6].

3. INVESTIGATION OF NEW SCALES

In the foregoing discussion we have seen that many systems have normal modes whose frequencies are described by one or more applications of equation (3) with suitable choices of parameters. This includes the sounds produced by "instruments" such as the spiral clock gong which are judged by most people to be pleasant and even musical despite the frequency intervals involved not being harmonies on the conventional Western system and in particular not involving octaves.

Obviously it will only be certain subsets of the multiple infinity of possibilities presented by equation (3) which will turn out to give pleasant chords. Work is proceeding on this front. However the knowledge we already have leads us to speculate that there may be no fundamental reason for scales to be based on octaves, other than that the majority of instruments rely on the normal modes of strings or pipes so any set of tones which do not produce too harsh a dissonance when sounded together could be used as the basis of a "musical" scale.

4. PROCEDURE

In order to choose a set of frequencies which could be used as the basis of a musical scale from the infinite possibilities, a series of experiments was carried out in which pure tones from 8 oscillators were mixed and fed through an audio system. Since equation 3 is known to be able to produce pleasant anharmonic sounds it was used to calculate the frequencies. For each test 8 frequencies were used. Their values were obtained using fixed values of k, c, b and m while varying n in integer increments with the starting value selected to give a lowest frequency of at least 60 Hz. This process was then repeated for various (fixed) values of k, c, b and m. The values used in this preliminary study were m = -1, 0, 1, 2, 3, 4; b = 1, 2, 3, 4, 5,; c = 5.62 (as observed in a previously-measured clock gong), 56, 112, 400. Steps of 0.1 from 0.4 up to 3.0 were used for k. The resulting sounds were judged for their "pleasantness" or otherwise by small groups (up to 6 each) of listeners. It soon became clear that only values of k between 0.5 and 2.2 could be considered to produce "not unpleasant" sounds and that the only acceptable values for b were 1 or 2 and for m were 0 or $\pm$ 1. The range of k was further restricted by the value of c, a low value of c (5.6) producing acceptable values of k from 1.4 to 2.2 and a higher value (56) being acceptable from 1.0 to 1.6, while 400 produced acceptable combinations for values of k from 0.5 to about 1.2.

Having identified series of frequencies which sound acceptable when played together, these were then used in the construction of musical scales. Each "note" produced by a musical instrument would normally consist of several harmonics with different amplitude ratios as well as the nominal "note" being played. However in our experimental procedure, each note was a pure tone with frequency obtained from equation 3. Music was produced by writing a BASIC program for each tune, which was then played by running the program on an Amiga 500 computer using its four sound channels to produce four musical parts feeding the audio output to an amplifier and speaker.

Two procedures were used for the actual production of the music. In the first a traditional melody in C Major (The Vicar of Bray) arranged as a 4-part song was transcribed into a number of Chladni Law-based scales by transforming the conventional scale intervals into the new scale. The result had no apparent connection with the original tune but was not unpleasant to the listeners, and was mainly described as "interesting". The second procedure was to use the scale as a basis for an original musical composition. This again was deemed to be interesting, and would appear to be the best method of demonstrating the scales. The results are demonstrated in the lecture. The main difficulty is that composers/musicians are naturally inclined to think in terms of their conventional scales, and therefore find it very difficult to work with a totally unfamiliar one. The process of composition was therefore very slow and proceeded on a trial and error basis. There is therefore an advantage in using simple adaptations of conventional music in which the pitch of each note is altered but all other factors (duration, relative positions in scale etc) retain their conventional meaning, but we do not believe that this procedure will prove to be the most productive process to adopt. We do, however, believe that the production of "interesting" music on totally non-harmonic scales has been successfully demonstrated.

5. CONCLUSION

Our investigations to date have concentrated solely on a small subset of the possibilities from equation (3). Other functions could equally well be investigated. This "liberation" from the octave which has been made possible by computer-generation of music should enable an enormous variety of new musical forms to be explored. Further investigations using larger samples of listeners and more systematic tests for subject response are planned.

6. REFERENCES

[1] R PERRIN, T CHARNLEY, G M SWALLOWE and H BANU, "Modes of the Spiral Clock Gong", J Sound & Vib (1993) (in press).
[2] LORD RAYLEIGH, "Theory of Sound", Dover, New York, 1945 re-issue of 1894 edition, see volume 1, chapter 8.
[3] E F F CHLADNI, "Die Akustik" Leipzig: Breitkopf and Härtel, 2nd edition, 1830.
[4] R PERRIN and T D ROSSING, "Vibrations of Bells", Applied Acoustics, 20, 41 (1987).
[5] R PERRIN, T CHARNLEY, H BANU and T D ROSSING, "Chladni's law and the modern English church bell", J Sound and Vib, 102, 11 (1985).
[6] R PERRIN and T CHARNLEY, "On the RIR modes of the modern English church bell", J Sound and Vib, 119, 243 (1987).

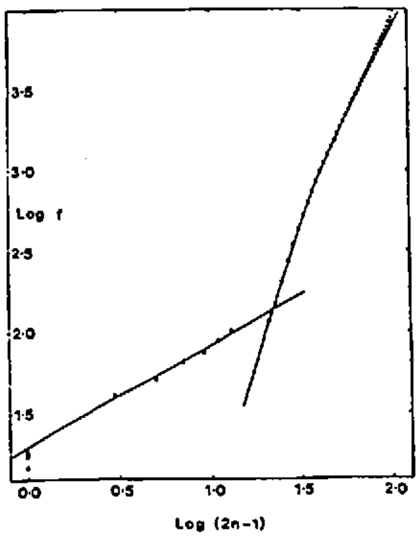

Figure 1: Log f versus log (2n-1). In plane modes for sprial clock gong. Crossover between curves is at n = 11.

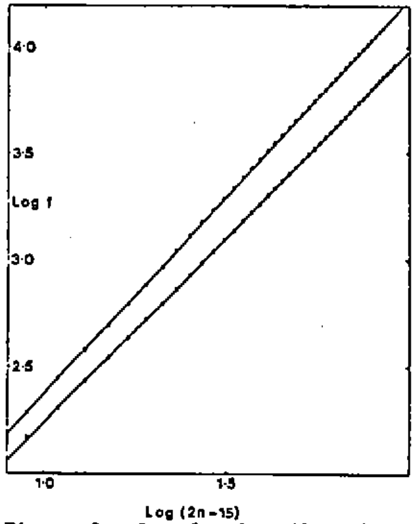

Figure 2: Log f v log (2n-15), spiral gong modes n>11. Lower line in-plane, upper line out-of-plane modes.

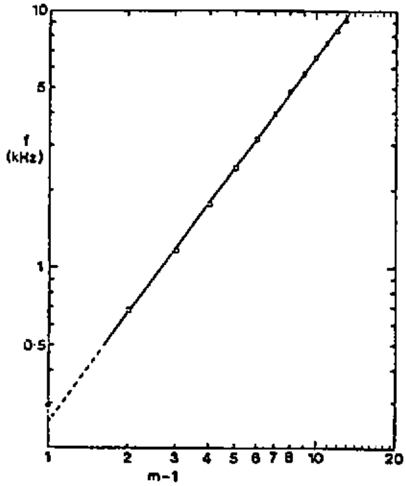

Figure 4: Log plot of f versus (m-1) for RIR church bell modes.

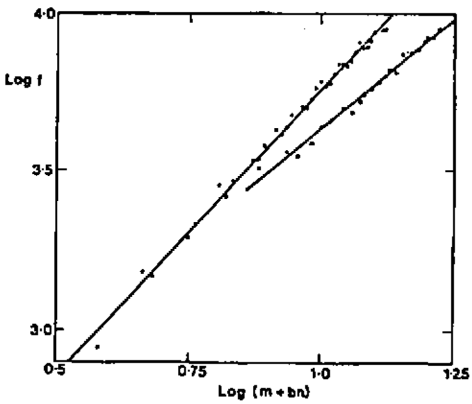

Figure 3: Log f versus log (m + bn) for shell driven bell modes, with and without crown participation.