Proceedings of The Institute of Acoustics

PREDICTIONS OF THE MODAL BEHAVIOUR OF VIOLIN PLATES BY THE FINITE ELEMENT METHOD

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1. Introduction

In recent years, workers in musical acoustics have recognised the possibilities of using finite element analysis to predict the modal properties of structures found on musical instruments. The first to appear was a short study of a guitar top by Schwab [1] and a more detailed study was given last year by Richardson & Roberts [2]. The method has also been used to analyse church bells [3] and trombone bells [4].

This paper reports on progress in applying the method to analyse violin plates.

2. Outline of the Finite Element Method

The finite element method is based on the knowledge that most sets of differential equations may be expressed as a corresponding variational principle. The most widely-read text on the finite element method is that by Zienkiewicz [5]. In the present problem it is necessary to solve the shell eigen-equations for a spruce or maple violin plate. Furthermore, these equations must be solved for each case of different shell geometry and boundary conditions.

The shell is divided into general quadrilaterals or triangles called 'finite elements' and the structural geometry is defined in terms of 'nodal points' or 'nodes' which are situated at the corners and mid-points of the element sides. N.B. There is an unfortunate clash of terminology between this use of 'node' and the usual meaning given to it in vibrational analysis (i.e. points of zero displacement.)

By assuming appropriate 'shape functions' for each element, the problem is reduced to a finite-dimensional eigen-problem. The unknowns in the equations are the eigen-frequencies and the relative displacements at the nodal points which give the mode shapes.

We have thus to solve:

$$[K]\{a\} = \lambda[M]\{a\}$$

where [K] is the stiffness matrix, [M] is the mass matrix and {a} the vector of nodal displacements.

The formation and solution of this equation is left to the computer. The formulation of the finite element method is ideally suited to modern computers as the various matrices may be built up an element at a time, so that essentially the same set of calculations are repeated for each element.

The results in this paper were obtained using a commercial finite element package called ASAS. This was supplied by Atkins Research and Development [6] and mounted on the SERC mainframe computers at Rutherford Appleton Laboratories (RAL) near Oxford. Data preparation and post-processing of results were carried out on the local SERC GEC4090 Multi-User-Mini.

3. Objectives of Analysis

This was a theoretical study. No attempt was made to perform a parallel experimental study and to match dimensions and material properties so as to compare closely the agreement between theory and experiment. Such a comparative study was previously undertaken for the guitar with some success [2]. However, this latter paper does outline some difficulties experienced with the ASAS code for accurate modelling of struts. The aim in the present study was to produce a general overview of the modal characteristics of violin plates modelled to be in various stages of construction. The results can be viewed in a general context and compared with holographic and acoustical experiments already published by other authors.

The plate dimensions were taken from Sacconi [7] and the finite element mesh for the top-plate is shown in Fig.1. This has been refined in several strategic places around the f-holes and at the corners so as not to overtly offend any luthiers! As can be seen, the mesh has been designed so that the positions of the element boundaries enable approximate modelling of the bass-bar and any attachment or boundary conditions at the blocks and at the soundpost.

The solution method utilised by ASAS would not allow the structure to be 'free-free', so modelling of the free plate modes required the restraint of the four beam elements shown in Fig.1. These were optimised to be as slack as possible so as to mimimise the rigid-body mode frequencies. (the starting criteria were the elastic constants for rubber.) With this proviso, investigation showed that they had a negligible effect on the results for the true plate modes. (Essentially they were elastic bands.)

For the purposes of this study, the modelling of a violin top-plate in its stages of construction was split into three parts:-

(a) arched spruce top-plate, without f-holes, without bass-bar

(b) with f-holes, without bass-bar

(c) with f-holes and bass-bar

At each stage the effect of changing various parameters was studied. The plate thickness was varied from 2mm to 4mm and the amount of arching was modelled from a completely flat plate to one with double Sacconi's recommended arching. At stage (c) the bass-bar was first modelled rough (6mm x 11mm), then shaped down ('tuned') to Sacconi's dimensions, then cut down further to a quarter of the recommended height. For the finished plate, clamped and hinged boundary conditions were incorporated.

For the maple back, different thicknesses and arching were modelled, and free, clamped and hinged boundary conditions studied.

The material properties for spruce and maple were taken from Haines [8].

4. Discussion of Results

4.1 Free plate modes

As an example of the finite element output, Fig. 2 shows a view of the 'ring' mode in the finished top-plate. More information may be obtained by studying contour diagrams of the mode shapes.

Figs. 4 and 5 show the mode shapes of the finished violin plates. For comparison, Fig. 3 shows a flat top-plate with neither f-holes nor bass-bar. These diagrams may be compared directly with experimental results obtained using holographic techniques [9] [10].

Table 1 shows the changes in modal frequencies as the top-plate is theoretically manufactured.

There is usually a direct correspondence between modes in the flat and arched plates. The flat top-plate mode which is most affected by the arching is the second one which, by analogy with the 'beam' modes of a square plate, may be designated (0,2). As the arching height is increased, its frequency is raised considerably and its shape transformed to become what is commonly called the ring-mode. The forming of the back-plate ring-mode is similar, but in this case it is the third mode (similar to a (2,0) beam mode) of the flat back-plate that is transformed.

The effect on a finished top-plate of changing only the arching height is given in Table 2, and Table 3 shows the effect of changing only its thickness.

Tables 4 and 5 give the corresponding frequency changes for the maple back-plate.

The most obvious effect of adding the bass-bar is to destroy modal symmetry and to increase the natural frequencies. Indeed, with a rough bass-bar (6mm x 11mm), the ring-mode almost reverts to a (0,2) mode shape. By shaping ('tuning') the bass-bar, frequencies are lowered, much symmetry is regained and the ring-mode becomes rounded once again.

4.2 Hinged Plate Modes

Figs. 6 and 7 show the modes of the finished top and back plates with hinged boundary conditions imposed at the edges and at the blocks. These may again be compared directly with experimental results obtained by other authors [11] [12].

It was found that the best model of the boundary conditions on an arched plate mounted on a completed instrument was obtained by imposing a hinged boundary condition. This was different from the corresponding results for the guitar [2] which was best modelled with a clamped edge.

Comparison of the top and back plate modes shows that the bass-bar tended to increase the vibrational amplitude in the upper-left part of the plate.

The maximum displacements of several of the top-plate modes occurred near the top of one of the f-holes. This was observed experimentally by Jansson et al [11] and suggests that the position of the f-holes and the thickness of the wood in the immediate area may have important effects on the complete instrument's behaviour. (This is, of course, in addition to their effects on the air modes.)

5. Conclusion

The general validity of the finite element method for the analysis of violin plates has been shown by the good agreement between the theoretical results given in this paper and those obtained by holographics and acoustical experiments by other authors [9][10][11][12].

The predictions of a finite element analysis can be invaluable as a guide for the experimental acoustician. The theoretical mode shapes can suggest the best positions for both the vibration drivers and supports, and combination modes may more easily be recognised and removed.

Finite element analysis is currently the only way to study the effect of a controlled change of parameters such as arching.

Currently the size limitations of ASAS as implemented at RAL have been reached, but other packages are under investigation and work is proceeding on a modal analysis of a complete violin body.

6. Acknowledgments

The finite element work in this paper was supported by a research grant from the SERC.

7. References

1. Schwab,H.L. 1974, CAS Newsletter #24 and #25. Finite Element Analysis of a Guitar Soundboard.

2. Richardson,B.E. and Roberts,G.W. 1983, Proceedings of the SMAC '83 Conference (to be published). The Adjustment of Mode Frequencies in Guitars: A Study by Means of Holographic Interferometry and Finite Element Analysis.

3. Charnley,T., Perrin,R., De Pont,J. 1981, Proceedings of the I.O.A. Spring Conference 1981, 309-312. Normal Modes of Church Bells.

4. Watkinson,P.S. and Bowsher,J.M. 1982, J. of Sound and Vibration 85(1),1-7. Vibration Characteristics of Brass Instrument Bells.

5. Zienkiewicz,O.C. 1977, McGraw-Hill. The Finite Element Method, 3rd ed.

6. Atkins Research and Development 1976, . ASAS User Manual.

7. Sacconi,F. , Cremona Library. The Secrets of Stradivari.

8. Haines,D.W. 1979, CAS Newsletter #31, 23-32. On Musical Instrument Wood.

9. Hutchins,C.M. 1981, CAS Newsletter #36, 16-17. A Note on Plate Theory and the Eigenmodes of Free Violin Plates.

10. Hutchins,C.M., Stetson,K.A. and Taylor,P.A. 1971, CAS Newsletter #16. Clarification of "Free Plate Tap Tones" by Hologram Interferometry.

11. Jansson,E., Molin,N.-E. and Sundin,H. 1970, Physica Scripta Vol. 2, 243-256. Resonances of a Violin Body Studied by Hologram Interferometry and Acoustical Methods.

12. Moral,J.A. and Jansson,E.V. 1980, CAS Newsletter #34, 29-32. Eigenmodes and the Function of the Violin.

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Table 1 Frequencies of free arched top-plate at different stages. The modes are ordered as in Fig.4.

Mode	(a)	(b)	(c) rough	(c) shaped
1	88.8	81.1	132.3	94.7
2	154.7	145.8	169.8	153.9
3	259.7	242.7	281.4	255.7
4	269.4	251.4	298.4	271.7
5	363.3	320.5	349.8	327.8
6	388.0	366.2	420.3	389.5
7	467.2	437.4	455.0	447.1
8	465.3	441.0	474.3	450.0
9	608.7	547.2	573.2	559.2
10	598.3	559.9	612.6	576.9

Table 2 Effect of arching on frequencies of free arched top-plate. The modes are ordered as in Fig.4.

Mode	flat	quarter	normal	double
1	76.1	78.8	94.7	110.9
2	116.5	123.2	153.9	184.2
3	191.7	201.4	255.7	301.3
4	214.5	219.2	271.7	335.6
5	137.4	154.0	327.8	482.5
6	262.8	274.1	389.5	579.3
7	347.9	354.5	447.1	485.7
8	390.7	393.9	450.0	573.4
9	427.7	437.1	559.2	602.0
10	457.6	466.1	576.9	712.7

Table 3 Effect of thickness on frequencies of free arched top-plate. The modes are ordered as in Fig.4.

Mode	4mm	3mm	2mm
1	109.9	94.7	80.2
2	184.7	153.9	122.4
3	308.8	255.7	201.3
4	322.5	271.7	218.5
5	352.9	327.8	295.0
6	447.8	389.5	318.2
7	530.6	447.1	359.9
8	548.1	450.0	362.1
9	664.9	559.2	445.2
10	692.5	576.9	469.0

Table 4 Effect of arching on frequencies of free arched back-plate. The modes are ordered as in Fig.5.

Mode	flat	quarter	normal	double
1	86.7	103.7	128.2	159.7
2	93.3	134.2	184.4	214.0
3	248.7	274.2	288.2	357.3
4	236.9	258.8	311.4	390.1
5	161.9	189.2	358.7	515.0
6	354.7	386.6	458.8	573.4
7	483.2	534.6	552.6	673.8
8	445.0	490.1	576.6	746.7
9	575.4	634.3	645.9	867.9

Table 5 Effect of thickness on frequencies of free arched back-plate. The modes are ordered as in Fig.5.

Mode	4mm	Sacconi	3mm	2mm
1	136.0	128.2	111.0	85.3
2	206.3	184.4	163.9	120.0
3	330.6	288.2	270.4	205.8
4	365.6	311.4	295.9	227.4
5	387.4	358.7	340.2	290.4
6	537.5	458.8	438.7	340.6
7	638.7	552.6	519.7	394.9
8	664.4	576.6	542.8	427.1
9	744.6	645.9	629.6	509.0
10	867.8	750.5	701.2	541.1

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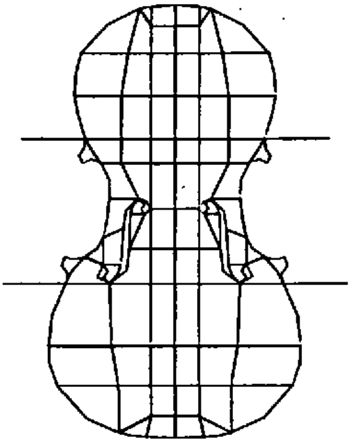


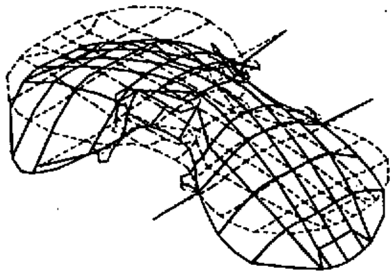


Figure 2 Top-plate ring-mode.

Figure 1 Mesh of top-plate.

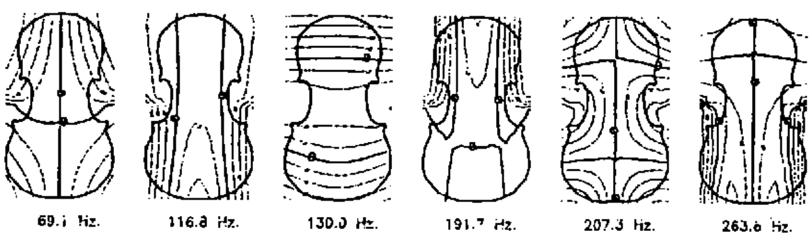


Figure 3 Free, flat top-plate, h = 3mm.

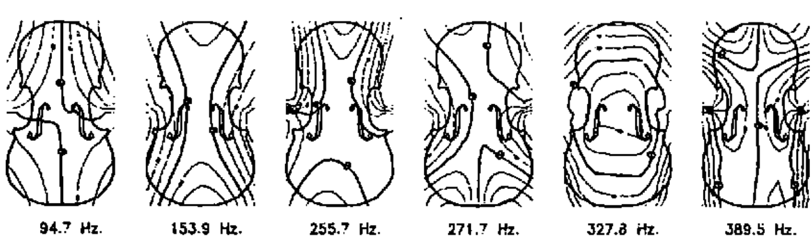
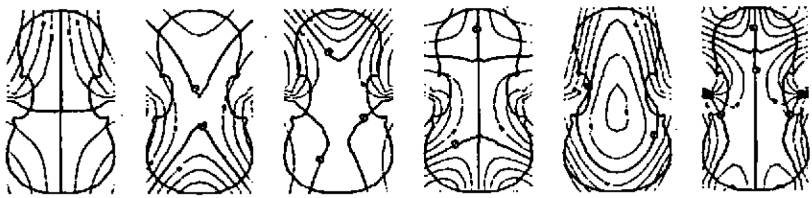
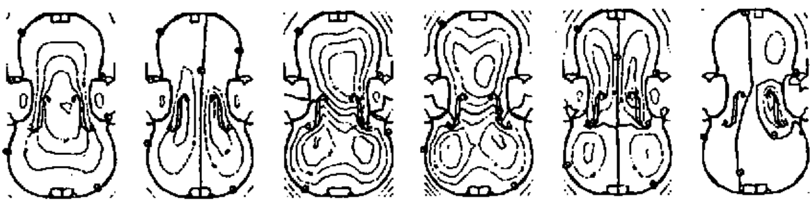


Figure 4 Free, finished arched top-plate, h=3mm.



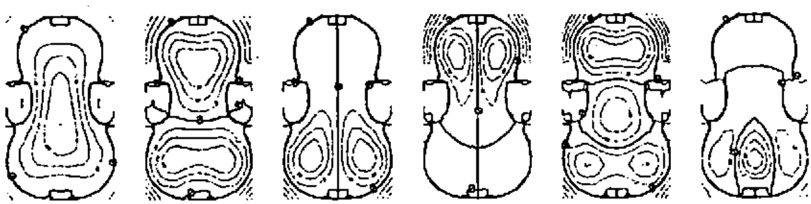
128.2 Hz. 184.4 Hz. 288.2 Hz. 311.4 Hz. 358.7 Hz. 458.8 Hz.

Figure 5 Free, finished arched back-plate, h varying.



332.0 Hz. 541.6 Hz. 614.1 Hz. 625.9 Hz. 790.7 Hz. 925.6 Hz.

Figure 6 Hinged top-plate.



402.3 Hz. 701.8 Hz. 740.8 Hz. 1005.9 Hz. 1068.1 Hz. 1234.8 Hz.

Figure 7 Hinged back-plate.