SCATTERING OF MULTIPLE SHALLOW-BURIED CAVITIES, INCLUSIONS AND FIXED SURFACE BY SH-WAVE IN ELASTIC SEMI-SPACE

Hongliang Li, Yu Wang , Zhanzhou Tang.

Department of Engineering Mechanics, College of Aerospace Engineering and Civil Engineering, Harbin Engineering University, Harbin, 150001, P.R. China
e-mail: leehl@sina.com

In natural medium, engineering materials and structures, it can be found that there are cavities and inclusions everywhere. Sometimes the surface of the structure is fixed, and it could be seen as a rigid surface. When structure is impacted by dynamic load, the scattering field will be produced because of the cavities, inclusions and the fixed surface, and it could cause dynamic stress concentration at the edge of the cavities or inclusions. In this paper, the solution of displacement field for elastic semi-space with fixed surface and multiple cylindrical cavities and inclusions by anti-plane SH-wave is constructed. In complex plane, considering the displacement boundary condition of the fixed surface, the displacement field aroused by the anti-plane SH-wave and the scattering displacement field impacted by the cylindrical cavities and inclusions comprised of Fourier-Bessel series with undetermined coefficients are constructed. Through applying the method of multi-polar coordinate system, the equations with unknown coefficients can be obtained by using the displacement and stress condition of the cavities or inclusions. According to orthogonality condition for trigonometric function, these equations can be reduced to a series of algebraic equations. Then the value of the unknown coefficients can be obtained by solving these algebraic equations. The total wave displacement field is the superposition of the displacement field aroused by the anti-plane SH-wave and the scattering displacement field. By using the expressions, an example is provided to show the effect of the change of relative location of the cylindrical cavities and inclusions. Based on this solution, the problem of interaction of multiple cylindrical cavities, inclusions and a linear crack in semi-space with fixed surface can be investigated further.
Keywords: cavity, inclusion, fixed surface, sh-wave, elastic semispace

1. Introduction

In natural medium, engineering materials and structures, it can be found that there are cavities and inclusions everywhere. Sometimes the surface of the structure is fixed, and it could be seen as a rigid line. When structure is impacted by dynamic load, the scattering field will be produced because of the cavities, the inclusions and the fixed surface, and it could cause dynamic stress concentration at the edge of the cavities and inclusions. In theory of elastic wave motion, cavity and inclusion are danger factors. Dynamic stress concentration could greatly decrease the bearing capacity of structure, and reduce the service life of structure.

In monograph of Pao(1973), it solved dynamic stress concentration problem in an infinite elastic space with a cavity by anti-plane SH wave, and it indicated that dynamic stress concentration factor is greater than static concentration factor. Datta(1974), Miklowitz(1978) and Moodie(1981) studied some correlative problems by different methods. The methods for solving such boundary value problems included wave function expansion, integral equation, integral transforms, matched asymptotic expansion. To regular shape cavity, wave function expansion method is more widely used. By applying the theory of complex function, Liu(1982) solved irregular shape cavity problem.

Sometimes, there are some complex engineering problems. For example, there are two or more underground pipelines in city. So it is important to study the problem of scattering of SH wave by multiple cylindrical cavities, cylindrical inclusions and fixed surface. It is one of the important and interesting questions in mechanical engineering for the latest decades. There are lots of materials obtained by theoretical research and damage investigation. This problem is complicated, because there are many factors influenced. Researchers solved these problems by analysis and numerical methods[1-4]. It is hard to obtain analytic solutions except for several simple conditions.

In this paper, the solution of displacement field for elastic semi-space with multiple cylindrical cavities, cylindrical inclusions and fixed surface by anti-plane SH-wave is constructed. The train of thoughts for this problem is that: In complex plane, considering the displacement boundary condition of the fixed surface, the displacement field aroused by the anti-plane SH-wave and the scattering displacement field impacted by the cylindrical cavities and cylindrical inclusions comprised of Fourier-Bessel series with undetermined coefficients are constructed. Through applying the method of multi-polar coordinate system, the equations with unknown coefficients can be obtained. According to orthogonality condition for trigonometric function, these equations can be reduced to a series of algebraic equations. Then the value of the unknown coefficients can be obtained by solving these algebraic equations.

After the expressions are obtained, an examples is provided to show the effect of the change of relative location of the cylindrical cavities and cylindrical inclusions.

2. Model and Governing equation

The model is shown as Fig. 1, an elastic semi-space containing multiple shallow-buried cavities and inclusions which bearing anti-plane SH-wave, and the surface is fixed. In this paper, the governing equation is studied, which is regarded as the displacement response to the elastic semi-space containing multiple shallow-buried cavities, inclusions and fixed surface impacted by SH wave. The dependence of the displacement function W on time t is $e^{-i\omega t}$. In complex plane, the displacement in the elastic semi-space is expressed as $W(z,\overline{z},t)$.The displacement in the inclusion is expressed as $W^{(sm)}(z,\overline{z},t)$. Based on the complex function theory, the governing equation of W can also be written as:

$$\frac{\partial^2 W}{\partial z \partial \overline{z}} + \frac{1}{4} k^2 W = 0 . \tag{1}$$

The displacement function $W^{(sm)}$ satisfies the following governing equations:

$$\frac{\partial^2 W^{(sm)}}{\partial z \partial \overline{z}} + \frac{1}{4} k_m^2 W^{(sm)} = 0 \quad (m = 1, 2, ..., N) . \tag{2}$$

The boundary conditions can be expressed as below:

$$w = 0 (where\ \theta = 0\ and\ \theta = \pi) . \tag{3}$$

$$\tau_{rz} = 0 (where\ \left| z - c_j \right| = R_j , j = 1, 2, ..., N) . \tag{4}$$

$$W = W^{(sm)} \quad (\left| z - C_k \right| = R'_k , k = 1, 2, ..., M) . \tag{5}$$

$$\tau_{rz} = \tau_{rz}^{(sm)} \quad (\left| z - C_k \right| = R'_k , k = 1, 2, ..., M) . \tag{6}$$

where N represents the number of cavities, M represents the number of inclusions. Eq. (3) represents fixed surface condition. Eq. (4) represents radial stress-free condition at the edge of cavities. Eqs. (5) and (6) represent displacement and stress condition at the edge of inclusions.

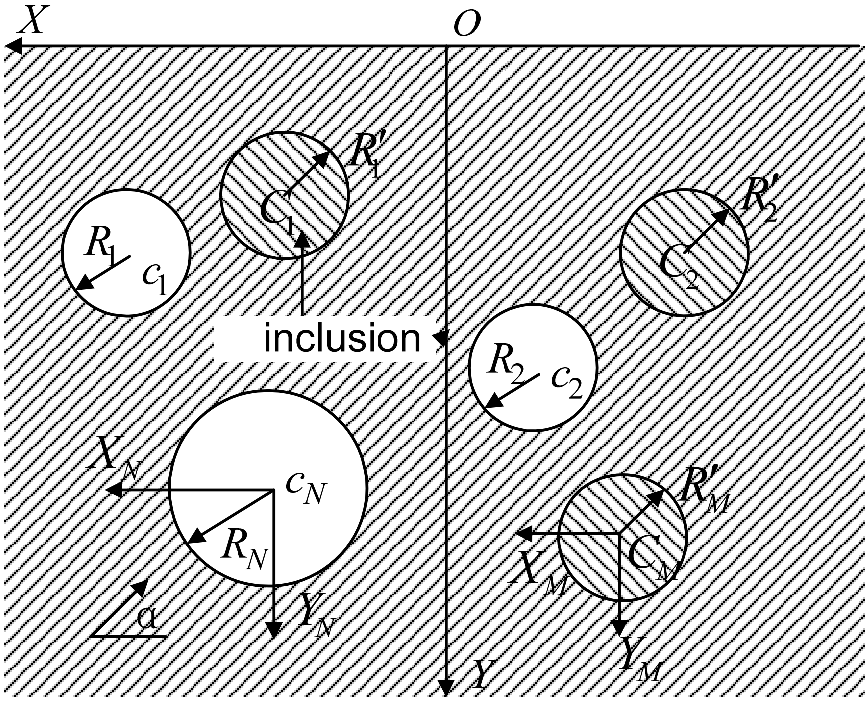

Figure 1: Model of problem

3. Solution of displacement field

The basic solution which satisfies the control equation Eq. (1) and the boundary conditions Eqs. (4)-(6) should include two parts of motion: the disturbance of incident SH-wave and the scattering wave incited by multiple shallow-buried cavities and inclusions. To satisfy the boundary condition Eq. (3), the wave displacement of the elastic semi-space due to the incident wave can be given as[5,6]:

$$W^{(i)} = W_0 (e^{\frac{ik}{2}(ze^{-i\alpha}+\bar{z}e^{i\alpha})} - e^{\frac{ik}{2}(ze^{i\alpha}+\bar{z}e^{-i\alpha})}) . \tag{7}$$

The scattering wave incited by multiple shallow-buried cavities and inclusions, and the displacement in the inclusion can be written as:

$$W^{(is1)} = \sum_{j=1}^{N} \sum_{n=-\infty}^{\infty} A_n^j [H_n^{(1)}(k|z-c_j|)(\frac{z-c_j}{|z-c_j|})^n - H_n^{(1)}(k|z-\bar{c}_j|)(\frac{z-\bar{c}_j}{|z-\bar{c}_j|})^{-n}] . \tag{8}$$

$$W^{(is2)} = \sum_{k=1}^{M} \sum_{n=-\infty}^{\infty} B_n^k [H_n^{(1)}(k|z-C_k|)(\frac{z-C_k}{|z-C_k|})^n - H_n^{(1)}(k|z-\bar{C}_k|)(\frac{z-\bar{C}_k}{|z-\bar{C}_k|})^{-n}] . \tag{9}$$

$$W^{(sm)} = \sum_{n=-\infty}^{\infty} C_n^k J_n(k_k|z-C_k|)(\frac{z-C_k}{|z-C_k|})^n \ (k=1,2,...,M) . \tag{10}$$

where A_n^j , B_n^k , C_n^k are unknown coefficients. c_j represents the center of the cavity and C_k represents the center of the inclusion. Where $H_n^{(1)}(*)$ is the first kind of Hankel function and n order.

Therefore, the wave field G can be written as:

$$W = W^{(i)} + W^{(is1)} + W^{(is2)} . \tag{11}$$

The wave field W must satisfy the stress or displacement condition on multiple shallow-buried cavities and inclusions, so by using the method of transferred coordinate, when the origin of coordinate is moved to c_m, W can be written as:

$$W = W_0(e^{\frac{ik}{2}(ze^{-i\alpha}+\bar{z}e^{i\alpha})} - e^{\frac{ik}{2}(ze^{i\alpha}+\bar{z}e^{-i\alpha})}) + \sum_{j=1}^{N}\sum_{n=-\infty}^{\infty} A_n^j [H_n^{(1)}(k|z_m - d_{mj}|)(\frac{z_m - d_{mj}}{|z_m - d_{mj}|})^n - H_n^{(1)}(k|z_m - d'_{mj}|)(\frac{z_m - d'_{mj}}{|z_m - d'_{mj}|})^{-n}] + \sum_{k=1}^{M}\sum_{n=-\infty}^{\infty} B_n^k [H_n^{(1)}(k|z - D_{mk}|)(\frac{z - D_{mk}}{|z - D_{mk}|})^n - H_n^{(1)}(k|z - D'_{mk}|)(\frac{z - D'_{mk}}{|z - D'_{mk}|})^{-n}]. \quad (12)$$

where $d_{mj} = c_j - c_m$, $d'_{mj} = \bar{c}_j - c_m$, $D_{mk} = C_k - c_m$, $D'_{mk} = \bar{C}_k - c_m$.

Substituting the wave filed W to the boundary conditions, it can be obtained that

$$\sum_{j=1}^{N}\sum_{n=-\infty}^{\infty} A_n^j \phi_{mn}^j + \sum_{k=1}^{M}\sum_{n=-\infty}^{\infty} B_n^k \phi_{mn}^k = \phi_m . \quad (13)$$

$$\sum_{j=1}^{N}\sum_{n=-\infty}^{\infty} A_n^j \phi'^j_{mn} + \sum_{k=1}^{M}\sum_{n=-\infty}^{\infty} B_n^k \phi'^k_{mn} + \sum_{n=-\infty}^{\infty} C_n^m \phi'_{mn} = \phi'_m . \quad (14)$$

$$\sum_{j=1}^{N}\sum_{n=-\infty}^{\infty} A_n^j \varphi_{mn}^j + \sum_{k=1}^{M}\sum_{n=-\infty}^{\infty} B_n^k \varphi_{mn}^k + \sum_{n=-\infty}^{\infty} C_n^m \varphi_{mn} = \varphi_m . \quad (15)$$

By multiplying both sides of Eqs. (13)-(15) with $e^{-im\theta_m}$ and integrating in interval $[-\pi, \pi]$, it can be obtained that

$$\sum_{j=1}^{N}\sum_{n=-\infty}^{\infty} A_n^j \Phi_{mn}^j + \sum_{k=1}^{M}\sum_{n=-\infty}^{\infty} B_n^k \Phi_{mn}^k = \Phi_m . \quad (16)$$

$$\sum_{j=1}^{N}\sum_{n=-\infty}^{\infty} A_n^j \Phi'^j_{mn} + \sum_{k=1}^{M}\sum_{n=-\infty}^{\infty} B_n^k \Phi'^k_{mn} + \sum_{n=-\infty}^{\infty} C_n^m \Phi'_{mn} = \Phi'_m . \quad (17)$$

$$\sum_{j=1}^{N}\sum_{n=-\infty}^{\infty} A_n^j \Psi_{mn}^j + \sum_{k=1}^{M}\sum_{n=-\infty}^{\infty} B_n^k \Psi_{mn}^k + \sum_{n=-\infty}^{\infty} C_n^m \Psi_{mn} = \Psi_m . \quad (18)$$

Eqs. (16)-(18) are a set of infinite algebraic equations for determining the coefficients A_n^j and B_n^k .Substituting the coefficients A_n^j and B_n^k to Eq. (11), the total wave field W of this problem can be obtained.

4. Example

In theory of elastic wave motion, dynamic stress concentration is a danger factor. In this paper, we pay attention to a representative kind of models. There is a cavity and a inclusion in elastic semi-space. The model is shown as Fig. 2. The radius of the shallow-buried cavity and inclusion equals 1. In Figure 3 and Figure 4, $k = 0.5$,and ratio of shear modulus in elastic semi-space to shear modulus in inclusion is $3/2$. The other parameters are shown in the figures. In Fig. 3, It shows the influence of burying depth to DSCF at the cavity edge. It can be found that when the burying depth is 3, semi-space surface has the greatest impact. In Fig. 4, it shows the influence of the left inclusion to DSCF at the right cavity edge. The distance between the centre of the right cavity and the centre of the left inclusion is changed from 3 to 9. It can be found that when the distance is 6, it has the biggst influence to DSCF of the right cavity. In Fig. 5, it shows the influence of k to DSCF of the left inclusion. It can be found that medium frequency has the most influence. In Fig. 6, it shows the influence of incident angle to DSCF at the left inclusion edge. It can be found that when $\alpha = 90^0$ there is the biggest influence.

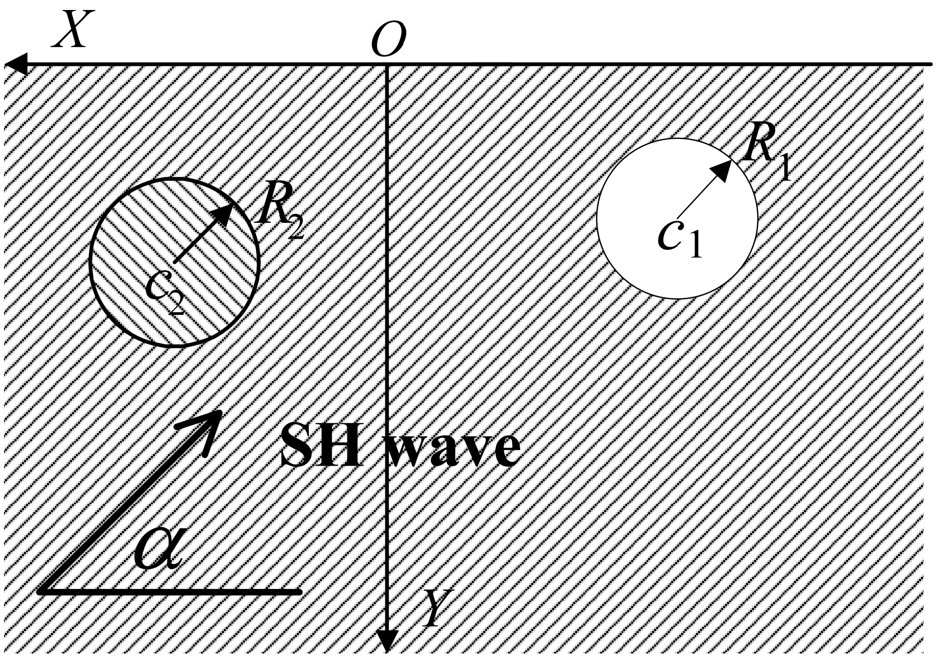

Figure 2: Calculating model

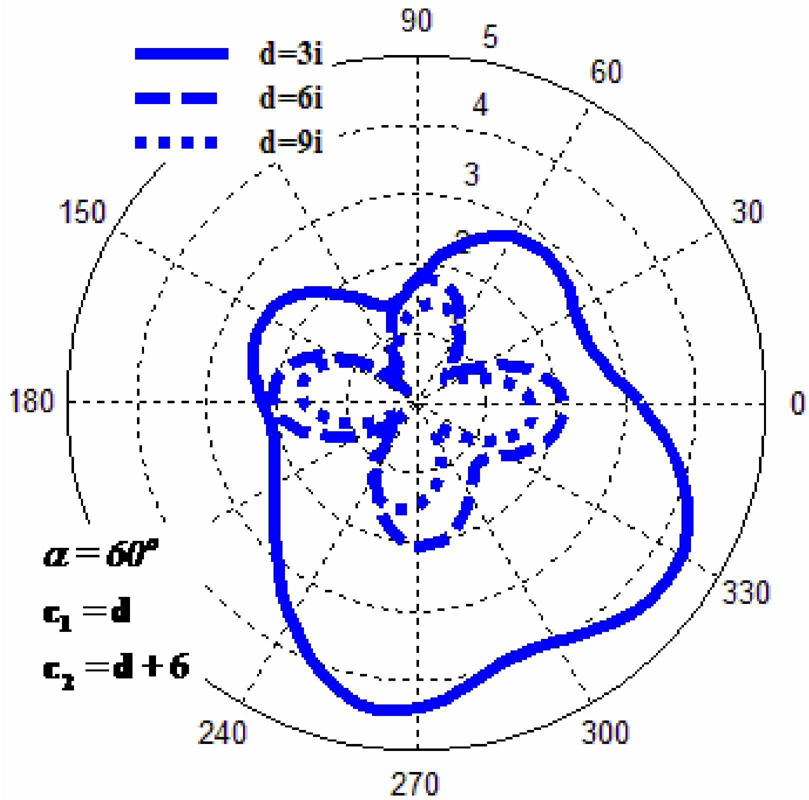

Figure 3: Influence of burying depth to DSCF at the cavity edge

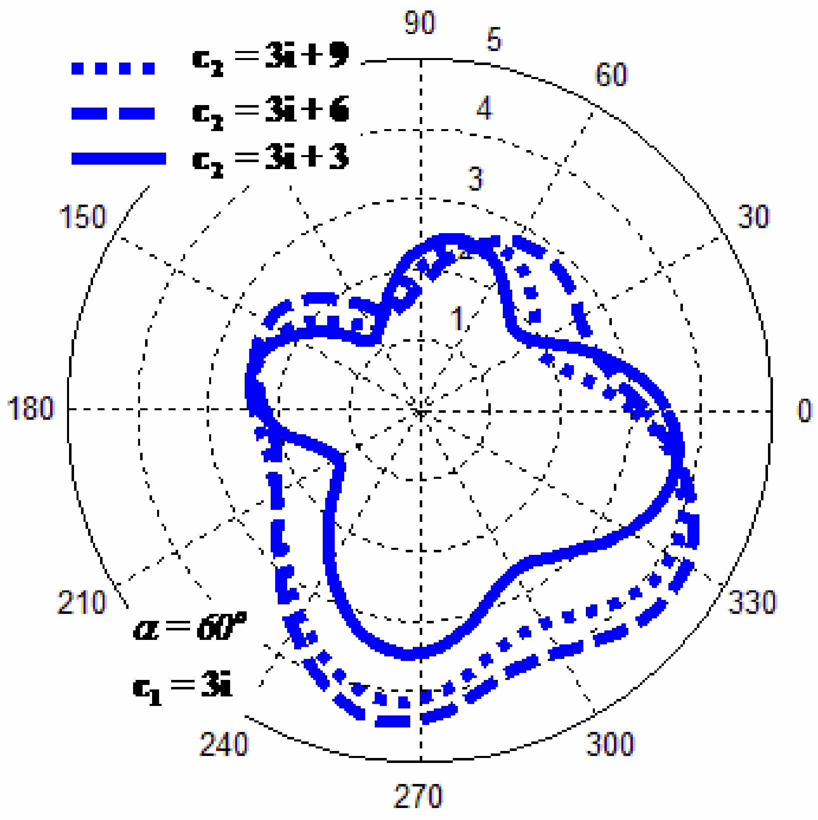

Figure 4: Influence of cavity to DSCF at the cavity edge

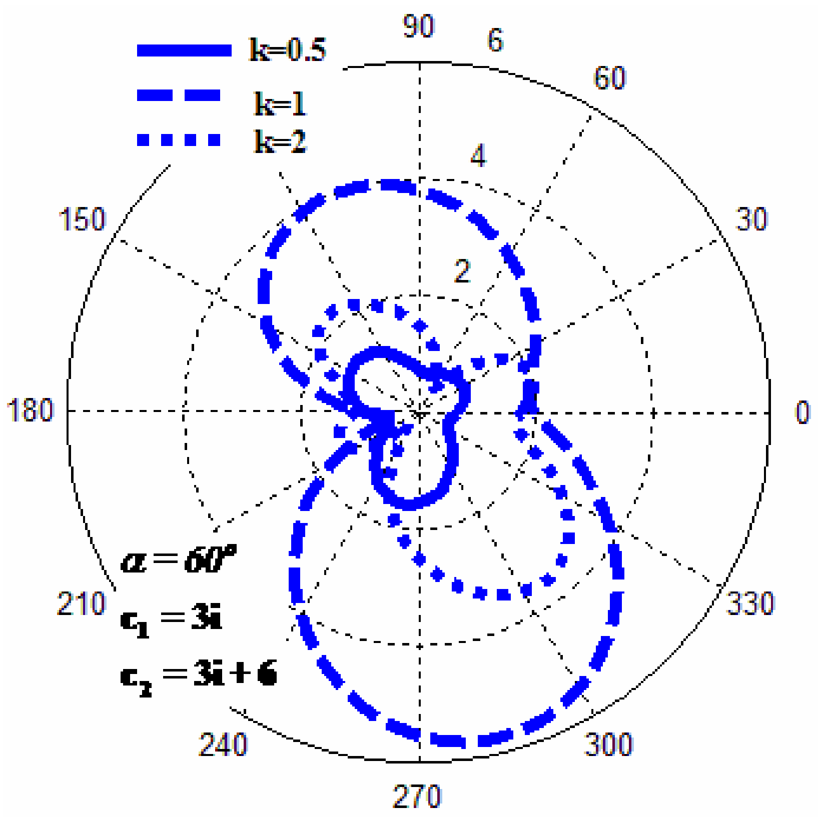

Figure 5: Influence of k to DSCF at the inclusion edge

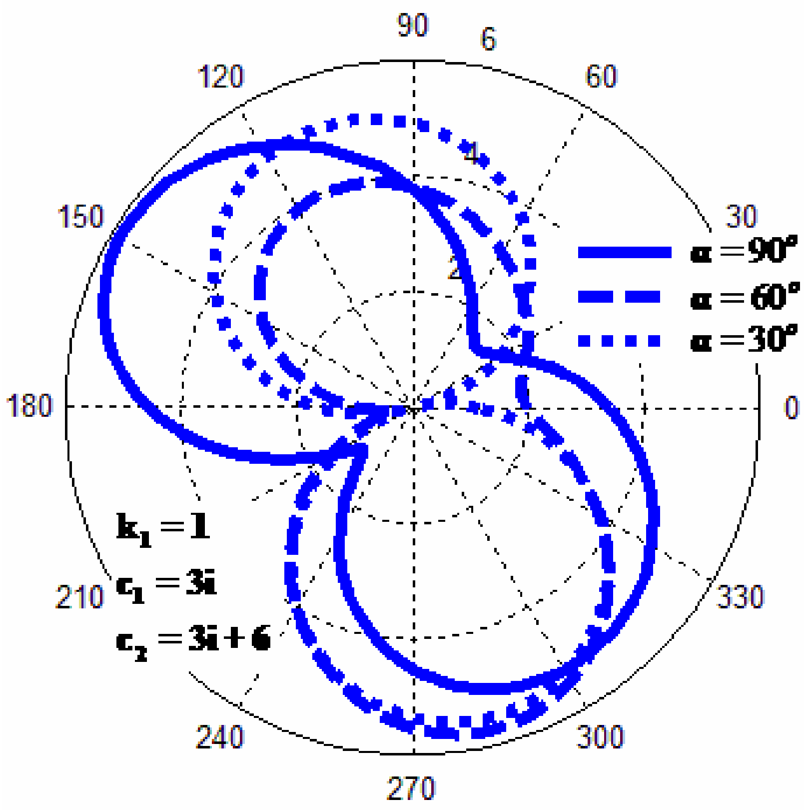

Figure 6: Influence of α to DSCF at the inclusion edge

5. Summary

In this paper, by using the method of complex function, the method of multi-polar coordinate system, the method of mirror image, the solution of displacement field of the elastic semi-space with fixed surface and multiple shallow-buried cavities and inclusions is got. Then an example is given. It can be found that the influence among the cavities and the inclusions is very complex. Based on this solution, the problem of interaction of multiple shallow-buried cavities, inclusions and a linear crack in semi-space with fixed surface can be investigated further.

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