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NOISE FROM AN ISOLATED ROTOR DUE TO INFLOW TURBULENCE

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1. INTRODUCTION

Measurement of noise radiated from subsonic rotors to on-axis points exhibit discrete tone levels above the broad-band, at blade passing frequency and its higher harmonics. Steady state or any periodic blade loading do not account for the noise levels measured. Hence, we directed our attention to the turbulence in the inflow velocity and the resulting random blade loading. The interaction of turbulence with rotor-related flow field also contributes to noise at on-axis points. In considering radiation from these sources, the effects of mean flow Mach number and duct walls are ignored. The results presented here are based on work performed under Contract No. NAS2-6401 for NASA Ames Research Center.

2. VELOCITY FLUCTUATIONS

Let us describe the fluctuating velocities in the flow, since they govern the blade loading and quadrupole strength within the fluid. Frozen-convected, isotropic turbulence with longitudinal space correlation function of the type $\exp(-|\xi|/\ell)$ is assumed in the inflow. The lift fluctuation on the blade is related to the relative velocity V and the turbulent velocity component v_n, normal to V. Liepmann (ref. 1) has shown that the spectrum of v_n can be written as

$$\phi_{v_n} = \overline{v^2}(\ell/\pi V)\{1+3(\ell\omega/V)^2\}\{1+(\ell\omega/V)^2\}^{-2} \qquad (1)$$

where $\overline{v^2}$ denotes the mean square value of turbulence. The correlation lengths in the axial, radial and tangential directions for the component v_n are respectively

$$\ell_1 = \tfrac{1}{2}\ell(\cos\lambda)^{-1} \;;\quad \ell_2 = \tfrac{1}{2}\ell \;;\quad \ell_3 = \tfrac{1}{2}\ell(\sin\lambda)^{-1} \;; \qquad (2)$$

where λ denotes the relative flow angles, as shown in Fig. (1).

For the noise at on-axis points we need to consider only longitudinal axial quadrupoles, which can be evaluated by means of the space-time correlation functions $R(\xi,\tau)$ of the axial components of velocities. Assuming that the turbulence is convected in the axial direction with velocity U_a, the axial velocity through the fan, we can write the correlation function for the axial component of turbulence as

$$R_{v_1}(\xi,\tau) = \overline{v^2}\cdot\exp\left\{-\frac{|\xi_1-U_a\tau|}{\ell} - 2\frac{|\xi_2|}{\ell} - 2\frac{|\xi_3|}{\ell}\right\}, \qquad (3)$$

The subscripts 1,2,3 on ξ denote respectively the axial, radial and tangential components of the separation vector ξ.

Potential flow field, related to blade lift and thickness, produces periodic axial velocity fluctuations u_1, which are of significance only in the region upstream of the rotor. Furthermore, the amplitude of the nth harmonic decreases rapidly as $\exp(-2n\pi\eta_1/d)$ where d is the blade spacing and η_1 is the axial distance upstream of rotor leading edge. For a rotor with B number of blades and circular frequency Ω, the space-time correlation of the nth component of u_1 can be written as

$$R_{u_{1,n}}(\xi,\tau) = \tfrac{1}{2}(\hat{u}_{1,n,0})^2\cdot\exp\{-\frac{2n\pi}{d}(2\eta_1+\xi_1)\} \times \cos(\frac{2n\pi\xi_3}{d} - nB\Omega\tau) \qquad (4)$$

where the amplitude of the nth component at rotor leading edge is denoted by $\hat{u}_{1,n,0}$.

3. DIPOLE RADIATION FROM RANDOM BLADE LOADING

We consider the B number of blade elements within an annular region $2\pi r\ell_2$. Each element experiences lift fluctuations related to the turbulence component v_n, relative speed V and Sears' lift-response function. Let ϕ^t denote the spectrum of thrust loading on the blade element of span ℓ_2. For blade spacing d smaller than ℓ_3 the number of blades that simultaneously interact with a coherent eddy can be represented by

β_1 = lowest integer larger than ℓ_3/d.

If the eddy is convected downstream rather slowly compared to the blade tangential speed Ωr, an additional number of blades will interact with the same eddy at subsequent times. The number of such blades can be represented by

β_3 = lowest integer larger than $\ell_1\Omega r/dU_a$.

Taking into account load fluctuations on the neighboring blades, as described above, we formulate the autocorrelation for the acoustic pressure in the radiated field. The resulting sound pressure spectrum at an axis point, distance R from the rotor, can be written as

$$\phi_p^d = \frac{\omega^2 B}{(4\pi Ra)^2}\cdot F(\beta_1,\beta_3,\omega,B\Omega)\cdot\phi^t \qquad (5)$$

where a = speed of sound and the superscript d is used to denote the contribution from dipole radiation. Our computations of the sound pressure spectrum for values of $\ell > d$ show prominent peaks at the blade passing frequency and its harmonics. Such behavior is also noted by Mani (ref 2) in his computations of acoustic power. Since the factor $\omega^2\phi^t$ in the above equation is a monotonically decreasing function in the neighborhood of $\omega = nB\Omega$, the peaks in the sound spectrum arise only from the behavior of the function F in this region. The function exhibits a sharp peak value of $(\beta_1+\beta_3-1)^2/\beta_1$ at $\omega = nB\Omega$.

4. RADIATION FROM RANDOM QUADRUPOLES

The turbulence in the inflow and the rotor-related potential flow fluctuations are unrelated. Consequently the space-time correlation function of $(u_1 v_1)$ can be written as the product of the individual correlation functions given by Eqs. (3) and (4). Using the spectral form of Lighthill's equation as given by Ribner (ref. 3), one can integrate the Fourier transform of the space-time correlation function of $(\rho u_1 v_1)$ over the coherent eddy volume to obtain the spectrum ϕ^q of the axial quadrupole strength. The source spectrum exhibits strong peaks at $B\Omega$ and higher harmonics, a feature also discussed by Williams and Hawkings (ref. 4), and Chandrashekhara in ref. (5). The spectrum of sound pressure at an axis point due to all the quadrupole sources can be written as

$$\phi_p^q = \frac{\omega^4}{(4\pi R a^2)^2} \int_{\text{vol.}} \phi^q \, d(\text{vol.}) \tag{6}$$

Since we defined u_1 only in the region upstream of the rotor, the volume integration in the above equation is limited to this region.

5. RESULTS AND CONCLUSIONS

Using the methods described above, we calculated the sound spectra at an axis point 5 ft. upstream of a 1 ft. diameter subsonic rotor having 15 blades. The details of the rotor and its operating conditions are described in ref. (6). In evaluating the thrust loading spectrum ϕ^t, we assumed turbulence intensity of 3% and parametric values of length scale ℓ. Results of our computations of radiation from the random blade loading are presented in Fig. 2(a) and the peaks in the spectra at blade passing frequency of 1.9 kHz become narrow as the ratio ℓ/d is increased. For the quadrupole source computations, we assumed the fundamental amplitude $\hat{u}_{1,1,0} = 0.4V$. Spectra of sound pressure calculated from the random quadrupole sources for various values of ℓ/d are presented in Fig. 2(b) and show narrow peaks at the blade passing frequency. In Fig. (3) is shown a comparison of sound pressure levels as computed from dipole and quadrupole radiation. It appears that the discrete tone noise from the typical subsonic rotor examined here is primarily from the random loading on the rotor blades resulting from the inflow turbulence.

8. REFERENCES

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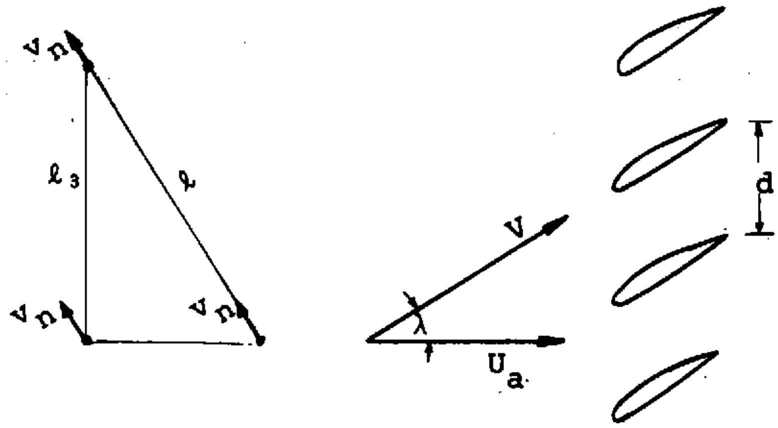

Fig. 1. Orientation of Blade Row and Velocity Field.

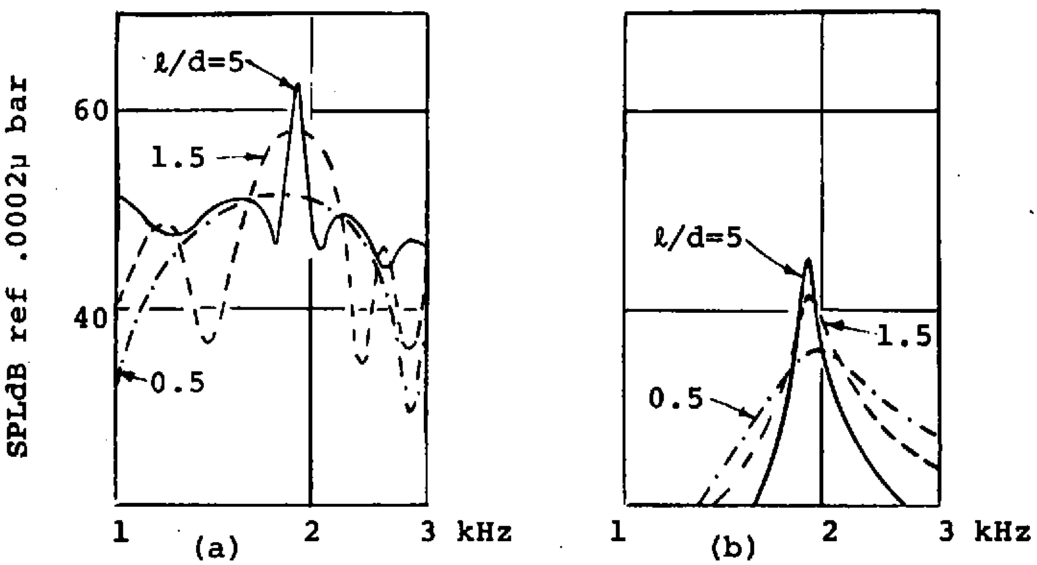

Fig. 2. Spectra of Sound Pressure Level Computed for Radiation from (a) Dipole Sources; and (b) Quadrupole Sources.

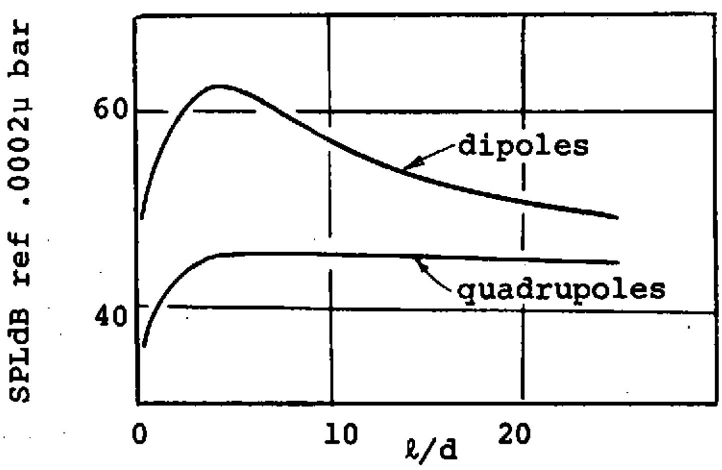

Fig. 3. Comparison of Dipole and Quadrupole Radiation at Blade Passing Frequency.