

# USING EXPERIMENTAL STATISTICAL ENERGY ANALYSIS TO DETERMINE LOSS FACTORS FOR COUPLED SPACE SUBSYSTEMS IN A CAR CABIN

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In automotive design, SEA is used as a prediction tool in the mid- and high-frequency ranges. To assess noise levels at driver and passenger positions it is potentially useful to subdivide the car cabin into several subsystems. However, these subsystems can be strongly coupled. To assess the viability of this approach the response in a car cabin is determined with FEM and ray tracing using a point source. SEA models for the coupled subsystems are determined using Experimental Statistical Energy Analysis (ESEA) in the form of General ESEA (GESEA) which includes indirect coupling between physically disconnected subsystems and Alternative ESEA (AESEA) which considers only direct coupling between adjacent subsystems. Four different SEA models are considered which comprise three, five and nine subsystems. The results show that with increasing frequency, SEA models using coupling loss factors determined from GESEA and AESEA are in close agreement with FEM and ray tracing. This indicates that indirect coupling is not significant. In general, results based on FEM in the mid-frequency range show that the three-subsystem model (front seat, back seat and boot) gives consistently low errors in each subsystem. However, in the high-frequency range the results based on ray tracing with diffuse reflections indicate that subdivision of the car cabin into three, five or nine subsystems are feasible.

Keywords: Statistical Energy Analysis (SEA), coupled volumes, car cabin

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## 1. Introduction

Prediction models for the mid- and high-frequency ranges in automotive vehicle cabins are important to assess acoustical comfort for driver and passengers [e.g. see 1]. For this purpose, Statistical Energy Analysis (SEA) is a powerful tool to predict the sound and vibration response [2]. However when predicting the interior acoustics, consideration needs to be taken of the different sound pressure level (SPL) at the head position compared to other positions in the cabin. Fahy [3] noted that the issue of subdividing the car cabin into separate coupled volumes representing SEA subsystems had been criticised and considered whether it might still be appropriate for a medium size saloon car above 200Hz. Fahy concluded that it could potentially be justifiable but this could only be proven through experimental work. Experimental work providing some evidence can be found in Musser *et al* [4] who predicted SPLs in a car cavity using a noise source near the side window and windshield. These measurements showed variations up to 18dB between different points in the cabin. This appears to have used over ten space subsystems for the cabin and boot in one-third octave bands above 500Hz and assumed a transmission coefficient of unity between the coupled volumes. The choice of subsystems was primarily chosen based on experience and practicality. For the automotive industry, Gagliardini *et al* [5,6] have used Frequency Response Functions as the basis to carry out Experimental SEA (ESEA) on car bodies. This work primarily focussed on an automatic approach to identify subsystems although the examples were primarily for structural subsystems.

For structural vibration of coupled plates, ESEA has previously been used with the output of Finite Element Methods (FEM) to extend the application of SEA models down to low-frequencies where mode counts and modal overlap are relatively low [7,8,9]. When subdividing a complex space such as a car cabin where the surfaces have widely varying absorption coefficients, ESEA can be used to give Coupling Loss Factors (CLFs) between subsystems as well as Internal Loss Factors (ILFs) for the subsystems. This avoids the need to estimate modal density for a subsystem representing a coupled volume.

This paper assesses the potential to use numerical models with ESEA as a basis on which to model the car cabin volume as several coupled subsystems in order to build an SEA model. The sound field in a cabin is predicted using FEM and ray tracing which provides the input data for ESEA to determine the CLFs and ILFs.

## 2. Prediction model and numerical experiments

### 2.1 Car cabin model

To provide a realistic cabin, the interior of a Porsche Cayenne (2009) is considered after removal of trivial protrusions to simplify the boundaries of the compartment as shown in Fig. 1 [10].

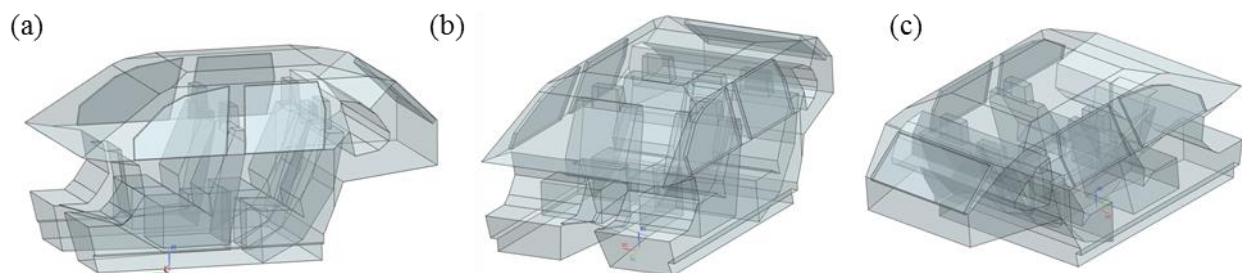


Figure 1: Car cabin model (a) side view of the car cabin, (b) front view of the car cabin, and (c) rear view of the car cabin

The car cabin model can be considered as being composed of three spaces: front seat volume, rear seat volume and boot volume. For practical purposes, further subdivision is required for the upper/lower and left/right sides at the front and rear seat because significant variations in level can exist as indicated by Musser [4]. The boot volume is not considered for further subdivision as there are no occupants.

In this study, four different SEA models are considered with the following division of the cabin into subsystems: (1) three subsystems ('front seat', 'rear seat' and 'boot'); (2) five subsystems with horizontal subdivision ('front upper', 'front lower', 'rear upper', 'rear lower' and 'boot'); (3) five subsystems with vertical subdivision ('front left', 'front right', 'rear left', 'rear right' and 'boot'); and (4) nine subsystems ('front left upper', 'front left lower', 'front right upper', 'front right lower', 'rear left upper', 'rear left lower', 'rear right upper', 'rear right lower' and 'boot').

### 2.2 Determination of parameters

Two forms of ESEA are described in the following sections, General ESEA (GESEA) and Alternative ESEA (AESEA).

### 2.2.1 General ESEA (GESEA)

GESEA includes indirect coupling loss factors between non-adjacent subsystems for which the matrix formulation is given by [11]

$$\begin{bmatrix} \sum_{n=1}^N \eta_{1n} & -\eta_{21} & -\eta_{31} & \cdots & -\eta_{N1} \\ -\eta_{12} & \sum_{n=1}^N \eta_{2n} & -\eta_{32} & & \vdots \\ -\eta_{13} & -\eta_{23} & \sum_{n=1}^N \eta_{3n} & & \vdots \\ \vdots & & & \ddots & \\ -\eta_{1N} & & & & \sum_{n=1}^N \eta_{Nn} \end{bmatrix} \begin{bmatrix} E_{11} & E_{12} & E_{13} & \cdots & E_{1N} \\ E_{21} & E_{22} & E_{23} & & \vdots \\ E_{31} & E_{32} & E_{33} & & \vdots \\ \vdots & & & \ddots & \\ E_{N1} & & & & E_{NN} \end{bmatrix} = \begin{bmatrix} \frac{W_{in}(1)}{\omega} & 0 & \cdots & 0 \\ 0 & \frac{W_{in}(2)}{\omega} & & \vdots \\ \vdots & 0 & \frac{W_{in}(3)}{\omega} & \vdots \\ \vdots & & & \ddots \\ 0 & & & & \frac{W_{in}(N)}{\omega} \end{bmatrix} \quad (1)$$

where  $E_{ij}$  is the energy of subsystem  $i$  with power input into subsystem  $j$ . The off-diagonal term and diagonal term of the loss factor matrix represent the CLF,  $\eta_{ij}$ , and the Total Loss Factor (TLF),  $\eta_i$  (NB ILF is denoted by  $\eta_{ii}$ ).  $W_{in}(n)$  denotes input power in  $n^{\text{th}}$  subsystem.

One problem with GESEA is that an ill-conditioned energy matrix can give negative CLFs which are not physically meaningful for an SEA model [7,8].

### 2.2.2 Alternative ESEA (AESEA)

In order to carry out ESEA with only direct coupling, an alternative version of ESEA (denoted here as AESEA) has been suggested by Lalor [12]. AESEA avoids the problem of ill-conditioned matrices allows calculation of CLFs and ILFs separately. CLFs are determined using

$$\begin{bmatrix} \eta_{1i} \\ \vdots \\ \eta_{ri} \\ \vdots \\ \eta_{Ni} \end{bmatrix}_{r \neq i} = \frac{W_i}{\omega E_{ii}} \begin{bmatrix} \left( \frac{E_{11}}{E_{i1}} - \frac{E_{1i}}{E_{ii}} \right) & \cdots & \left( \frac{E_{r1}}{E_{i1}} - \frac{E_{ri}}{E_{ii}} \right) & \cdots & \left( \frac{E_{N1}}{E_{i1}} - \frac{E_{Ni}}{E_{ii}} \right) \\ \vdots & \ddots & & & \vdots \\ \vdots & & \left( \frac{E_{rr}}{E_{ir}} - \frac{E_{ri}}{E_{ii}} \right) & & \vdots \\ \vdots & & & \ddots & \vdots \\ \left( \frac{E_{1N}}{E_{iN}} - \frac{E_{1i}}{E_{ii}} \right) & \cdots & \left( \frac{E_{rN}}{E_{ir}} - \frac{E_{ri}}{E_{ii}} \right) & \cdots & \left( \frac{E_{NN}}{E_{iN}} - \frac{E_{Ni}}{E_{ii}} \right) \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ \vdots \\ \vdots \\ \vdots \\ 1 \end{bmatrix} \quad (2)$$

and ILFs using

$$\begin{bmatrix} \eta_{11} \\ \vdots \\ \eta_{NN} \end{bmatrix} = \frac{1}{\omega} \begin{bmatrix} E_{11} & \cdots & E_{N1} \\ \vdots & & \vdots \\ E_{1N} & \cdots & E_{NN} \end{bmatrix}^{-1} \begin{bmatrix} W_1 \\ \vdots \\ W_N \end{bmatrix} \quad (3)$$

Lalor [13] noted that to ensure that CLFs are always positive, an approximate equation to calculate the CLFs can also be used, given by

$$\eta_{ij} \approx \frac{1}{\omega} \left( \frac{E_{ji}}{E_{ii}} \right) \left( \frac{W_i}{E_{jj}} \right) \quad (4)$$

In this paper, both full matrix AESEA (Eq. (2)) and approximate AESEA (Eq. (4)) are used to determine direct CLFs as differences between two approaches occur when there is non-negligible indirect coupling between non-adjacent subsystems, for example due to a direct field.

## 2.3 Numerical experiments

This paper uses two different numerical models to generate the spatial-average energy data in each subsystem. FEM modelling uses Abaqus for low- and mid-frequencies (defined as being up to the 1kHz octave band). Ray tracing uses ODEON for high frequencies (defined as being above the 1kHz octave band). A single point source is used to excite each subsystem as shown in Fig. 2 where each source is positioned at least 0.2m away from boundaries in the left and right side volumes.

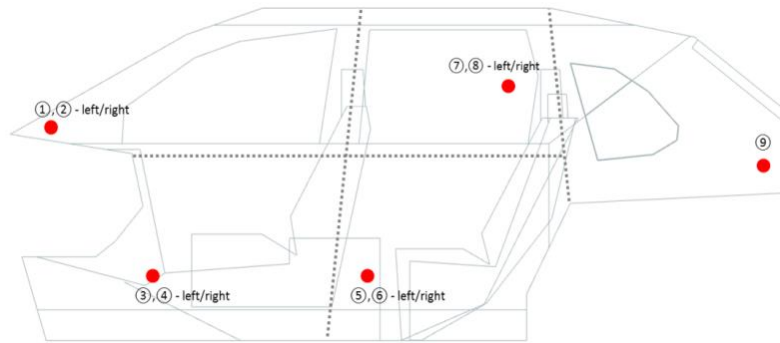


Figure 2: Subdivisions for SEA subsystems and source positions for PIM

Ray tracing and FEM incorporate absorption coefficients for seats, windows and other parts which are assigned as  $\alpha=0.9$ ,  $\alpha=0.03$  and  $\alpha=0.5$  respectively. The absorption coefficient for the seats is based on experimentally determined values in the literature [14,15] and the coefficient for windows is obtained from the ODEON material database (see Table 1).

Table 1: Absorption coefficient from ODEON material database for window glass in octave bands

Frequency (Hz)	63	125	250	500	1000	2000	4000	8000
Single pane of glass	0.18	0.18	0.06	0.04	0.03	0.02	0.02	0.02

The spatial average SPL in each subsystem is calculated over a regular grid with points at 0.1m spacing in x-, y- and z-directions that are at least 0.2m away from the boundaries of the cabin.

## 3. Results

Fig. 3 shows the mode count in octave bands determined from FEM eigenfrequencies and a statistical mode count based purely on the cabin volume. With increasing frequency the deterministic mode count tends towards the statistical estimate. At and above 125Hz, there are a sufficient number of modes to apply SEA in octave bands for the entire cabin volume, and the following results are used to assess whether it is also reasonable to apply SEA when the entire cabin is subdivided.

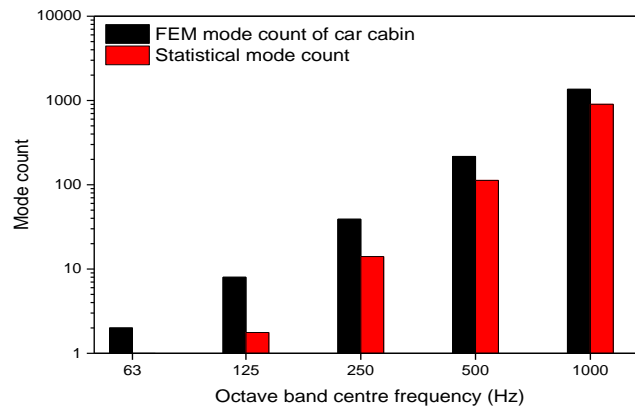


Figure 3: Mode count for the car cabin in octave bands

Based on FEM data in the 500Hz octave band, Fig. 4 shows that for the three- and five-subsystems models with vertical subdivision, SEA using GESEA or AESEA CLFs gives close agreement with FEM, whereas for the five-subsystem model with horizontal subdivision and the nine-subsystem model, SEA using AESEA CLFs (full matrix or approximate) gives closer agreement with FEM than SEA using GESEA CLFs. However, SEA using GESEA CLFs still gives a reasonable estimate ( $< 5$ dB).

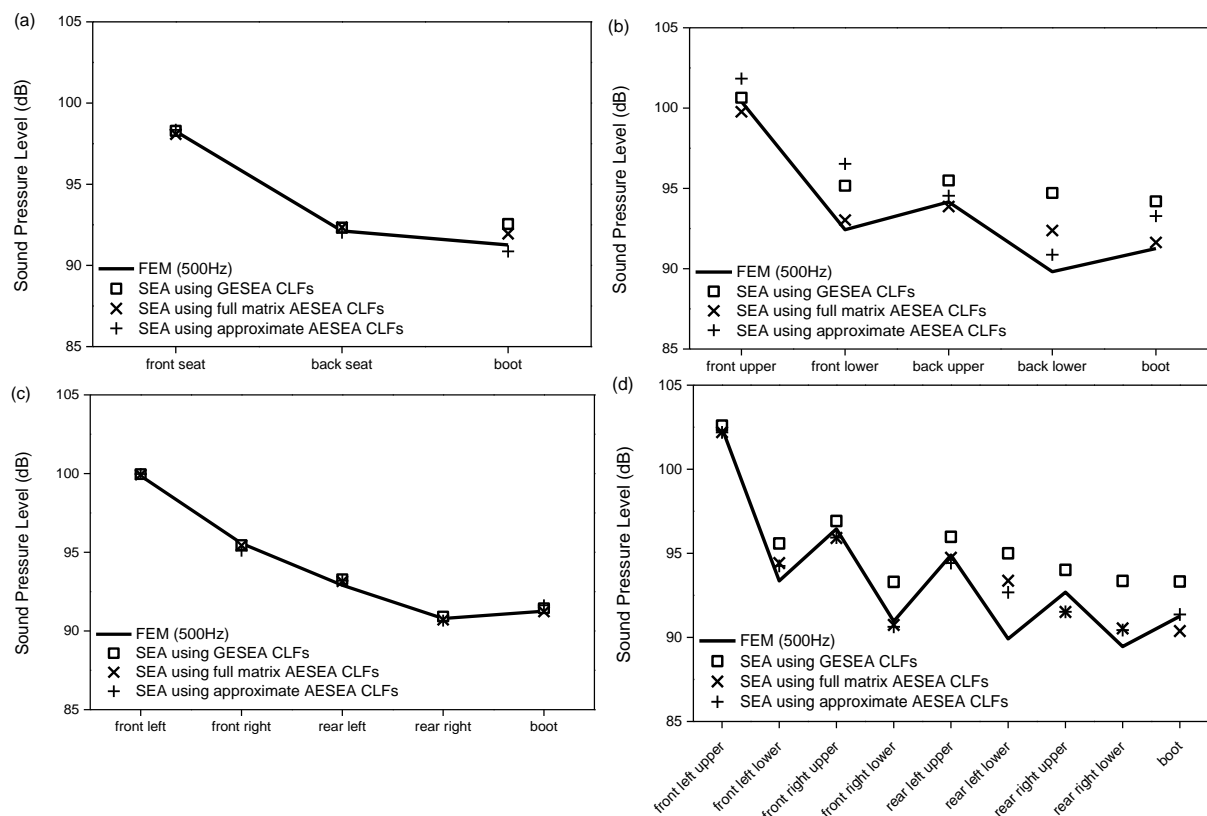


Figure 4: 500Hz octave band: SPL in each subsystem estimated from SEA using ESEA CLFs and FEM in terms of (a) three subsystems, (b) five subsystems with horizontal subdivision, (c) five subsystem with vertical subdivision and (d) nine subsystems. Point source is in the front left subsystem.

Ray tracing data is used to give a frequency-independent result to represent the high frequency range above 1000Hz where the absorption coefficients are frequency-independent (refer back to Table 1). The SPL estimated from SEA using GESEA and AESEA CLFs and ray tracing assuming diffuse reflections is shown in Fig. 5. The results show that SEA using GESEA CLFs gives closer

agreement with ray tracing than SEA using AESEA CLF. For the nine- and five subsystems with horizontal subdivisions, no data is shown from SEA using AESEA CLFs because AESEA gave negative CLFs. Fig. 5 (c) shows that the rear right seat subsystem gives different estimate of SPL between ray tracing and SEA using AESEA CLFs. This subsystem is diagonally opposite the source subsystem; hence the direct field from the point source could affect that receiving subsystem. This indicates that direct field can be incorporated by considering indirect coupling with GESEA. Whilst this improves the prediction model it is necessary to question whether its inclusion is reasonable for the actual source that might be considered in the final SEA model (e.g. sound radiating from the windscreen).

Fig. 4 and Fig. 5 illustrate the general trend that SEA using GESEA CLFs tends to always give a working SEA model regardless of the subdivision of the cabin whereas this is not always possible with AESEA due to the existence of negative ILFs. Hence using GESEA to include indirect coupling seems to be a more useful approach.

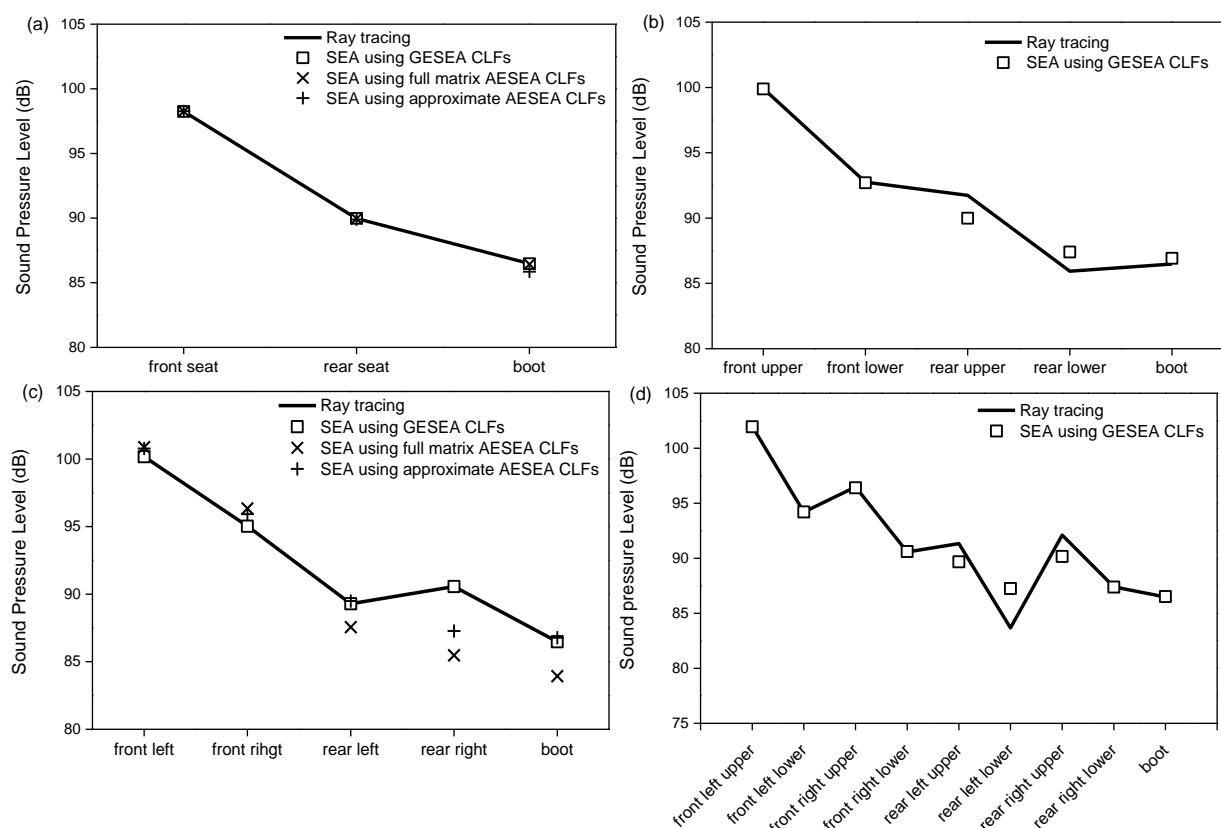


Figure 5: SPL in each subsystem estimated from SEA using ESEA CLFs and ray tracing in terms of (a) three subsystems, (b) five subsystems with horizontal subdivision, (c) five subsystem with vertical subdivision and (d) nine subsystems. Point source is in the front left subsystem.

## 4. Conclusion

In this paper, the division of a realistic car cabin into subsystems representing coupled spaces has been considered in order to model interior noise levels using SEA. Loss factors were determined using two forms of ESEA: GESEA and two forms of AESEA. In general, SEA using CLFs determined from GESEA which allows indirect coupling gave a suitable working model whereas AESEA often gave negative ILFs in terms of subdivision method.

The two different five-subsystem models (horizontal and vertical subdivision) indicate an important issue about subsystem definition by an experimenter, namely that it is not always intuitive. In this case, the vertical subdivision might be considered unintuitive because of the significantly different absorbing surfaces that form each subsystem, whereas the horizontal subdivision seems



more logical because the absorbing surfaces are more similar. However, for the five-subsystem model with horizontal subdivision (and the nine-subsystem model) AESEA gave invalid negative ILFs and therefore did not provide a working model. For this reason when carrying out ESEA it is always worth using a grid of response points that can be grouped in different ways to test different subsystem definitions (i.e. avoid carrying out spatial averages in rigidly defined subsystems where there is no scope to calculate energy average responses from slightly different volumes).

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