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MODELLING AIR-CAVITY MODES OF STRINGED INSTRUMENTS

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The finite element method has previously been used extensively to model the structure of the violin and guitar [1]. These models included only the structure of the body and effectively gave the natural modes in a vacuum.

Experimental investigations reported by several authors have shown that the vibrations of the internal air cavities of violins and guitars have an important effect on the behaviour of the complete instrument.

The governing linearised equation for pressure in a fluid is the wave equation

$$\nabla^2 = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2},$$

where c is the sound velocity (given by $c^2 = B/\rho$, where B is the adiabatic bulk modulus and ρ is the density).

For an eigenvalue analysis, $p = pe^{i\omega t}$ is substituted into the wave equation and this gives the Helmholtz equation

$$\nabla^2 p + \frac{\omega^2}{c^2}p = 0.$$

The boundary conditions are

$$\frac{\partial p}{\partial n} = 0 \quad \text{at a rigid boundary,}$$

$$p = 0 \quad \text{at a free boundary.}$$

Consider a rectangular cavity of dimensions L_x, L_y, L_z with rigid walls. The wave equation may be solved by separation of variables to give the result that the standing wave is comprised of cosines

$$P_{ijk} = A_{ijk} \cos k_i x \cos k_j y \cos k_k z,$$

with

$$k_i = \frac{i\pi}{L_x}, \qquad k_j = \frac{j\pi}{L_y}, \qquad k_k = \frac{k\pi}{L_z}.$$

The natural frequencies are given by

$$f_{ijk} = \frac{c}{2\pi}\sqrt{k_i^2 + k_j^2 + k_k^2}$$

It is impossible to obtain analytical solutions of the wave equation for general boundaries as found on musical instruments. The finite element was therefore used to calculate the internal air modes of cavities of various shapes. This work was a purely acoustical model and assumed rigid cavity walls. The material properties used for air were

$$c = 343ms^{-1}, \quad \rho = 1.21kgm^{-3}.$$

Using the normal finite element approximations [2], the problem reduces to

$$([S] - \omega^2[M])\{p\} = 0,$$

where {p} are the pressure amplitudes at the grid points, and [S] and [M] are the acoustic stiffness and mass matrices.

The natural boundary condition at a rigid boundary, $\frac{\partial p}{\partial n} = 0$, is satisfied by not specifying any boundary condition at the finite element grid points. The computation and graphical output was performed on a GEC 4090 minicomputer.

To check the accuracy of the finite element program, a rectangular cavity was modelled and results compaed with the exact analytical results presented earlier. Table 1 gives the two sets of results for a shallow cavity measuring 0.35 x 0.16 x 0.04m (which is about the same size as a violin). The comparison is very close for all the modes modelled.

A violin-shaped cavity was then analysed. The mesh comprised of three-dimensional acoustic elements having corner modes and single mid-side nodes and is shown in Figure 1.

Figure 2 shows the predicted modes in the form of contour plots of the pressure distribution at the top plate. These are in close agreement with experimental results obtained by Jansson [3] and a comparison of frequencies is given in Table 2. The computed results showed that, for the frequency range considered, the modes were two-dimensinal in nature, with the same pressure distribution occuring throughout the thickness of the cavity.

In order to assess the effect of the f-holes on the internal air modes, the f-holes were approximated by specifying a boundary condition of zero pressure along a line in the finite element mesh lying along the f-hole positions (shown dark in Figure 1). The resulting modes are shown in Figure 3 and a comparison of frequencies with those obtained by Jansson is given in Table 3. Here the agreement is not as close as for the sealed cavity. The most accurate predictions are for modes 1 and 5 which would in any case have pressure nodal lines in the vicinity of the f-holes. All the modes obtained were again essentially two-dimensional in character, although there was some perturbation in the pressure field near the top plate in the region of the f-holes.

With open f-holes, a new mode is obtained which approximates to a Helmholtz resonance. The finite element method does predict a new resonance, but its frequency is higher than that obtained experimentally. This inaccuracy is not surprising since no mass loading or radiation impedance is modelled and this also causes loss of accuracy for several of the higher modes.

A guitar-shaped cavity was also modelled and the predicted modes are shown in Figure 4. Jansson also measured the modes of a guitar cavity, but his was a half-scale model. To compare with the present results, his experimental frequencies have been halved and the resulting close comparison is given in Table 4. The finite element model predicts an air mode (mode 4) which was not obtained experimentally; it is probably difficult to excite because its frequency is so near to that of mode 3. Again the modes are two-dimensional in the frequency range modelled.

Finally, the guitar cavity was modelled with a zero pressure boundary condition at the location of the soundhole. This gave the modes shown in Figure 5 and the frequencies are compared with Jansson's experimental results in Table 5. The same comments apply for these as for the open violin-shaped cavity. For symmetric modes, the pressure distribution at the top plate was affected considerably by the open soundhole, although the modes were still easily recognisable. The pressure distribution at the back plate was hardly changed from the sealed case. For antisymmetric modes, a nodal line passes through the soundhole centre in any case, so forcing zero pressure at the soundhole did not cause much change.

The next stage would be to allow coupling between the acoustic and structural motions. It is possible to form a coupling matrix which defines the connection between the acoustic and structural elements at the fluid-solid interface, thus combining the model presented here with that given in [1]. Unfortunately, complications arise because the acoustic model is inaccurate unless all the degrees of freedom are retained. This makes the combined model very large as the usual methods for obtaining an efficient solution to the structural motion cannot be utilised.

REFERENCES

[1] Roberts, G.W., 'Vibrations of shells and their relevance to musical instruments', PhD dissertation, University College Cardiff, (1986).

[2] Zienkiewicz, O.C., 'The Finite Element Method', 3rd ed., McGraw-Hill, (1977).

[3] Jansson, E.V., 'Acoustical properties of complex cavities', Acustica 37, 211-221, (1977).

TABLES

Table 1 Comparison of analytic and finite element results for the natural frequencies of a shallow rectangular cavity of dimensions 0.16m x 0.35m x 0.04m.

Mode i,k,l	Exact	Finite element
0,1,0	490	490
0,2,0	980	980
1,0,0	1072	1072
1,1,0	1179	1179
1,2,0	1452	1453
0,3,0	1470	1472

Table 2 Comparison of experimental [1] and finite element results for a rigid violin-shaped cavity.

Mode	Experimental	Finite element
1	475	481
2	1050	1049
3	1110	1078
4	1290	1309
5	1570	1523
6	1770	1773
7	1900	1809

Table 3 Comparison of experimental [1] and finite element results for a rigid violin-shaped cavity with a zero-pressure boundary condition at the f-holes.

Mode	Experimental	Finite element
0	290	432
1	500	550
2	1090	1125
3	1190	1334
4	1290	1318
5	1610	1609
6	-	2006
7	1910	1829

Table 4 Comparison of experimental [1] and finite element results for a rigid guitar-shaped cavity.

Mode	Experimental	Finite element
1	372	370
2	540	568
3	760	765
4	-	765
5	980	1012
6	1000	1039

Table 5 Comparison of experimental [1] and finite element results for a rigid guitar-shaped cavity with a zero-pressure boundary condition at the soundhole.

Mode	Experimental	Finite element
0	121	199
1	395	461
2	545	572
3	770	785
4	-	788
5	985	1031
6	1005	1039

FIGURES

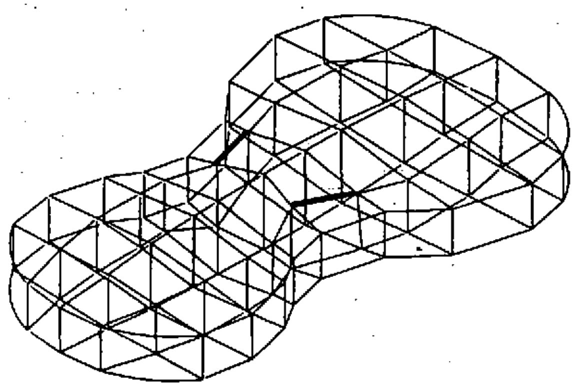

Figure 1 Mesh of a violin-shaped cavity which comprises of 20-noded hexahedral and 15-noded pentahedral acoustic elements.

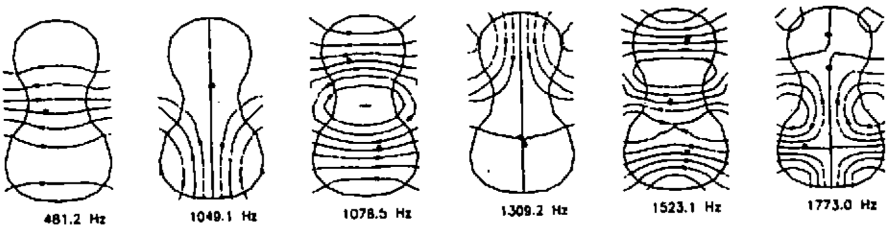

Figure 2   Finite element mode shapes for the pressure distribution within a violin-shaped cavity.

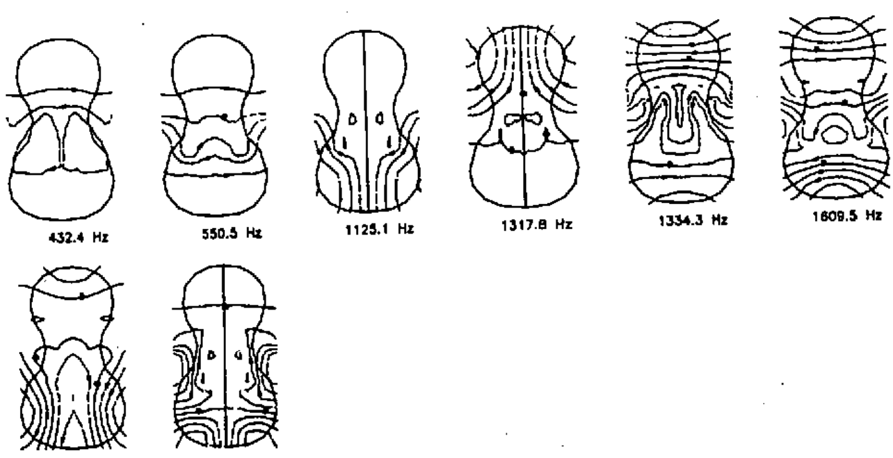

Figure 3   Finite element mode shapes for the pressure distribution within a violin-shaped cavity with a zero-pressure boundary condition at the f-holes.

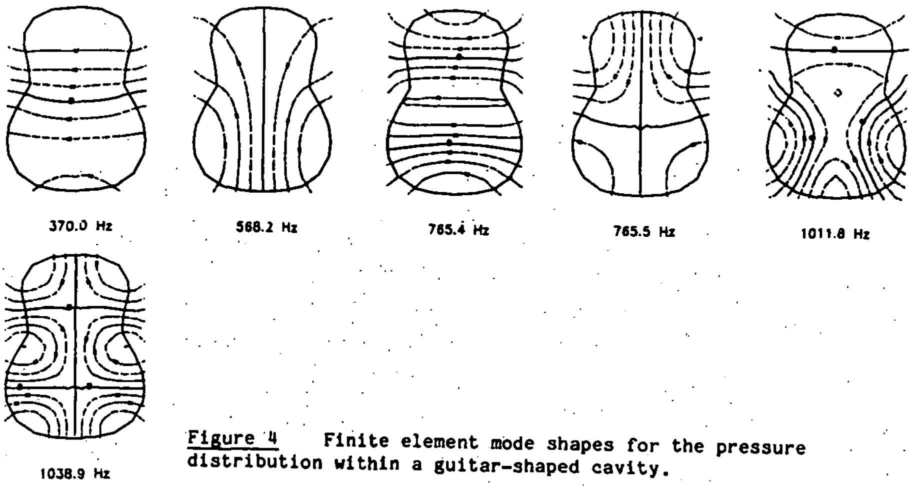

Figure 4 Finite element mode shapes for the pressure distribution within a guitar-shaped cavity.

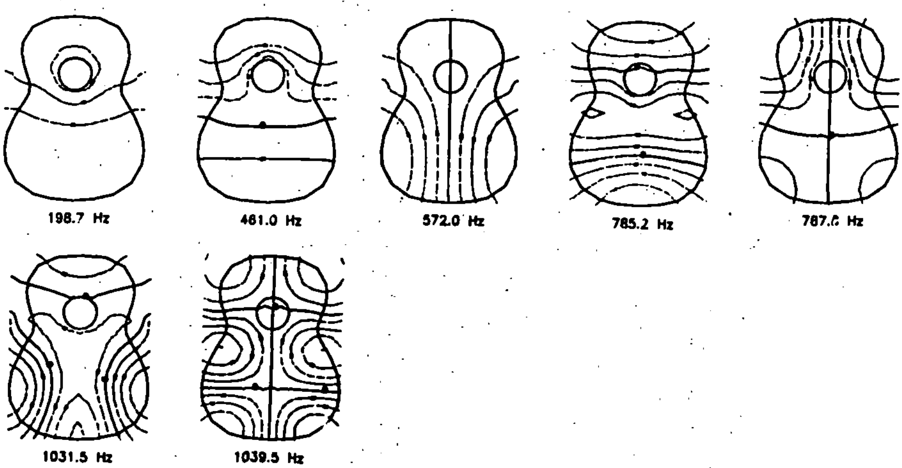

Figure 5 Finite element mode shapes for the pressure distribution within a guitar-shaped cavity with a zero-pressure boundary condition at the soundhole.