WAVE ABSORPTION OF A FLEXIBLE BEAM FOR BAND-LIMITED SPECTRA USING DIRECT FEEDBACK

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This study presents direct-feedback-based approach for the wave absorption of a flexible beam. The purpose of this study is to expand the conventional method which specifies the control at a single frequency to the method for band-limited spectra. First, wave dynamics of a flexible beam are described by a transfer matrix method. This is followed by the derivation of an optimal wave control law. Next, parameter tuning technique of a direct feedback system is presented in order to minimize reflected waves for band-limited spectra. Finally, some numerical simulations are carried out, demonstrating the validity of the proposed method.

Keywords: active wave control, flexible beam, direct feedback

1. Introduction

Due to the demands on energy efficiency of engineering systems, recent mechanical systems are constructed with light and thin components. Consequently, such systems tend to be flexible, and hence prone to vibration. Many attempts of active vibration suppression have been presented, and those are divided into two groups; modal-based control and wave-based control. The former is a widely used technique since it is easily possible to introduce many concepts proposed in control engineering field such as H-infinity control and so on [1]. However, if the target system is what is called modally-rich structure, some difficulties such as complexity of the control system may arise. On the other hand, wave-based control which is based on the mechanism of resonant phenomena can suppress the structural vibration with simple control architecture.

Reviewing literature on feedback wave control, most of the control methods have been proposed for one-dimensional structures such as a flexible beam. For example, von Flotow and Schafer proposed some types of wave absorbing controllers using half-differentiator approximately realized by electrical circuits [2]. Tanaka and Kikushima presented the perfect wave absorption at a designated single frequency using direct feedback [3]. Mei et al. proposed the hybrid control using modal and wave concepts, and direct feedback control and the approximation using a finite impulse response (FIR) filter are presented for wave controller [4]. The latter approach can reduce progressive waves in relatively wide frequency range; however, the relationship between the approximation accuracy and the stability is not clear.

This study proposes an alternative approach which realizes the sufficient wave absorption for band limited spectra using direct feedback. First, wave dynamics of a flexible beam are described by a transfer matrix method. This is followed by the derivation of an optimal wave control law. Next, parameter tuning technique of a direct feedback system is presented in order to minimize reflected waves for band-limited spectra. Finally, some numerical simulations are carried out, demonstrating the validity of the proposed method.

2. Theoretical development of a transfer matrix method

In order to understand and treat wave dynamics of a flexible beam, a transfer matrix method is introduced in this paper. Assuming that shear deformation and rotary inertia are negligible, governing equation of a flexible beam is described as

$$EI\frac{\partial^4 \xi(x,t)}{\partial x^4} + \rho A\frac{\partial^2 \xi(x,t)}{\partial t^2} = f(x,t)\,, \tag{1}$$

where E, I, A, ρ and $f(x,t)$ are Young's modulus, area moment of inertia, cross-sectional area, mass density and applied force per unit length, respectively. If the beam vibration is harmonic, the spatial component of the displacement is described in the region where no external force is applied as follows:

$$\frac{d^4 \xi(x)}{\partial x^4} - k^4 \xi(x) = 0\,, \tag{2}$$

where k is a wave number and defined as

$$k = \left(\frac{\rho A}{EI}\right)^{\frac{1}{4}} \omega^{\frac{1}{2}}\,. \tag{3}$$

A general solution of Eq. (00) is derived as

$$-\xi(x) = c_1 \mathrm{e}^{-\mathrm{j}kx} + c_2 \mathrm{e}^{-kx} + c_3 \mathrm{e}^{\mathrm{j}kx} + c_4 \mathrm{e}^{kx}\,, \tag{4}$$

where c_1, c_2, c_3 and c_4 are the positive traveling wave amplitude, the near-field amplitude decaying from the left boundary, the negative traveling wave amplitude and the near-field amplitude decaying from the right boundary, respectively. Differentiating the bending displacement, the slope θ, the internal bending moment M and internal shear force Q are obtained, and then the state vector z(x,s) is defined as

$$\mathbf{z}(x) = \left(-\bar{\xi}(x) \quad \bar{\theta}(x) \quad \bar{M}(x)/EI \quad \bar{Q}(x)/EI\right)^{\mathrm{T}}\,, \tag{5}$$

where T denotes the transpose of the expression.

Next, consider the beam element whose length is r as shown in Fig.1. Defining the left and right end of the element as node i-1 and i, respectively, the relation between two state vectors at these nodes is described as

$$\mathbf{z}_i = \mathbf{T}_{i,i-1}(r)\mathbf{z}_{i-1}\,, \tag{6}$$

where T is the transfer matrix between the two nodes, and is defined as

$$\mathbf{T}_{i,i-1}(l) = \mathbf{K}\mathbf{D}(l)\mathbf{K}^{-1}\,, \tag{7}$$

where

$$\mathbf{K} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ -\mathrm{j}k & -k & \mathrm{j}k & k \\ -k^2 & k^2 & -k^2 & k^2 \\ \mathrm{j}k^3 & -k^3 & -\mathrm{j}k^3 & k^3 \end{pmatrix}, \tag{8}$$

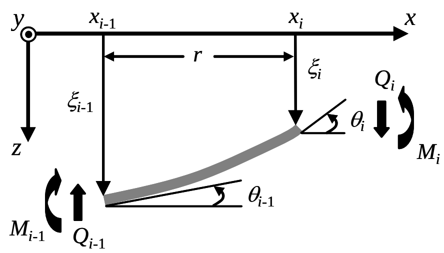

Figure 1: Beam element and its state variables.

$$\mathbf{D}(l) = \begin{pmatrix} e^{-jkr} & 0 & 0 & 0 \\ 0 & e^{-kr} & 0 & 0 \\ 0 & 0 & e^{jkr} & 0 \\ 0 & 0 & 0 & e^{kr} \end{pmatrix}. \tag{9}$$

Assuming that a disturbance force and a control force are applied to a flexible beam as shown in Fig. 2, the sate equation is described as

$$\mathbf{z}_3 = \mathbf{T}_{30}\mathbf{z}_0 + \mathbf{T}_{31}\mathbf{f}_c + \mathbf{T}_{32}\mathbf{f}_d \,, \tag{10}$$

where

$$\mathbf{f}_c = \begin{pmatrix} 0 & 0 & 0 & f_c / EI \end{pmatrix}^{\mathrm{T}}, \tag{11}$$

$$\mathbf{f}_d = \begin{pmatrix} 0 & 0 & 0 & f_d / EI \end{pmatrix}^{\mathrm{T}}. \tag{12}$$

Here, the arbitrary boundary conditions at right end of the beam are determined by setting two out of four state variables in the state vector to be zero. Supposing that the *i*th and *j*th state variables at right end are zero, Eq. (12) expands to the following equations;

$$0 = {}_{30}t_{im} z_{0,m} + {}_{30}t_{in} z_{0,n} + {}_{31}t_{i4} f_c / EI + {}_{32}t_{i4} f_d / EI \,, \tag{13}$$

$$0 = {}_{30}t_{jm} z_{0,m} + {}_{30}t_{jn} z_{0,n} + {}_{31}t_{j4} f_c / EI + {}_{32}t_{j4} f_d / EI \,, \tag{14}$$

where ${}_{ij}t_{kl}$ denotes the *k*th row and *l*th column variable in the transfer matrix $\mathbf{T}_{ij}$. Next, from Eqs. (17) and (18), the non-zero variables at the left end (node 0) are given by

$$z_{0,m} = -\left(\alpha_{11} f_c + \alpha_{12} f_d\right) / EI\Delta \,, \tag{15}$$

$$z_{0,n} = -\left(\alpha_{21} f_c + \alpha_{22} f_d\right) / EI\Delta \,, \tag{16}$$

where

$$\alpha_{11} = {}_{30}t_{in\,31}t_{j4} - {}_{30}t_{jn\,31}t_{i4} \,, \tag{17}$$

$$\alpha_{12} = {}_{30}t_{in\,32}t_{j4} - {}_{30}t_{jn\,32}t_{i4} \,, \tag{18}$$

$$\alpha_{21} = {}_{30}t_{jm\,31}t_{i4} - {}_{30}t_{im\,31}t_{j4} \,, \tag{19}$$

$$\alpha_{22} = {}_{30}t_{jm\,32}t_{i4} - {}_{30}t_{im\,32}t_{j4} \,, \tag{20}$$

$$\Delta = {}_{30}t_{im\,30}t_{jn} - {}_{30}t_{in\,30}t_{jm} \,. \tag{21}$$

Since the non-zero values of the initial state vector are obtained as written in Eqs. (13) and (14), a state vector at an arbitrary point can be calculated.

3. Derivation of an ideal wave control law

Wave absorption of a flexible beam shown in Fig. 2 indicates the suppression of the bending waves that positively propagate in element 2. For this purpose, feedback approach is employed in this study. Defining the sensor point as node *a*, the displacement sensor output is described as

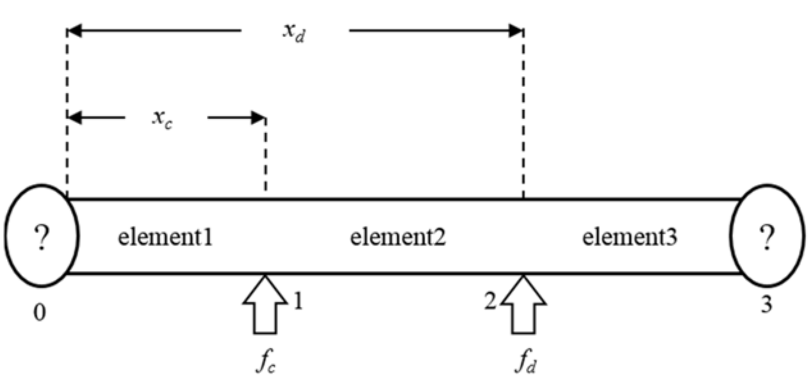

Figure 2: Target beam with disturbance and control forces.

$$\begin{aligned}\xi_a &= \begin{pmatrix}1 & 0 & 0 & 0\end{pmatrix}\mathbf{z}_a \\ &= {}_{a0}t_{1m}z_{0,m} + {}_{a0}t_{1n}z_{0,n}\end{aligned}. \tag{22}$$

The feedback control force is then described as

$$\begin{aligned}f_c &= EIG_w\xi_a \\ &= EIG_w\left({}_{a0}t_{1m}z_{0,m} + {}_{a0}t_{1n}z_{0,n}\right)\end{aligned}, \tag{23}$$

where G is an ideal wave control law. Substituting the above equation into Eqs. (15) and (16), The non-zero values of the initial state vector is rewritten in the vector form as follows:

$$\begin{pmatrix} z_{0,m} \\ z_{0,n} \end{pmatrix} = -\mathbf{A}^{-1}\mathbf{b}f_d / EI \,, \tag{24}$$

where

$$\mathbf{A} = \begin{pmatrix} \Delta + \alpha_{11}G_{wa0}t_{1m} & \alpha_{11}G_{wa0}t_{1n} \\ \alpha_{21}G_{wa0}t_{1m} & \Delta + \alpha_{21}G_{wa0}t_{1n} \end{pmatrix}, \tag{25}$$
$$\mathbf{b} = (\alpha_{12} \quad \alpha_{22})^T .$$

Next, an ideal control law for the wave absorption is derived. The positively-traveling wave in element 2 at node 1 is defined as

$$\begin{aligned}w_{1pp} &= (1 \quad 0 \quad 0 \quad 0)\mathbf{K}^{-1}\mathbf{z}_1 \\ &= (1 \quad 0 \quad 0 \quad 0)\mathbf{K}^{-1}\left(\mathbf{T}_{10}\mathbf{z}_0 + \mathbf{f}_c\right)\end{aligned}. \tag{26}$$

Furthermore, the state vector at node 1 is expanded to

$$\mathbf{T}_{10}\mathbf{z}_0 + \mathbf{f}_c = \begin{pmatrix} {}_{10}t_{1m} & {}_{1L}t_{1n} \\ {}_{10}t_{2m} & {}_{1L}t_{2n} \\ {}_{10}t_{3m} & {}_{1L}t_{3n} \\ {}_{10}t_{4m} + G_{wa0}t_{1m} & {}_{10}t_{4n} + G_{wa0}t_{1n} \end{pmatrix}\begin{pmatrix} z_{0,m} \\ z_{0,n} \end{pmatrix}. \tag{27}$$

Substituting Eqs. (24) and (27) into Eq. (26), and putting the resultant equation to zero, an ideal control law for the wave absorption is derived as

$$G_w = \frac{(\alpha_{12}\gamma_1 + \alpha_{22}\gamma_2)\Delta}{(\gamma_{1a0}t_{1n} - \gamma_{2a0}t_{1m})(\alpha_{11}\alpha_{22} - \alpha_{12}\alpha_{21}) + \mathrm{j}\Delta(\alpha_{12a0}t_{1m} + \alpha_{22a0}t_{1n})}, \tag{28}$$

where

$$\gamma_1 = k^3\,{}_{10}t_{1m} + \mathrm{j}k^2\,{}_{10}t_{2m} - k\,{}_{10}t_{3m} - \mathrm{j}_{10}t_{4m}\,, \tag{29}$$

$$\gamma_2 = k^3\,{}_{10}t_{1n} + \mathrm{j}k^2\,{}_{10}t_{2n} - k\,{}_{10}t_{3n} - \mathrm{j}_{10}t_{4n}\,. \tag{30}$$

4. Approximated realization of wave absorption for band-limited spectra

The ideal wave control law derived in the previous section is non-causal, and hence its approximated realization is required in practice. This study employs direct feedback approach which is based on collocation of sensors and actuators. Since the ideal control law, G_w, is described as a complex function of ω, it is written as

$$G_w = \mathrm{Re}\left[G_w(\omega)\right] + \mathrm{j}\omega\,\mathrm{Im}\left[G_w(\omega)/\omega\right] = G_d(\omega) + \mathrm{j}\omega G_v(\omega). \tag{31}$$

As $\mathrm{j}\omega$ is a differential operator in the frequency domain, G_v indicates velocity feedback gain while G_d is displacement feedback gain. In the conventional method, the perfect wave absorption is realized at a designated single frequency. This study proposes an alternative approach for the wave absorption for band limited spectra. For this purpose, the mean square of the target wave in the frequency band between ω_1 and ω_2 is introduced as the cost function, that is,

$$J = \frac{1}{\omega_2 - \omega_1} \int_{\omega_1}^{\omega_2} w_{1pp}^2 d\omega \,. \tag{32}$$

By numerically finding the velocity and displacement feedback gains that minimizes the above cost function, active wave control system that realize the wave absorption for designated frequency band can be constructed.

5. Numerical simulations

Some numerical simulations are carried out in this section. The specification of the target structure is as follows: length is 1 m, width is 4 cm, thickness is 3 mm and the material is stainless steel. The boundary condition at both ends is pinned support. A disturbance force is applied at the distance of 0.1 m from the right end, and a control force is applied at the symmetrical position. The target frequency band is from 10 Hz to 100 Hz.

Shown in Fig. 3 is the value of the cost function versus displacement and velocity feedback gains. As shown in the figure, the cost function becomes the minimum value when G_d is 1283 and G_v is -6.8. Figure 4 shows the driving point compliance with and without control. When no control is ap-

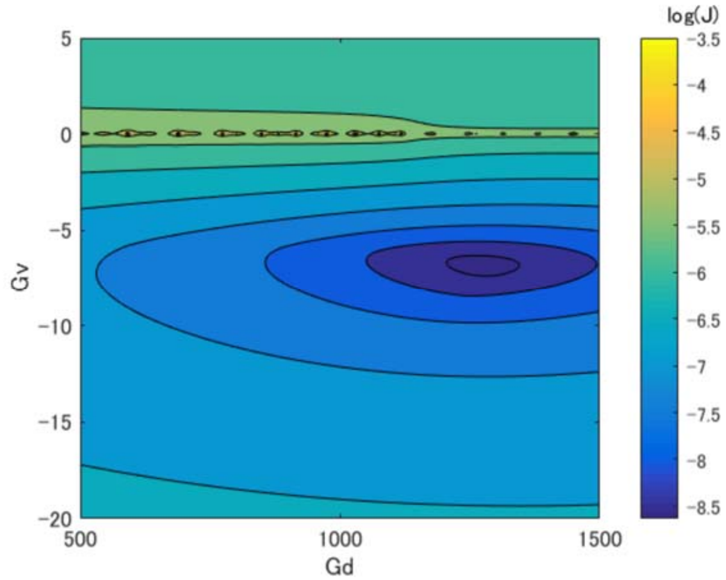

Figure 3: Contour map of the cost function versus displacement and velocity feedback gains.

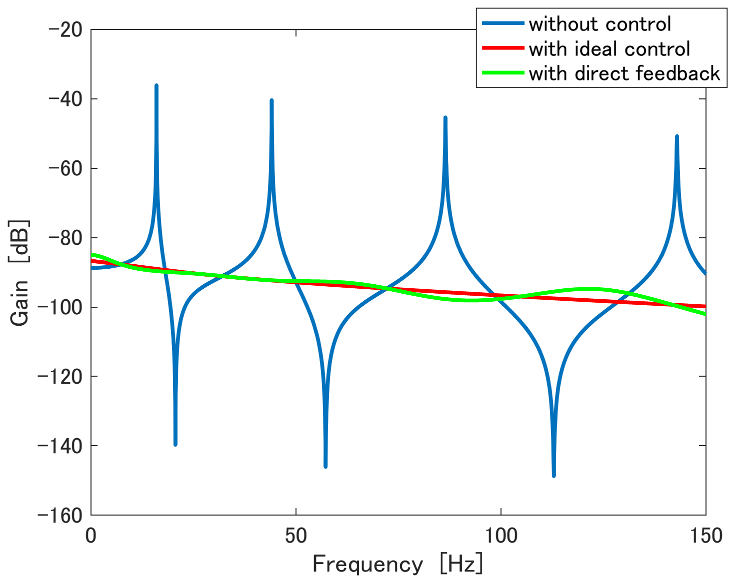

Figure 3: Driving point compliance with and without control.

plied to the beam, there are four resonant peaks in the frequency range up to 150 Hz. In contrast, if an ideal control that realizes the perfect wave absorption is applied to the beam, all peaks and notches disappear, and the gain curve converges to the asymptotic line. This result indicates that all vibration modes are made inactive by the ideal control. On the other hand, when the direct feedback that is optimized in the frequency range between 10 Hz and 100 Hz is applied to the beam, the gain curve almost converges to the asymptotic line, and hence the control effect of the porposed method is close to that of the ideal control. Especially at the first and second modal frequencies, the difference between the ideal control and proposed method is minute. This is because the levels of those modes are higher than those of the third and fourth modes when no control is applied.

6. Conclusions

This study has presented direct-feedback-based approach for the wave absorption of a flexible beam for band-limited spectra. First, wave dynamics of a flexible beam were described by a transfer matrix method. This was followed by the derivation of an optimal wave control law. Next, the cost function which is a mean square of the target wave in the designated frequency band is introduced. The velocity and displacement feedback gains were numerically found that minimizes the cost function. Finally, some numerical simulations on the feedback wave control were carried out, demonstrating that sufficient wave absorption was succeeded by the proposed method.

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