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THE ACCURACY OF AN INTERFEROMETRIC SIDESCAN SONAR

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ABSTRACT

Interferometric sidescan sonars have been developed in the recent past for seabed surveying purposes *[1,2]*. These sonars provide not only range and amplitude information as a function of time but continuous estimates of the angle of arrival of the wavefronts in the vertical plane. The angle of incidence is usually estimated by measuring the phase-difference between the signals from a pair of transducers mounted above one another. The resolution of such a system is set primarily by the frequency bandwidth and acoustic beam widths; the accuracy by factors such as the signal-to-additive-noise ratio and the degree of temporal, and spatial, coherence.

An expression will be given for the effective correlation coefficient between the signals from two transducers when additive noise, temporal incoherence and spatial incoherence contribute. The probability distribution of the instantaneous phase-difference will be related to this effective correlation coefficient.

A novel form of signal pre-processing which dramatically reduces the tails of the phase-difference probability distribution will be briefly discussed. Plots of the distributions resulting from a Monte-Carlo simulation will be presented and the benefits of this form of pre-processing for interferometric sidescan sonar made clear.

INTRODUCTION

There has been commercial interest shown recently in a new sonar technique for seabed surveying *[3]*. The technique permits the rapid, dense surveying of broad swathes of seabed in much the same way that conventional sidescan provides images of the backscattered acoustic intensity from a broad swathe. Figures 1 and 2 depict the underlying principle of the technique.

Identical transducers with beams that are narrow ($\lesssim 1^{\circ}$) in azimuth and broad ($\gtrsim 60^{\circ}$) in the vertical plane are mounted above one another on the two sides of a 'fish' (towed body) thus permitting the simultaneous surveying of both sides of the fish. Since the system is symmetrical and each side of the fish is ideally independent, one needs only to consider a single side of the fish to understand the principle of operation.

An acoustic pulse many cycles long (narrow band) is transmitted from one of the transducers and the backscattered signals from the seabed received by all three. By measuring the phase-differences between the arriving signals at the three transducers as a function of time, the angle of incidence of the signals upon the transducers can be determined unambiguously at any instant (even if the transducer separations are greater than the acoustic wavelength λ) and combined with range information to calculate a continuous height profile for the seabed. If a single pair of transducers only are used, and these have a large separation ($d > \lambda$), the phase-difference cannot be unambiguously determined. Further information such as that provided by a third transducer is required to remove the ambiguity.

The height profile of the seabed will be determined with respect to the towed body's frame of reference and a practical surveying system must include equipment to provide the absolute position and orientation of the towed body with time. Whilst inaccuracies in the absolute position and orientation of the fish will cause depth measurement errors, the concern of this paper is to deduce the magnitude of the expected random errors due to additive noise and partial incoherence in the acoustic system alone. For the purpose of this paper, sidelobes will be assumed to be negligible.

THE RESOLUTION

The limits to the resolution of the depth-measuring system are set by the azimuthal beam-width and the acoustic pulse length which, in turn, will be determined by the frequency band-width of the system. In general it is not possible to specify the depth resolution of the system since it depends upon the instantaneous geometry of the situation and the seabed topography. For example, at any instant of time, an acoustic pulse of a given length would insonify a larger area of flat seabed at a close range than at a distant range (see figure 2). Of course, any post-detection averaging that is used to decrease random errors will result in a degradation of the resolution.

RANDOM ERRORS

The effect of additive noise and partial incoherence is best established by determining their effect upon the accuracy of the phase-difference measurements. Uncertainties in phase-difference measurements may then be readily converted into uncertainties in depth measurements when the geometry of the situation is known. To proceed further it must be assumed that the ambiguity in phase-difference which exists when $d > \lambda$ has been removed so that phase-difference errors lie within the range $\pm 180^{\circ}$. Once the phase-difference ambiguity has been removed, the depth may be determined from a single pair of transducers and thus to determine the ultimate accuracy one needs only to consider a single pair of transducers (see figure 2).

The statistical nature of the signals

The electrical signals from the two receivers will be made up of two major components which can be further subdivided: the backscattered acoustic signals from the seabed and additive noise. At a frequency of 300 kHz, a commonly used frequency for this work, the additive noise would be expected to be largely introduced by the front ends of the receiving systems whilst at low frequencies the additive noise would be predominantly produced by water-borne acoustic sources. It must be assumed that in a practical system the two receiving channels would be highly isolated such that the level of cross-talk would be negligibly small. This ensures that there would be no coherent additive noise resulting from the electrical components. The spatial separation of the transducers together with spatial incoherence should ensure the statistical independence of additive acoustic noise.

The backscattered acoustic signals are noise-like in character and, for our purposes, can be modelled by the Rice representation for narrow-band noise [4,5,6,7]. When received by the two transducers they will not be identical, again due to the spatial separation of the transducers, and the incoherent component of the signals will behave in a manner which is indistinguishable from independent additive noise. One may thus identify three components in the electrical signals from the receivers: a coherent component of the acoustic signals, an incoherent component of the acoustic signals and an independent additive noise component. All three components are noise-like and the incoherent signal component can be lumped together with the additive noise and thought of as a single, effective noise component.

The effective correlation coefficient between the signals from the receivers as a result of the effects discussed above is given by

$$\rho_e = P_c/(P_s + P_a) \tag{1}$$

where P_c is the coherent component of the signal power P_s, and P_a is the additive noise power. If additive noise were not present, the correlation coefficient due to incoherence of the signals would be given by

$$\rho_s = P_c/P_s \tag{2}$$

Also, if the backscattered acoustic signals were perfectly coherent any decorrelation would be due to additive noise alone and the correlation coefficient would be given by

$$\rho_a = P_s/(P_s + P_a) \tag{3}$$

From (1), (2) and (3) it can be seen that

$$\rho_e = \rho_s \rho_a \tag{4}$$

and hence that the effective correlation coefficient is the product of the correlation coefficients due to two unrelated physical processes. Equation 3 can be expressed in terms of the signal-to-additive-noise ratio R as follows:

$$\rho_a = R/(1 + R). \tag{5}$$

It is also possible to define an effective signal-to-noise ratio R_e:

$$R_e = P_c/(P_s - P_c + P_a). \tag{6}$$

The frequency characteristics of the signals and additive noise

A narrow-band system would generally apply a filter that is matched to the expected frequency spectrum of the signals in order to maximise the signal-to-additive-noise ratio. This has the effect of band-limiting the signal and additive noise in the same way irrespective of the source of the additive noise. In practice, narrow-band transducers would be employed that band-pass filter any acoustic noise before it is match-filtered in the receiving system thus tending to make the frequency characteristics of the signals and acoustic additive noise more alike. The incoherent component of the signals would, of course, have an identical frequency spectrum to that of the coherent component.

In determining the probability distribution of the phase-difference error, any dissimilarities in the frequency spectra of the signals and additive noise have no effect when the phase-difference is measured by using an instantaneous sampling technique. The precise forms of the frequency spectra would be expected to have some effect however if time averaging is used at any stage in the processing. Time averaging is used in a technique that will be described later and this will be further discussed then.

A method that is commonly used to measure phase-difference is the 'zero-crossing technique'. The time lag between detections of zeros in the two signals is measured and converted into phase-difference. This time lag will be within the range $\pm 1/2f$ where f is the carrier frequency and, since the acoustic pulse will typically be several carrier cycles long (narrow band), the determination of phase-difference in this manner can be considered to be instantaneous.

The probability distribution (PD) of the instantaneous phase-difference error

The probability distribution (PD) of the instantaneous phase-difference error has been derived in a particularly relevant text by Ol'shevskii *[7]*:

$$P(\Delta\phi_e) = \frac{\left(1 - \rho_e^2\right)}{360(1 - \beta^2)} \left[1 + \frac{\beta}{\left(1 - \beta^2\right)^{\frac{1}{2}}} \left\{ \frac{\pi}{2} + \text{arc}\sin\beta \right\}\right] \qquad (7)$$

where $\beta = \rho_e \cos\Delta\phi_e$ and $-180^\circ \leqslant \Delta\phi_e \leqslant +180^\circ$. Clearly (7) is an even function of $\Delta\phi_e$. Some plots of this distribution are compared with similar plots which result from a different, non-instantaneous pre-processing technique (discussed later) in figure 3. Note that the PD given in (7) is characterised by a single variable only; the effective correlation coefficient. It can be seen that as ρ_e tends towards zero the PD becomes a uniform distribution over $\pm 180^\circ$.

From equation (4) it is clear that, besides requiring a knowledge of the signal-to-additive-noise ratio R, ρ_s must be calculated in order to determine ρ_e. Consider a pair of transducers that receive backscattered acoustic signals $S_1(t)$ and $S_2(t)$. It will be shown elsewhere *[8]* that the correlation coefficient between $S_1(t)$ and $S_2(t+t')$ can be approximated by

$$\rho_s(t,t') \approx \frac{\int E(t-\tau)E\left(t-\tau+t'-\frac{d\sin\theta}{V}\right) \langle F^2(\tau)\rangle\, e^{\frac{i2\pi\, d\cos\theta}{\lambda}\theta'(\tau)}\, d\tau}{\int E^2(t-\tau)\ \langle F^2(\tau)\rangle\, d\tau} \qquad (8)$$

where E(t) is the envelope of the transmitted pulse. θ is the angle of incidence subtended by the centre of the insonified area at time t and θ' is the angle measured from θ (see figure 2) of an element of seabed at range $r = V\tau/2$. $\langle F^2(\tau)\rangle$ represents the ensemble averaged power of the scattering by the element of seabed $Vd\tau/2$ at time τ. For most circumstances $\langle F^2(\tau)\rangle$ will be constant over the range of the integral. The limits on the integral are set by the pulse duration since E(t) is zero for t < 0 and t > T.

It is of interest to investigate equation (8) for two particularly relevant cases. The first case is when the insonified area which is common to both transducers at time t subtends a very small angle. If the angle is sufficiently small such that the exponential term in the integral is approximately unity for the range of the integral, then equation (8) becomes

$$\rho_s(t,t') \approx \frac{\int E(t-\tau)E\left(t-\tau+t'-\frac{d\sin\theta}{V}\right) \langle F^2(\tau)\rangle\, d\tau}{\int E^2(t-\tau)\ \langle F^2(\tau)\rangle\, d\tau}\,. \qquad (9)$$

In traditional terms this represents the degree of temporal coherence. If $\langle F^2(\tau)\rangle$ is constant then ρ_s is simply the autocorrelation function of the envelope.

The second case is when the time delay t' is made equal to $d\sin\theta/V$. When this is so, equation (9) represents the degree of spatial coherence alone and can be re-expressed as the Fourier transform of the angular spectrum of the power incident upon the transducers at time t. This is an analogue of the Van Cittert-Zernike theorem of optics.

AN IMPROVED SIGNAL PRE-PROCESSING TECHNIQUE

We have developed a pre-processing technique for this application which results in phase-difference measurements that are more accurate than those obtained by an instantaneous sampling technique. The technique produces the 'in-phase' and 'quadrature' components of the phase-difference as follows:

$$\int_{t-T/2}^{t+T/2} A(t)B(t)\cos\Delta\phi(t)\,dt \approx \overline{A(t)B(t)}\cos\Delta\phi(t)$$
$$\int_{t-T/2}^{t+T/2} A(t)B(t)\sin\Delta\phi(t)\,dt \approx \overline{A(t)B(t)}\sin\Delta\phi(t) \qquad (10)$$

A and B are the signal amplitudes and the bar denotes the integration over a time corresponding to the pulse duration T. Clearly the phase-difference can be calculated from these terms together with a measure of the signal intensity. Very little resolution is lost because of the limit on the duration of the integration and, because the process results in a power-weighted average for the phase-difference, the probability of obtaining a wildly erroneous measurement of phase-difference due to the coherent signal level being low at the time of the sample is greatly reduced. This technique will be described in greater detail separately *[8]*.

A Monte-Carlo simulation of the statistics

We have not yet found an analytic expression for the PD of the phase-difference when it is measured by the improved technique and have therefore simulated the PD by the Monte-Carlo method. 1.75×10^6 independent samples of phase-difference error were calculated by each of the techniques (the instantaneous and improved techniques) from the same simulated signals. Some examples of the PDs resulting from this simulation are shown in figure 3. In addition, simulated PDs corresponding to an instantaneous technique are compared with the expected analytic expression (equation (7)) in figure 3 as a check on the reliability of the simulation. The new pre-processing technique yields PDs with significantly lower tails.

Figure 4 shows the probability of obtaining a measurement which is in error by more than a given angle of phase-difference (cumulative distributions). It appears that, not only does the non-instantaneous technique provide more-reliable results for low signal-to-noise levels, the benefit of the technique increases with the effective signal-to-noise ratio.

The log of the standard deviation of the phase-difference error is plotted against the effective signal-to-noise ratio in dB in figure 5. Attention is drawn to the fact that the probability of obtaining an error greater than the standard deviation is not constant, as it is with the normal (Gaussian) distribution, but is a function of the effective signal-to-noise ratio. It also depends upon the processing technique and extreme care must be taken when interpreting this plot. Clearly, as the signal-to-noise ratio deteriorates the standard deviation will approach 103.9° corresponding to a uniform distribution over $\pm 180^{\circ}$.

It was mentioned earlier that the precise forms of the spectra of the signals and additive noise would be expected to have some effect upon the PD of the phase-difference error when time averaging is used. The present simulation corresponds to the case where a tophat acoustic pulse of duration T has been transmitted and the spectrum of the additive noise is identical to that of the signals. For the reasons outlined earlier, it is thought that this should represent a good approximation for any practical narrow-band system.

CONCLUSIONS

When the phase-difference between backscattered signals arriving at two transducers is measured by using an instantaneous technique, the probability distribution of the random error is characterised by a single parameter: the effective correlation coefficient between the voltages. The effective correlation coefficient is the product of the coefficients due to additive noise and partial coherence of the backscattered signals. Expressions have been given for these coefficients and the resulting probability distribution. It has been shown that a significant reduction in error can be achieved by simply changing the method by which the phase-difference is measured. In particular, it is possible to greatly reduce the number of wildly erroneous measurements thus producing much lower tails in the distribution of the errors.

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REFERENCES

[1] R.L. Cloet, S.L. Hurst, C.R. Edwards, P.S. Phillips and A.J. Duncan, 'A Sideways-looking Towed Depth-measuring System', J.R. Inst. Navigation, Vol.35, 411-420, (1982).

[2] P.N. Denbigh, 'Stereoscopic visualization and contour mapping of the sea bed using a bathymetric sidescan sonar (BASS)', The Radio and Electronic Engineer, Vol.53, no.7/8, 301-307, (1983).

[3] R.L. Cloet, C.R. Edwards and A.J. Duncan, 'The University of Bath Swathe Sounding System', Proceedings of this conference, Institute of Acoustics, (1985).

[4] J.S. Bendat, 'Principles and Applications of Random Noise Theory', John Wiley & Sons, Inc., 138-141, (1958).

[5] S.O. Rice, 'Mathematical Analysis of Random Noise', Bell System Tech. J., Vol.23, 282-332, (1944).

[6] S.O. Rice, 'Mathematical Analysis of Random Noise', Bell System Tech. J., Vol.24, 46-156, (1945).

[7] V.V. Ol'shevskii, 'Characteristics of Sea Reverberation', Consultants Bureau, New York, 133-137, (1967).

[8] G.R. Gapper and T. Hollis, In preparation.

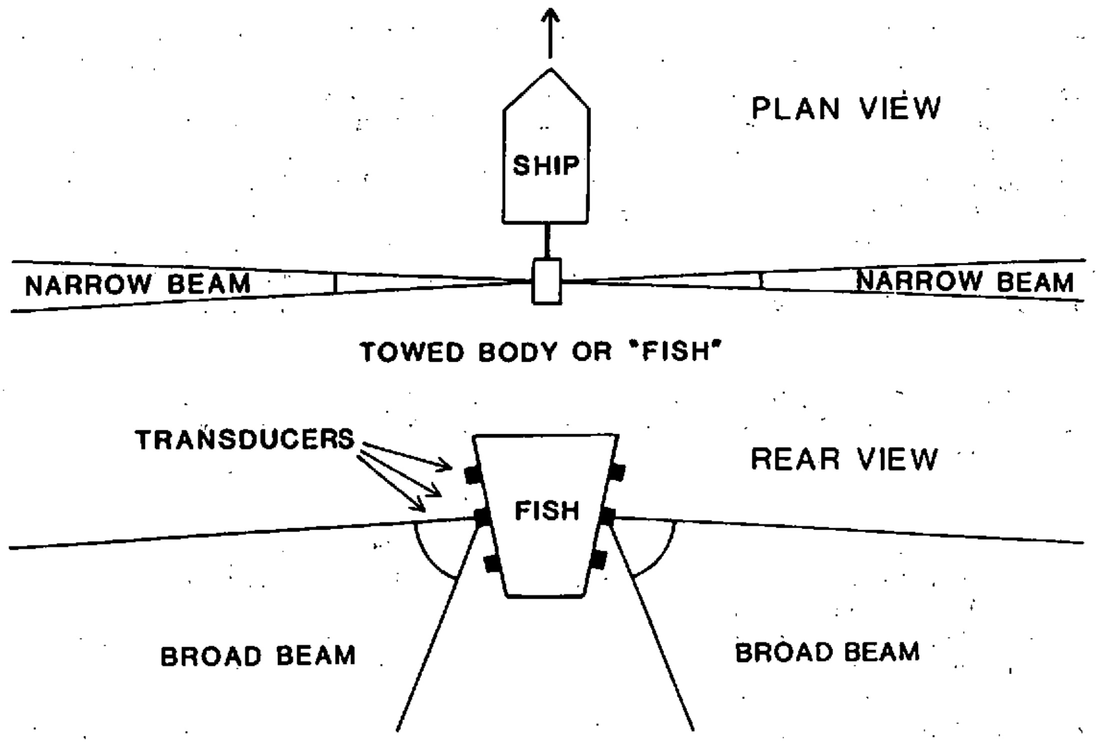

Figure 1. A schematic representation of the operating principle.

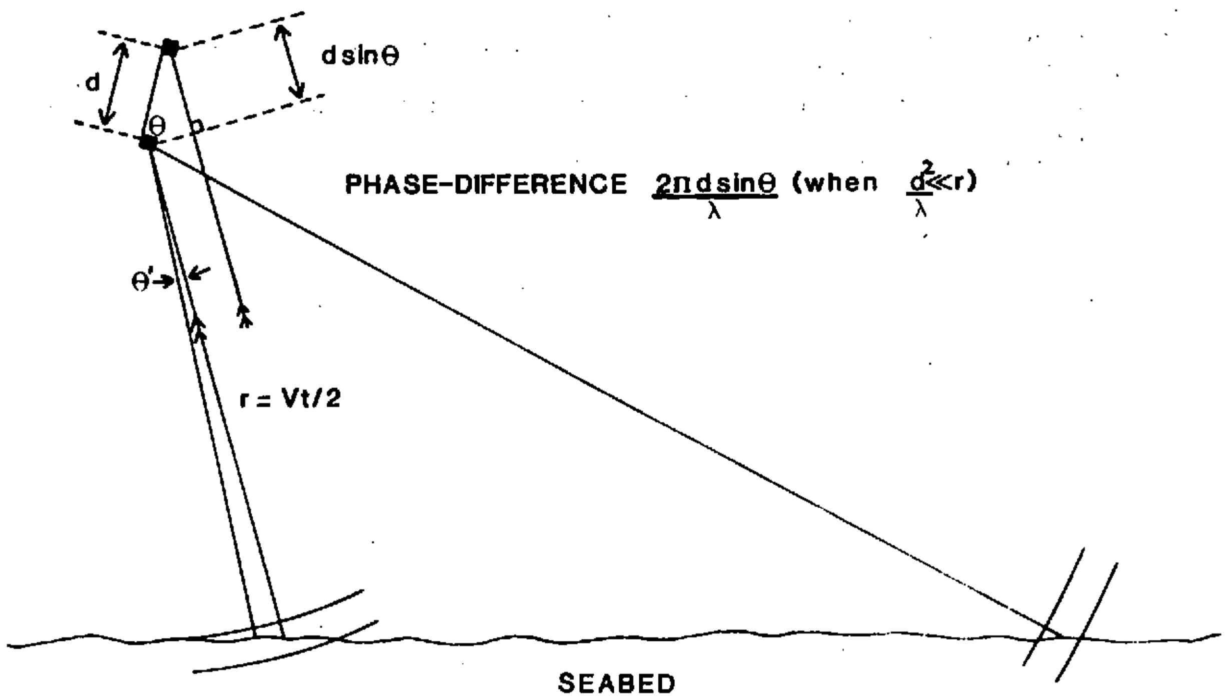

Figure 2. Waves that are backscattered in the far-field of the transducers appear as plane waves to the transducers and lead to a simple relationship between phase-difference and angle of incidence. The resolution of the technique depends upon the instantaneous geometry of the situation.

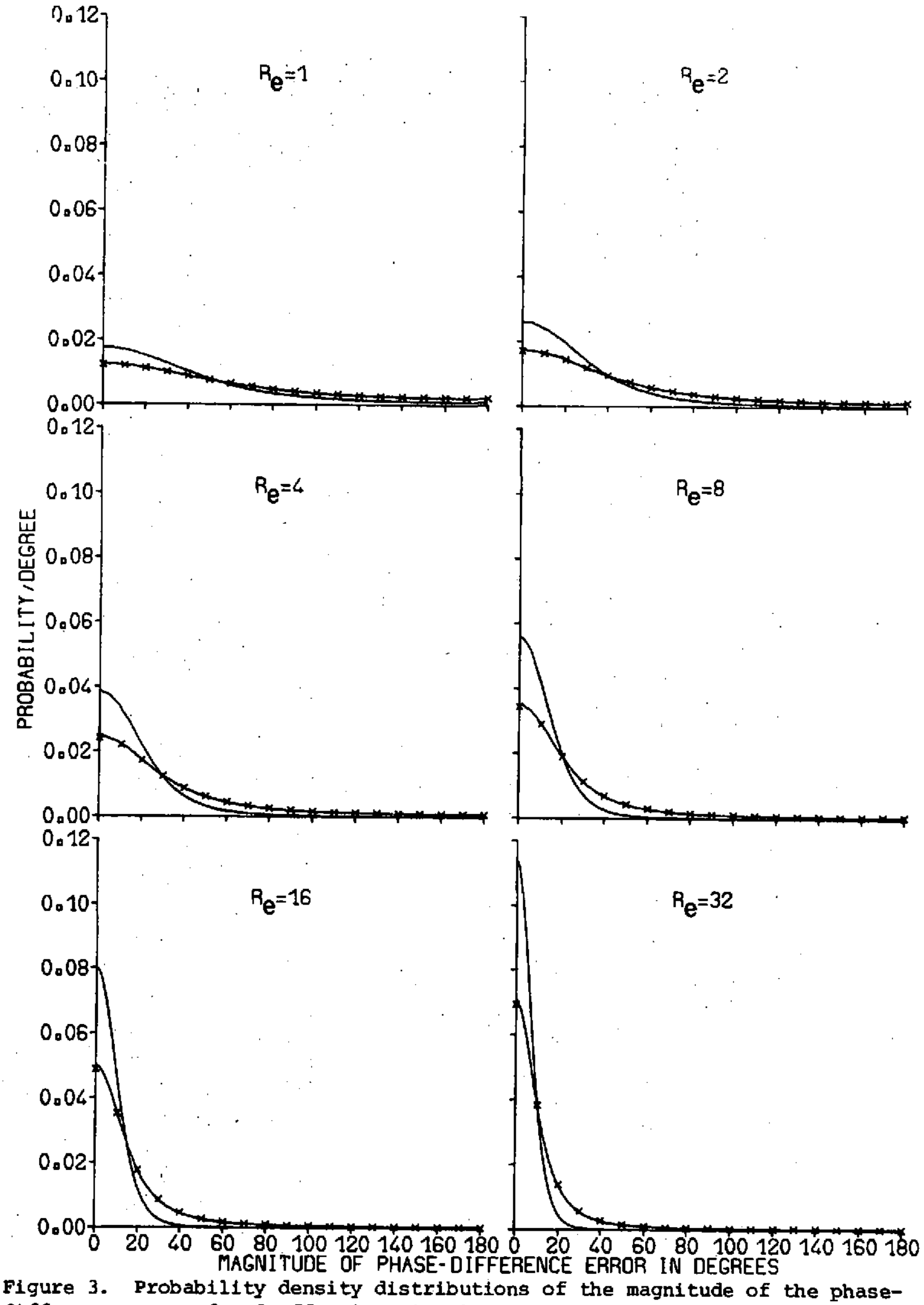

Figure 3. Probability density distributions of the magnitude of the phase-difference error for 6 effective signal-to-noise ratios. The crosses result from the analytic expression (7); the pairs of lines from the Monte-Carlo simulation.

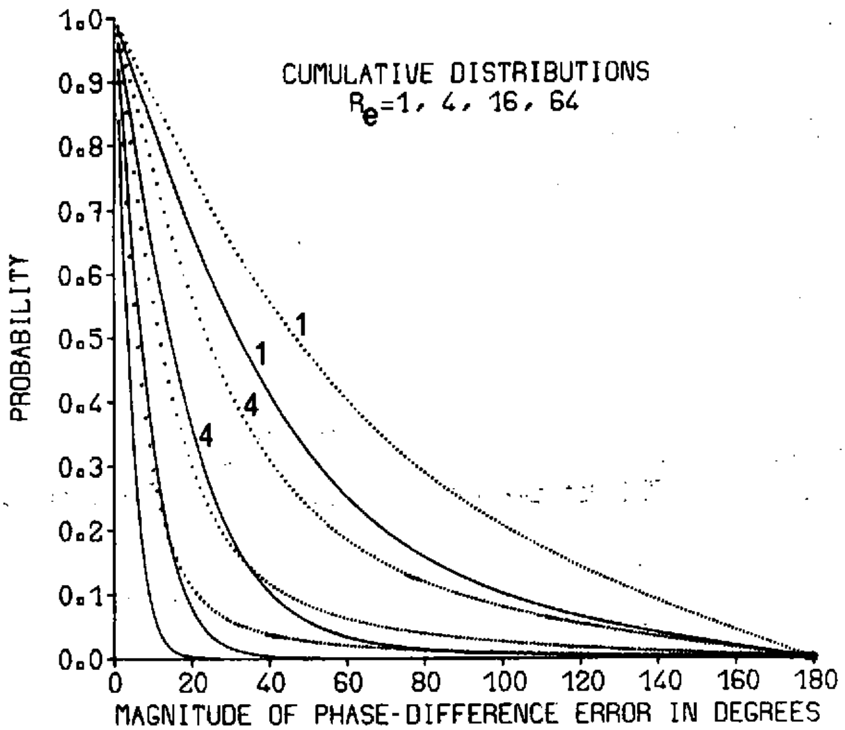

Figure 4. The probability of obtaining a phase-difference error greater in magnitude than a given angle for various effective signal-to-noise ratios. The dotted lines correspond to an instantaneous technique.

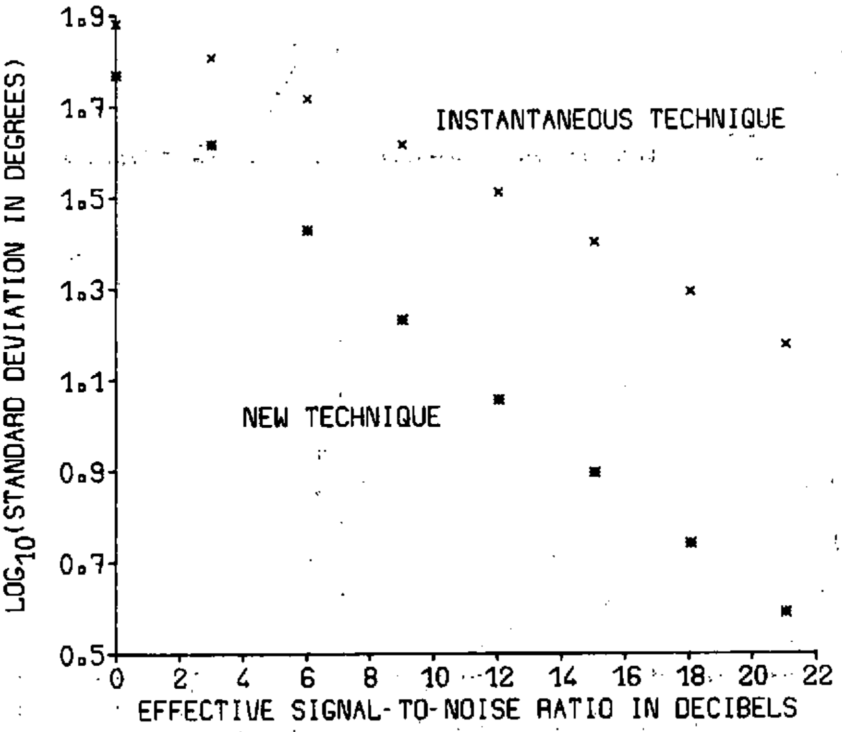

Figure 5. The log of the standard deviation of the phase-difference error versus the effective signal-to-noise ratio in decibels for the two techniques.