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THE SCATTERING FUNCTION OF SOUND SCATTERED BY THE OCEAN AND WIND WAVE INTERACTIONS

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LIST OF SYMBOLS

$\alpha, \theta, \phi, \theta_o, \phi_o$	angles
k	wave number of sound passing at the sea surface
q	azimuth angle
θ	direction of the incident wave
ϕ	direction of the scattered wave
θ_a	direction of the wind
ϕ_a	direction of the scattered wind
θ_o	direction of the observation plane with the incident wave
ϕ_o	direction of the observation plane with the scattered wave
X	Scattered sea wave vector
(X_o, θ_o, ϕ_o)	indicates observation directions
k_a	wave vector for the wind propagation
$\delta\phi_1, \delta\phi_2$	shifts in the spread along the two wind directions
$\Delta(\phi_1), \Delta(\phi_2)$	shifts in the two wind directions as influenced by Doppler shifts
θ_c	critical angle
ρ	density of sea water

1 INTRODUCTION

A project is in progress at Newcastle University to study the following aspects of sound scattering:
(1) The specific characteristics of the general expression for the coefficient of sound scattering by the disturbed ocean surface.
(2) The shape of the angular scattering function (Normalized scattering coefficient).
(3) The positions of its maxima and minima of the angular scattering function as the angular widths of the maxima are of considerable interest.

(4) This study provides initial parameters for the solution of other problems such as the investigation of multiple scattering in the wave guide transmission of sound in the ocean with a disturbed surface for long range reverberation.
(5) We provide the numerical calculations of the angular scattering function and so obtain simple estimates of the angular widths of its maxima and their shift from the direction of specular reflection.
(6) The average displacement of the angular scattering function is equal to zero in the specular direction with the zero displacement. A component with infinite wavelength responsible for scattering in the backward direction does not exist in the spectrum. We now obtain estimates for the shift of the scattering maxima from the direction of specular reflection and then the angular widths.

This paper presents a new derivation of the effects of wind and water surface interactions on the scattering function cross section.

2 THE SCATTERING COEFFICIENT

The sound scattering coefficient at angle α, at small values of the Rayleigh parameter is given by the equation:

$$m_s(\theta,\phi)=4k^4\ [Cos(\theta_0)]^2\ G(X,\alpha) \tag{1}$$

Here $G(X,\alpha)$ is the spatial sea wave spectrum as a function of scattered sea wave vector X, where X has the form:

$$X=k\ [\sin^2(\theta)\ +\ Sin^2(\theta_0)- 2\ Sin(\theta)\ Sin(\theta_a)\ Cos(q-q_0)] \tag{2}$$

Here k is the wave number of the incident sound wave and q is the azimuth angle of the resonance harmonics. The terms θ_o, ϕ_o and θ and ϕ are the angles representing the direction of propagation of the incident and scattered sound waves. θ_a is the wind direction, such that:

$$0<\ \phi_0,\ \phi\leq 2\pi \qquad -\frac{\pi}{2}\ <\ \theta_0,\ \theta<\frac{\pi}{2} \tag{3}$$

For the Pierson-Moskovitz spectrum (1964), we find the sound scattering coefficient as:

$$m_s(\theta,q)=1.62\times10^{-4}Cos^2(\theta_a)Cos^2(\theta)\exp[-[\frac{X_0}{X}]^2] \tag{4}$$

such that:

$$\phi_0=\frac{0.86g}{V^2} \tag{5}$$

where V is the wind speed, such that in the specular direction:

$$m_s(\theta_0,\phi_0)=0 \qquad \phi\to 0 \tag{6}$$

3 SCATTERING COEFFICIENT IN THE PRESENCE OF WIND DOPPLER SPREAD ε

This provides:

$$m_s(\theta,\phi)=4k^4 Cos^2(\theta)Cos^2(\theta_0)G(X,\alpha)$$
$$X^2=k^2[Sin^2(\theta)+Sin^2(\theta_0)$$
$$-2Sin(\theta)Sin(\theta_0)Cos(\phi-\phi_0)]^2 \quad (7)$$

Such that:

$$\tan(\alpha)=\frac{A}{B} \quad (8)$$

where A and B are given by:

$$A=Sin(\theta)Sin(\phi)-Sin(\theta_0)Sin(\phi_0)$$
$$B=Sin(\theta)Cos(\phi)-Sin(\theta_0)Cos(\phi_0) \quad (9)$$

With this substitution:

$$m_s(\theta,\phi)=1.62\times10^{-4}[\frac{k}{X}]^4 Cos^2(\theta)\times$$
$$Cos^2(\theta_0)\exp[-[\frac{X_0}{X}]^2] \quad (10)$$

Since:

$$m_s(\theta_0,\phi_0)=0,\ \phi=0 \quad (11)$$

Which provides:

$$[[\frac{X_0}{X}]^2-2]Cos[\frac{\phi-\phi_0}{2}]=0 \quad (12)$$

The wave vector for the wind propagation is given by:

$$k_a=2kSin(\theta_0)Sin[\frac{\phi-\phi_0}{2}]$$
$$X_a=\pm\frac{X_0}{\sqrt{2}} \quad (13)$$

Which indicates:

$$Sin[\frac{\phi_a-\phi_0}{2}]=\pm\epsilon \quad (14)$$

Where:

$$\epsilon=\frac{X_0}{\sqrt{8}\ k\ Sin(\theta_0)} = \frac{0.3g}{kV^2 Sin(\theta_0)} \qquad (15)$$

Which provides:

$$x^2\exp\left(\frac{1}{x}\right)=\frac{\epsilon^3}{4} \quad x_1=1.65 \quad x_2=0.212 \qquad (16)$$

$$\phi_1=\phi_0+2Sin^{-1}(\epsilon\sqrt{2x_1}) \quad \phi_2=\phi_0+2Sin^{-1}(\epsilon\sqrt{2x_2}) \qquad (17)$$

$$\delta\phi_1=2[Sin^{-1}(1.81\epsilon)-Sin^{-1}(\epsilon)] \quad \phi_1>\phi_a \qquad (18)$$

$$\delta\phi_2=2[Sin^{-1}(0.65\epsilon)-Sin^{-1}(\epsilon)] \quad \phi_2<\phi_a \qquad (19)$$

Here ϕ_1, ϕ_2 are the scattered angles for two wind directions and $\delta\phi_1$, $\delta\phi_2$ are the shifts of the scattered wave maxima from the directions of specular observation.

$$\left[\frac{X_0}{X}\right]^2-2=\frac{X\ Sin(\theta_a)}{k\ Cos^2(\theta_a)} \quad Sin(\theta_a)=Sin(\theta)\pm\frac{X_0}{\sqrt{2}\ k} \qquad (20)$$

The critical angle θ_c is given by:

$$\theta_c=\pm\frac{X_0}{\sqrt{2}\ k\ Cos(\theta_0)} \quad \pm\left[\frac{8}{3}\right]^{0.25}\left[\frac{X_o}{k\ Sin(\theta_o)}\right]^{\frac{1}{2}} \qquad (21)$$

$$\Delta(\theta_1)=Sin^{-1}\ [Sin(\theta_0)-\frac{0.461x_0}{k}]-\theta_a \quad \theta_a<\theta_0 \quad \Delta(\theta_2)=\theta_a-Sin^{-1}\ [Sin(\theta_0)-\frac{1.285x_0}{k}] \quad \theta_a<\theta_0 \qquad (22)$$

Here $\Delta(\theta_1)$ and $\Delta(\theta_2)$ are the spread along two wind directions as effected by Doppler shifts.

These shifts have different values depending on the wind direction (θ_a).

$$\Delta(\theta_1)=Sin^{-1}[Sin(\theta_0)+\frac{1.285x_0}{k}]-\theta_a$$
$$\theta_a>\theta_0 \qquad (23)$$
$$\Delta(\theta_2)=\theta_a-Sin^{-1}[Sin(\theta_0)+\frac{0.461x_0}{k}]$$
$$\theta_a>\theta_0$$

These equations provide the following results:

$$x^{1.5}\exp[\frac{3}{2x}]=e^{2.5}$$
$$x_1=4.164,\ x_2=0.379 \qquad (24)$$
$$x=[\frac{\Delta(\theta)\pm\theta_c}{\theta_c}]^4$$

as is shown in equations (22 and 23). This indicates the widths at the critical angle θ_c as:

$$\Delta(\theta_1)=0.215(\theta_c)$$
$$\Delta(\theta_2)=0.429(\theta_c) \qquad (25)$$

which provides the condition as:

$$k\rho_0[Sin(\theta)-Sin(\theta_0)]<1 \qquad (26)$$

where ρ_o is the density of sea water at N.T.P (Normal Temperature and Pressure). such that at the critical angle, the wind direction is given by:

$$Cos(\theta_a)=\frac{k[1-Sin(\theta_0)]^{1.5}}{x_0},\ \theta_a>0$$
$$Cos(\theta_a)=\frac{-k[1+Sin(\theta_0)]^{1.5}}{x_0},\ \theta_a<0 \qquad (27)$$

at the critical angle θ_c, the spread in the wind directions as effected by wind Doppler shifts is given by:

$$\Delta(\theta_1)=\Delta(\theta_2)=Cos^{-1}[\frac{Cos(\theta_a)}{\sqrt{\epsilon}}] \qquad (28)$$

4 WIND PROFILES

The wind speed S(z) at height z above the water surface is given by the quadratic polynomial of the form :

$$S(z)=D\ [Ln\ z]^2+E\ [Ln\ z]+F \qquad (29)$$

The wind speed is continuous for all heights z, but due to the presence of Log term (Ln), it is discontinuous at z=0. The terms D, E, F are adjustable parameters.

The Monin-Obukhov similarity theory (1990) for the turbulent atmospheric surface layer states that the wind speed and temperature profiles can be described using the two parameters connected with the interaction on the sea surface. The two parameters under consideration are:

(1) Frictional Velocity U^*.

(2) Scaling temperature T^*.

Here U^* is the wind speed touching at the sea surface and T^* is the air temperature at the contact with the sea surface. L is the Monin-Obukhov stability parameter, incorporating the effects of the entire atmospheric, such that:

$$\frac{K\, z\, [\frac{\partial V}{\partial z}]}{U^*} = \Phi_m(\frac{z}{L}) \tag{30}$$

Here Φ_m is a universal function of non dimensional height z/L. It indicates that:

$$\frac{K\, T\, [\frac{\partial T}{\partial z}+\Gamma]}{T^*} = \Phi_H(\frac{z}{L}) \qquad \frac{\partial \Theta}{\partial z} = \frac{\partial T}{\partial z}+\Gamma \tag{31}$$

Here T is the air temperature in the atmosphere and:

$$L = \frac{[U^*]^2\; T_0}{gKT^*} \tag{32}$$

Here T_0 is the air temperature at the sea surface in degrees Kelvin. K is the Von-Karman turbulence constant arising from the constant circulation (Batchelor 1976) for the atmospheric environment whose value (K) is about 0.4. The term Θ is the potential temperature in absolute degrees, Γ is the dry adiabatic lapse rate arising from the layering of wind waves. The subscript m is the momentum and subscript H is the heat enthalpy constant of the atmospheric system near the sea surface as is well known that during adiabatic processes the total heat (heat enthalpy) remains constant.

For unstable stratification of the atmospheric air layers over the sea surface:

$$\frac{z}{L} < 0 \tag{33}$$

which provides:

$$\Phi_m(\frac{z}{L}) = [1-\frac{16z}{L}]^{-0.25} \qquad \Phi_H(\frac{z}{L}) = [1-\frac{16z}{L}]^{-0.5} \tag{34}$$

For stable stratification:

$$\frac{z}{L} \geq 0 \tag{35}$$

which provides:

$$\Phi_m(\frac{z}{L})=\Phi_H(\frac{z}{L})\equiv 1+\frac{5z}{L} \qquad (36)$$

In this case the temperature profile is adiabatic. For neutral stratification, we have:

$$T^*=0, \frac{z}{L}=0$$
$$\frac{\partial T}{\partial z}=-\Gamma \qquad (37)$$

In the neutral stratification case, the wind velocity is given by:

$$V(z)=\frac{U^*[Ln(\frac{z}{Z_0})]}{K} \qquad (38)$$

$$\phi_m(0)=\phi_H(0)\equiv 1 \qquad (39)$$

Here Z_0 is the sea roughness length. The Monin-Obukhov similarity theory is only valid for $z \geq Z_0$ due to the presence of logarithmic discontinuity in the wind profile. The boundary condition for the wind profile is given by:

$$V=0, \ z=Z_0 \qquad ((40)$$

The value of Z_0 is about $10^{(-5)}$ metres for the sea surface covered with ice and several metres for the sea surface close to the sea shore.

The Monin-Obukhov theory is based on the physical processes existing in the turbulent surface layer as introduced by the above derivation of the effects of wind and the turbulence on high frequency sound propagation.

5 CONCLUSION

We have shown that the effect of wind direction and strength at different heights above the sea water surface upon the scattering coefficients may be calculated by the method outlined above. The calculation includes the effects of Doppler shift as happens during wind flow.

In the wind profiles considerations, we have also included the adiabatic effects created by the layering of wind motions. The turbulence effects caused by the wind circulations are also taken into consideration.

6 REFERENCES

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