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A FAST PREDICTION METHOD OF ACOUSTIC RADIATION FROM ELASTIC CYLINDRICAL SHELL IN SHALLOW WATER

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In order to predict acoustic radiation from an elastic structure in shallow water, a method based on wave superposition is proposed, in which the free-space Green function is used to match the strength of equivalent sources. Necessary treatment is conducted in order to neglect the effect of sound reflection from boundaries, which makes the method more efficient. Moreover, the method is combined with the sound propagation algorithms to predict the sound radiated from a elastic cylindrical shell in Pekeris waveguide. The cylindrical shell with hemi-cap is taken as an example to illustrate the validity of the proposed method. Numerical simulations show that the proposed method has robust and efficient in computation.

Keywords: acoustic radiation, fast prediction, shallow water, wave superposition

1. Introduction

As the noise control and dynamic hiding are taken more and more seriously, it is very important to predict acoustic radiation accurately and quickly. Because of the shape complexity of structures, the numerical algorithms, such as finite element method (FEM) and boundary element method (BEM), have become a mainstream of acoustic radiation research methods. After these numerical methods, Koopmann proposed the wave superposition method (WSM) based on simple source substitution [1]. The WSM reduces the difficulty of numerical implementation greatly, and claimed to be more suitable for engineering application [2].

The wave superposition method is also known as the equivalent source method (ESM) [3], source simulation method [4], etc. It has been proven to be an effective method for acoustic radiation, scattering and acoustic holography by several researchers [5-7]. In the method, the sound field is approximated by an array of equivalent sources placed inside a vibrating structure and the strengths of the equivalent sources are determined by matching the boundary conditions on the surface of structure. Therefore, the prediction for structure-borne noise in non-free spaces can be realized by changing the Green function of the equivalent sources.

The difficulty lies in describing the analytical Green function that acts as a vibro-acoustic transfer function between the structure and the surrounding acoustic medium. Machens proposed a method of comparative sources and a critical appraisal for acoustic radiation prediction [8]. Favre proposed an in-situ vibro-acoustic transfer function to approach Green function and it contains the acoustic radiation from the structure and the reflection contributions by boundaries as well [9].

Since the Green function of structure in shallow water is hard to be gotten, there are few public dissertations on structure- borne noise prediction in shallow water, The BEM has been applied to acoustic radiation from a three-dimensional structure submerged in the perfect waveguide [10-11], but BEM needs the three-dimensional derivation of Green function on the surface of structure, which can be hardly calculated except the perfect waveguide with perfectly soft or rigid boundaries. In this paper, the WSM is extended into half-space for avoiding the derivation of Green function in waveguide. Furthermore, the proposed method is combined with the sound propagation algorithms to achieve the prediction of acoustic radiation from a structure in shallow water.

2. Method of wave superposition

The superposition method is based on the idea that the acoustic field of a complex radiator can be constructed as a superposition of fields generated by an array of simple sources enclosed within the radiator [1]. In matrix form:

$$\boldsymbol{P} = \boldsymbol{M}\boldsymbol{D}^{-1}\boldsymbol{U}_{\boldsymbol{n}} \,. \tag{1}$$

The matrix $\boldsymbol{M}$ and $\boldsymbol{D}$ is the calculation of the free-space Green function:

$$\begin{aligned} \boldsymbol{M} &= \mathrm{j}\omega\rho_0 \cdot \boldsymbol{G} \\ \boldsymbol{D} &= \nabla_n \boldsymbol{G} \end{aligned} \,, \tag{2}$$

Where ω is the angular frequency, $\mathrm{j}^2 = -1$ and ρ_0 the mean density of the medium. Eq. (1) can be rewritten as

$$\boldsymbol{p} = \mathrm{j}\omega\rho_0 \cdot \boldsymbol{G} \cdot \left(\nabla_n \boldsymbol{G}\right)^{-1} \boldsymbol{U}_{\boldsymbol{n}} \,. \tag{3}$$

It means that the sound field will be predicted from a structure by vibratory data, which is given as the known variable.

3. Wave superposition with boundary

Since the relationship between acoustic pressure vector and normal velocity is given by Eq. (3), the prediction for structure-borne noise with boundaries can be realized by changing the Green function, as the structure has been equivalent to an array of equivalent sources. When there is a boundary, by considering images of equivalent source, the Green function in WSM becomes the half-space Green function

$$\boldsymbol{g} = \boldsymbol{G} + \eta \cdot \boldsymbol{G}' \,. \tag{4}$$

where η represents the reflection coefficient. The sound pressure vector can be written by

$$\boldsymbol{p} = \mathrm{j}\omega\rho_0 \boldsymbol{g} \left(\nabla_n \boldsymbol{g}\right)^{-1} \boldsymbol{U} \,. \tag{5}$$

As Fig. 1 shows, the sound field in half-space is composed of direct sound and the reflection, which can be considered as the acoustic radiation from an image. The relationship between equivalent sources and the images is similarly given by

$$\begin{cases} \boldsymbol{U}_{\boldsymbol{n}}' = \eta \cdot \boldsymbol{U}_{\boldsymbol{n}} \\ \boldsymbol{Q}' = \eta \cdot \boldsymbol{Q} \end{cases} , \tag{6}$$

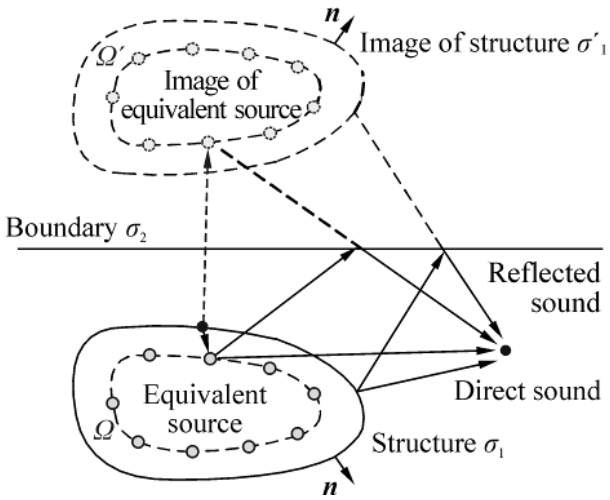


Figure 1: The illustration of acoustic radiation with a boundary.

$\boldsymbol{U}_n'$, $\boldsymbol{Q}'$ are the images of normal velocity and equivalent sources, respectively. Similarly, the normal velocity on surface σ_1 is composed of the acoustic radiation of structure σ_1 and its image σ'_1:

$$\boldsymbol{U} = \boldsymbol{U}_n + \boldsymbol{v}_n', \tag{7}$$

Both velocities in Eq. (7) can be matched by the equivalent sources in the wave superposition solution:

$$\begin{cases} \boldsymbol{U}_n = (\nabla_n \boldsymbol{G})\boldsymbol{Q} \\ \boldsymbol{v}_n' = (\nabla_n \boldsymbol{G}')\boldsymbol{Q}' \end{cases}, \tag{8}$$

Obviously, the strength of the equivalent sources can be calculated by either expression of Eqs.(8). The source vector $\boldsymbol{Q}$ is given by $(\nabla_n \boldsymbol{G})^{-1}\boldsymbol{U}_n$, thus Eq. (5) becomes

$$\boldsymbol{P} = \mathrm{j}\omega\rho_0 \boldsymbol{g} (\nabla_n \boldsymbol{G})^{-1} \boldsymbol{U}_n . \tag{9}$$

The advantage of Eq. (9) is that the relation between equivalent sources and normal velocity is the derivative of free-space Green function instead of half-space Green function, which makes more simplifications, since the three-dimensional derivation of Green function in shallow water is hard to be gotten.

To predict acoustic radiation by Eq. (9), the normal velocity of structure needs a treatment, as the velocity on the surface of the structure is not $\boldsymbol{U}_n$, but $\boldsymbol{U}$. Suppose the maximum linear dimension of a structure is D and the distance between structure (geometrical center) and boundary is L. Evidently, the mean distance between structure surface σ_1 and equivalent sources $\boldsymbol{Q}$ is less than $D/2$, and in contrast, the mean distance between images of equivalent sources $\boldsymbol{Q}'$ and σ_1 is more than $2L$. Since the structure has been equivalent to an array of monopoles by WSM, and the monopole's attenuation meets the rule of spherical wave: $1/r$ which applies to all wave numbers (where r is the sound propagation distance), it is not difficult to conclude that $\boldsymbol{v}_n'$ is one order of magnitude less than $\boldsymbol{U}_n$ when it satisfies the following inequality

$$L > D. \tag{10}$$

By doing so, Eq. (9) is further simplified to

$$\boldsymbol{P} = \mathrm{j}\omega\rho_0 \boldsymbol{g}\left(\nabla_n \boldsymbol{G}\right)^{-1} \boldsymbol{U} . \tag{11}$$

Eq. (11) shows that the boundary has a strong effect on sound propagation, but weak effect on the strength of equivalent sources, if Eq. (10) is satisfied.

4. The Green function in waveguide

The proposed WSM can predict sound radiation in shallow water by changing the g of Eq. (11) into the Green function of shallow water. Since the Green function of the equivalent source in WSM is given by the sound propagation algorithms, both the normal-mode and ray methods are used.

4.1 Method of normal-mode

Consider the simple case that has been first studied by Pekeris, plane $z=0$ denotes the surface of waveguide in cylindrical coordinate system, and the point source (monopole) is located on z-axis, with the ordinate value z_0. The depth of the waveguide is H and its bottom is considered as another kind of fluid medium, so there is no shear wave. The density of water and bottom medium are ρ_1 and ρ_2, and the sound speeds are c_1 and c_2, respectively. The sound pressure in Pekeris waveguide has been deduced:

$$p(r,z) = \mathrm{j}\sum_{n=1}^{N}\sqrt{\frac{2\pi}{\zeta_n r}}A_n^2 \,\boldsymbol{sin}(k_{zn}z_0)\,\boldsymbol{sin}(k_{zn}z)\times \boldsymbol{exp}\left[\mathrm{j}\left(\zeta_n r-\frac{\pi}{4}\right)\right] \quad 0\le z\le H . \tag{12}$$

Where $k_{zn}^2 = \left(\omega/c_1\right)^2 - \zeta_n^2$, ζ_n is the eigenvalue of each normal mode and A_n^2 is given by

$$A_n^2 = \frac{2k_{zn}}{k_{zn}H - \boldsymbol{cos}(k_{zn}H)\,\boldsymbol{sin}(k_{zn}H) - \left(\dfrac{\rho_1}{\rho_2}\right)^2 \boldsymbol{sin}^2(k_{zn}\mathrm{H})\,\boldsymbol{tan}(k_{zn}H)} . \tag{13}$$

After the calculation of sound pressure in waveguide, the Green function of WSM can be written as Eq. (14). Then the sound radiation of structure in shallow water is a straightforward calculation from Eq. (11).

$$g = p(r,z)/4\pi . \tag{14}$$

4.2 Solution by ray method

As the environment assumed is the standard Pekeris waveguide, the total field at a receiver can also be expressed as a sum of ray fields due to the source and its images. The field is constructed with the rays from an infinite number of image sources. Since the top of Pekeris waveguide is a soft boundary, the top reflection coefficient is −1. The pressure field becomes

$$\begin{aligned} p(r,z) = \sum_{n=0}^{\infty} & \left(-V_{n1}\right)^n \frac{\boldsymbol{exp}\left(\mathrm{j}kR_{n1}\right)}{R_{n1}} - \left(-V_{n2}\right)^{n+1} \frac{\boldsymbol{exp}\left(\mathrm{j}kR_{n2}\right)}{R_{n2}} - \\ & \left(-V_{n3}\right)^{n+1} \frac{\boldsymbol{exp}\left(\mathrm{j}kR_{n3}\right)}{R_{n3}} + \left(-V_{n4}\right)^{n+1} \frac{\boldsymbol{exp}\left(\mathrm{j}kR_{n4}\right)}{R_{n4}} \end{aligned} . \tag{15}$$

Where R_{ni} is the distance between images and field points:

$$R_{ni} = \sqrt{r^2 + z_{ni}^2}, \qquad n = 0,1,2...,\infty \qquad i = 1,2,3,4 . \tag{16}$$

The geometry is specified by the horizontal range r and the source depth z_{ni}, which is given by Eq. (17). z_0, z are the depth of source and field respectively.

$$\begin{cases} z_{n1} = 2Hn + z_0 - z \\ z_{n2} = 2H(n+1) - z_0 - z \\ z_{n3} = 2Hn + z_0 + z \\ z_{n4} = 2H(n+1) - z_0 + z \end{cases} . \tag{17}$$

The reflection coefficient of the bottom is calculated by

$$V_{ni}(\theta_{ni}) = \frac{(\rho_2/\rho_1)\sin\theta_{ni} - \sqrt{(c_1/c_2)^2 - \cos^2\theta_{ni}}}{(\rho_2/\rho_1)\sin\theta_{ni} + \sqrt{(c_1/c_2)^2 - \cos^2\theta_{ni}}} . \tag{18}$$

Where θ_{ni} is the grazing angle of images. It should be noted that Eq. (18) is the plane wave reflection coefficient that is used to approximate the spherical wave when $ka > 1$ [12], where a is the distance of structure away from the bottom. It is more appropriate to formulate reflection coefficient by changing the spherical sound radiation into integrals of plane wave expansion [13].

5. Sound radiation prediction in shallow water

As shown in Fig. 2, the model of shallow water is analyzed. Since the influence of reflected sound on the structural vibration is small when the distance between the structure and the boundaries exceeds the maximum linear dimension of structure, the surface velocity of structure is calculated by sound radiation of a monopole in free-field by using the free-space Green function.

The elastic cylindrical shell with hemi-cap is taken as an example to illustrate the validity of the proposed method. The radius of the elastic structure is 0.06m and the total length is 0.48m. Its thickness is 0.002m. The density of the elastic material is 7800kg/m^3. Yang's modulus is 2.1×1011N/m2 and the poisson ratio is 0.3. As a reference, FEM was used to calculate the normal velocity on the surface of structure and field pressure in elements of water. The total element number on the surface of the cylindrical shell is 602, and there is a 1N excitation on force at the end of the hemi-cap.

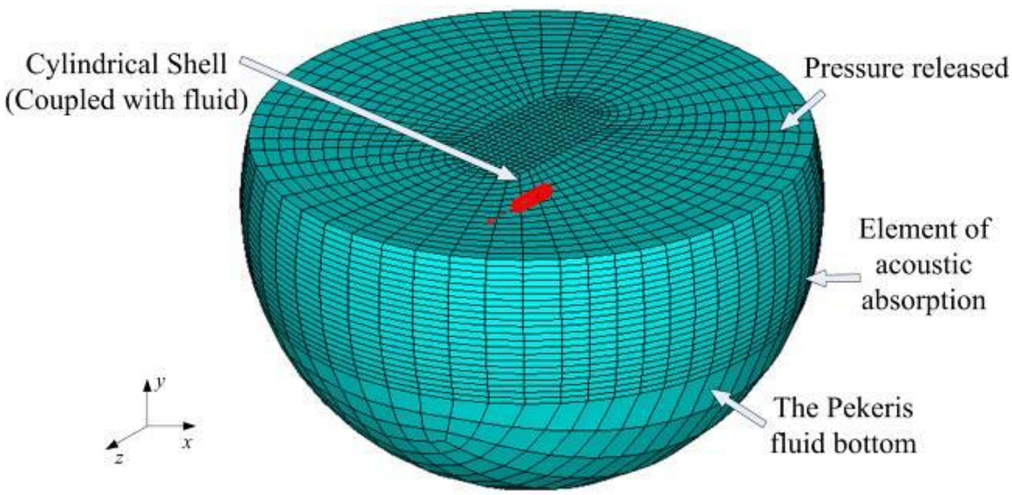
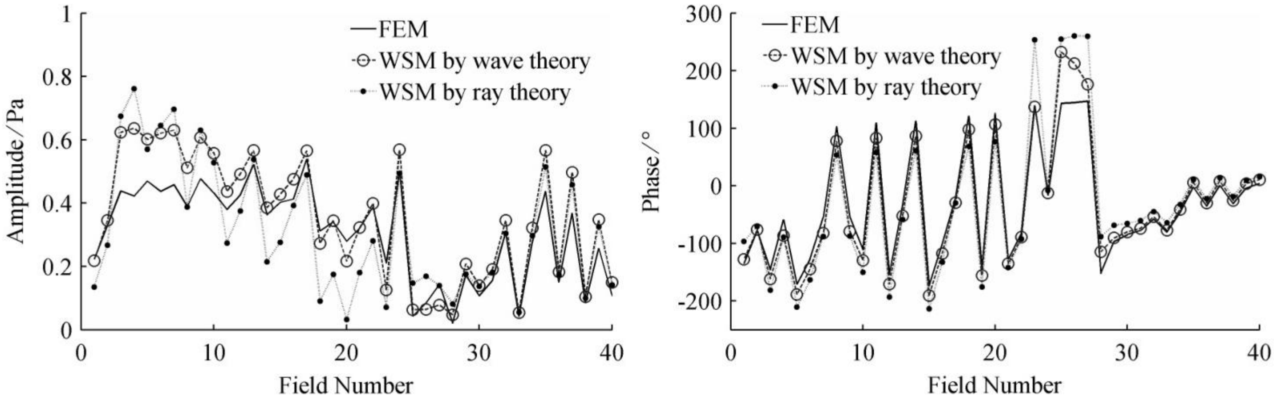


Figure 2: Diagram of sound prediction in Pekeris waveguide.

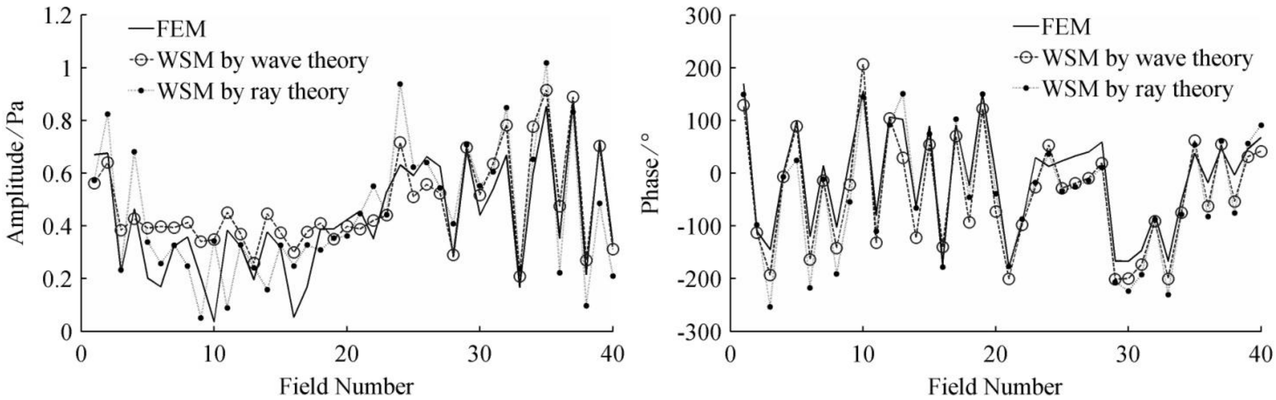
The surface of Pekeris waveguide is soft and the bottom is considered as another kind of fluid medium with the following parameters: ρ_2=2000 kg/m3, c_2=2000 m/s2, different from the parameters of water: ρ_1=1000 kg/m3, c_1=1500 m/s2, as shown in Fig. 2. The depth of the structure is 1 m, which meets Eq. (10). Hence, Eq. (11) can be used to predict acoustic radiation of the elastic cylindrical shell in shallow water.

The following numerical experiments were calculated by both wave theory and ray theory. In Fig. 3 and Fig. 4, the “WSM by wave theory” and the “WSM by ray theory” curves correspond to the radiation of the monopole calculated by both wave theory and ray theory. The results showed that most WSM are approximate to the FEM solution, which means that the sound prediction on the structure in shallow water is realized as the structure has been equivalent to an array of equivalent sources.



(a) The comparison of amplitude (b) The comparison of phase

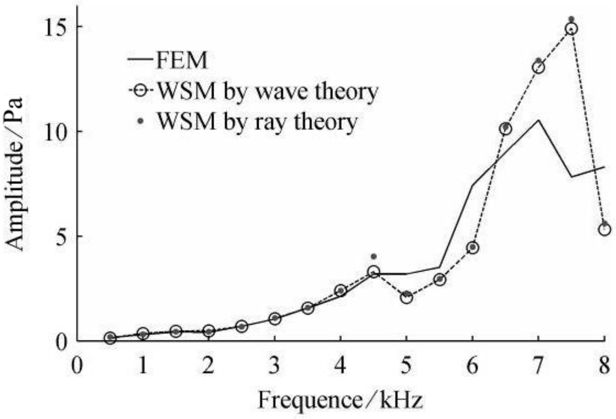
Figure 3: Acoustic radiation prediction in shallow water (1kHz).



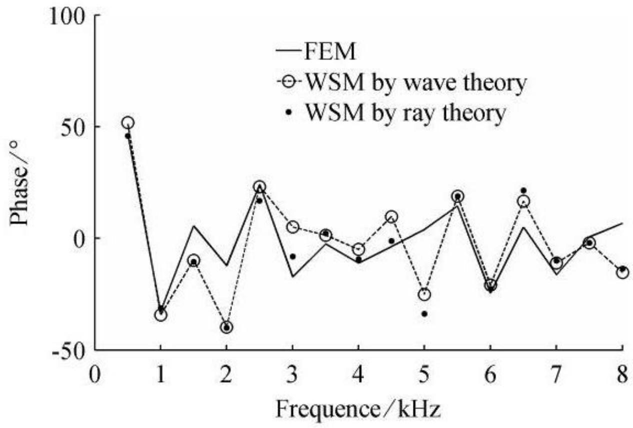
(a) The comparison of amplitude (b) The comparison of phase

Figure 4: Acoustic radiation prediction in shallow water (2kHz).

By taking the sound radiation of the FEM as the standard value, the acoustic radiation of the calculated by wave theory and ray theory is compared. Fig. 5 shows the spectra of acoustic radiation pressure within the range of 1 kHz–8 kHz. It is concluded that the WSM by normal mode method and the ray method can precisely predict the acoustic radiation in the shallow water. Practically, since the element is not enough, the FEM get more error than the WSM. The proposed method has more suitability for the prediction of acoustic radiation in the higher frequency band. Appropriate propagation algorithm should be chosen for different cases, and additional studies are needed to provide more working guidelines.



(a) The comparison of amplitude



(b) The comparison of phase

Figure 5: Spectra of acoustic radiation in Pekeris waveguide.

6. Conclusions

The analysis of the elastic cylindrical shell shows that the proposed method using the free-space Green function to match the strength of equivalent sources gains robustness and computational efficiency, and can be used to predict acoustic radiation in shallow water by combining with propagation algorithms of monopoles. The effect of sound reflections on the proposed method can be neglected when the distance between the structure and the boundaries exceeds the maximum linear dimension of the structure. The reliability of acoustic radiation prediction in shallow water mainly depends on the sound propagation algorithms. Appropriate propagation algorithm should be chosen for specific cases, and additional studies are needed to provide working guidelines for cases where more complicated waveguides are to be treated.

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