inter-noise 83

A PROBABILISTIC EVALUATION METHOD OF PEAK VALUES FOR THE ARBITRARY RANDOM NOISE AND VIBRATION AND ITS APPLICATION TO ROAD TRAFFIC NOISE

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INTRODUCTION

In the noise and vibration environmental systems, the statistical properties of peak values and the expected number of level-crossings are as important as the statistics connected with usual amplitude distribution such as mean value, variance, Lx, Leq etc. for the random noise and vibration, especially from the viewpoints of psychological evaluation of random fluctuation and/or noise and vibration control. As is well-known, the expected number of level-crossings can be principally determined by only the two characteristic quantities such as level and its velocity. On the other hand, three characteristic quantities such as level, velocity and acceleration are essentially necessary to evaluate a peak distribution. From the above fundamental viewpoint, the statistical peak distribution is difficult to evaluate in a general form, but in a typical case of a Gaussian random process with narrow frequency band, it is well-known that the peak values are distributed according to a Rayleigh distribution [1]. Furthermore, after once introducing a simplified peak evaluation method based on only the expected number of level-crossings, a general expression of the probability density function for peak values has already been derived in a generalized expansion form of distribution applicable to any type of non-Gaussian random process with narrow frequency band [2].

In the previous paper, we have shown that the probability density function of the instantaneous level value of noise and vibration can be expressed in terms of an orthogonal expansion form of statistical Hermite series connected with a Gaussian distribution. However, the fluctuation of any actual random noise and vibration level values measured in dB scale varies principally only over a positive region. If we pay our attention to the above fundamental property, it will be more rational to adopt a statistical Laguerre expansion series connected with a well-known Gamma distribution as a level distribution of arbitrary random noise and vibration.

In the present paper, we will first introduce a bivariate joint probability density function of instantaneous value and its velocity in the mixed orthogonal expansion form of a statistical Laguerre series and a statistical Hermite series. Then, by use of the above-mentioned simplified peak evaluation method, a probability distribution expression for the peak values is concretely derived in a generalized expansion form applicable to any type of random noise and vibration fluctuating over a positive region. Finally, the validity of the proposed theory has been experimentally confirmed by applying it to observed actual traffic noise.

THEORETICAL CONSIDERATIONS

Derivation of the expected number of level-crossings

Let x and $\dot{x}$ be the instantaneous amplitude and its velocity for a general type of stationary random process $x(t)$ with an arbitrary probability distribution. Then, the expected number, $N(x_0)$, of x per second crossing over an arbitrary value, x_0, of level x with only positive slope is given by [3]

$$N(x_0)=\int_0^{\infty}\dot{x}\,P(x_0,\dot{x})\,d\dot{x}\,, \tag{1}$$

where $p(x,\dot{x})$ is a joint probability density function of x and $\dot{x}$. In the special case with a stationary random property for the noise and vibration fluctuations, as a joint probability density function, $P(x,\dot{x})$, in Eq. (1), it is natural to adopt a mixed orthogonal expansion series of the statistical Laguerre series connected with a Gamma distribution for x and the statistical Hermite series connected with a Gaussian distribution for $\dot{x}$, because of the following situations:

(i) The actual noise and vibration level values, x's, are usually distributed only over a positive region.

(ii) The velocity, $\dot{x}$, is approximately distributed symmetrically with respect to zero level under the condition of the stationary property.

Therefore, $P(x,\dot{x})$ can be first expressed as follows:

$$\left.\begin{aligned} P(x,\dot{x}) &= \frac{x^{m-1}}{\Gamma(m)\,s^{m}}e^{-\frac{x}{s}}\frac{1}{\sqrt{2\pi}\,\sigma_{\dot{x}}}e^{-\frac{\dot{x}^2}{2\sigma_{\dot{x}}^2}}\Big\{1+\sum_{n,K}{}' B(n,K)\,L_n^{(m-1)}\Big(\frac{x}{s}\Big)H_K\Big(\frac{\dot{x}}{\sigma_{\dot{x}}}\Big)\Big\},\\ \text{with}\quad B(n,K) &\triangleq \Big\langle L_n^{(m-1)}\Big(\frac{x}{s}\Big)H_K\Big(\frac{\dot{x}}{\sigma_{\dot{x}}}\Big)\Big\rangle\frac{n!\,\Gamma(m)}{\Gamma(n+m)\,K!}\,. \end{aligned}\right\} \tag{2}$$

In the above Eq. (2), $B(n,K)$ is an expansion coefficient reflecting the linear and higher order correlations between x and $\dot{x}$. Σ' denotes the total sum for all sets of (n,K) except $(0,0)$; s, m and $\sigma_{\dot{x}}$ are arbitrary constant values; $H_n(\ast)$ denotes a Hermite polynomial and $L_n^{(m-1)}(\ast)$ denotes a Laguerre polynomial. Substituting Eq. (2) into Eq. (1), we easily see

$$N(x)=\frac{x^{m-1}}{\Gamma(m)\,s^{m}}e^{-\frac{x}{s}}\frac{1}{\sqrt{2\pi}\,\sigma_{\dot{x}}}\sum_{n=0}^{\infty}\sum_{K=0}^{\infty}B(n,K)\,L_n^{(m-1)}\Big(\frac{x}{s}\Big)\int_0^{\infty}\dot{x}\,H_K\Big(\frac{\dot{x}}{\sigma_{\dot{x}}}\Big)e^{-\frac{\dot{x}^2}{2\sigma_{\dot{x}}^2}}d\dot{x}\,. \tag{3}$$

Now we use an integral relationship with respect to Hermite polynomial:

$$\int_0^{\infty}\dot{x}\,H_K\Big(\frac{\dot{x}}{\sigma_{\dot{x}}}\Big)e^{-\frac{\dot{x}^2}{2\sigma_{\dot{x}}^2}}d\dot{x}=\begin{cases}\sigma_{\dot{x}}^2 & (K=0)\\ \sqrt{2\pi}\,\sigma_{\dot{x}}^2/2 & (K=1)\\ H_{K-2}(0)\,\sigma_{\dot{x}}^2 & (K\geq 2)\,,\end{cases} \tag{4}$$

the general expression of the expected number, $N(x)$, of x per second crossing over a level x can be obtained from Eq. (3) by

$$N(x)=\frac{x^{m-1}}{\Gamma(m)s^m}e^{-\frac{x}{s}}\frac{\sigma_{\dot{x}}}{\sqrt{2\pi}}\Big\{\sum_{n=0}^{\infty}B(n,0)\,L_n^{(m-1)}\Big(\frac{x}{s}\Big)+\frac{\sqrt{2\pi}}{2}\sum_{n=0}^{\infty}B(n,1)L_n^{(m-1)}\Big(\frac{x}{s}\Big)+\sum_{K=2}^{\infty}H_{K-2}(0)\sum_{n=0}^{\infty}B(n,K)L_n^{(m-1)}\Big(\frac{x}{s}\Big)\Big\}. \quad (5)$$

Derivation of peak distribution based on N(x)

If a random noise or vibration has a narrow frequency band, the expected number of level-crossings can be, for practical purposes, approximated by the expected number of peaks distributed over its level [4]. Accordingly, a cumulative peak distribution can be evaluated by use of the expected number, $N(x)$, as follows:

$$Q(x)=1-N(x)/N(\mu). \quad (6)$$

Based on this, we assume that the peak values are distributed only over the region $\mu\le x<\infty$, and then $N(\mu)$ denotes the expected number of all peaks distributed over the same region. Therefore, a probability density function of peak values is obtained by differentiating Eq. (6) with respect to x as follows:

$$P(x)=-\frac{1}{N(\mu)}\frac{dN(x)}{dx}. \quad (7)$$

Substituting Eq. (5) into Eq. (7), and using the differential relationship with respect to Laguerre polynomial:

$$\frac{d}{dx}\Big\{\frac{x^{m-1}}{\Gamma(m)s^m}e^{-x/s}L_n^{(m-1)}\Big(\frac{x}{s}\Big)\Big\}=\frac{e^{-x/s}x^{m-2}}{\Gamma(m)s^m}L_{n+1}^{(m-2)}\Big(\frac{x}{s}\Big)(n+1), \quad (8)$$

a general expression on statistical distribution of peaks, $P(x)$, for the arbitrary random noise and vibration with a narrow frequency band can be consequently derived as follows:

$$P(x)=\Big[e^{-\frac{(x-\mu)}{s}}x^{m-2}(-1)\sum_{n=0}^{\infty}(n+1)L_{n+1}^{(m-2)}\Big(\frac{x}{s}\Big)\Big\{B(n,0)+\frac{\sqrt{2\pi}}{2}B(n,1)+\sum_{K=2}^{\infty}H_{K-2}(0)B(n,K)\Big\}\Big]\Big/ \quad (9)$$

$$\Big[\mu^{m-1}\Big\{\sum_{n=0}^{\infty}B(n,0)L_n^{(m-1)}\Big(\frac{\mu}{s}\Big)+\frac{\sqrt{2\pi}}{2}\sum_{n=0}^{\infty}B(n,1)L_n^{(m-1)}\Big(\frac{\mu}{s}\Big)+\sum_{K=2}^{\infty}H_{K-2}(0)\sum_{n=0}^{\infty}B(n,K)L_n^{(m-1)}\Big(\frac{\mu}{s}\Big)\Big\}\Big].$$

Especially, if the probability density function of the velocity is approximated by a Gaussian distribution, the above peak distribution can be rewritten immediately as follows:

$$P(x)=\Big\{e^{-(x-\mu)/s}x^{m-2}(-1)\sum_{n=0}^{\infty}(n+1)L_{n+1}^{(m-2)}\Big(\frac{x}{s}\Big)B(n,0)\Big\}\Big/\Big\{\mu^{m-1}\sum_{n=0}^{\infty}L_n^{(m-1)}\Big(\frac{\mu}{s}\Big)B(n,0)\Big\}. \quad (10)$$

Furthermore, if the probability density function of the amplitude level is approximated by a standard Gamma distribution, we can use the first expansion term in Eq. (9) as this peak distribution expression in a simplified form:

$$P(x)=e^{-(x-\mu)/s}\frac{x^{m-2}}{\mu^{m-1}}\Big(\frac{x}{s}-m+1\Big),\quad (\mu\triangleq\langle x\rangle-s,\ \mu\le x<\infty). \quad (11)$$

Hereupon, we can arbitrarily adopt the values of σ_x, m and s matched to the actual noise and vibration data. But if we wish to get a good approximation with only the first expansion term in Eq. (11), we had better to choose $m=\langle x\rangle^2/\sigma_x^2$, $s=\langle x\rangle/m$ and $\sigma_x^2=\langle (x-\langle x\rangle)^2\rangle$ by use of the moment method. As is well-known for large values of m (>>1), the Gamma distribution can be approximated by a Gaussian type, and for the special case with m=1, the above Gamma distribution shows an exponential distribution form. Thus, the general expression of peak values in Eq. (9) is very useful for any random noise and vibration with an arbitrary amplitude distribution type.

EXPERIMENTAL CONSIDERATIONS

In this section, we have confirmed experimentally the validity of the general expression (9) of peak distribution by applying it to an actual random traffic noise wave observed by a usual sound level meter at a typical urban district in Hiroshima City. Fig. 1 gives the comparison between experimentally sampled points and theoretically estimated curves in the form of cumulative distribution. From this figure, we can find a good agreement between experiment and theory even if adopting only the first expansion term in Eq. (11). Furthermore, it is obvious that the successive addition of higher expansion terms moves the theoretical curves closer to the experimentally sampled points.

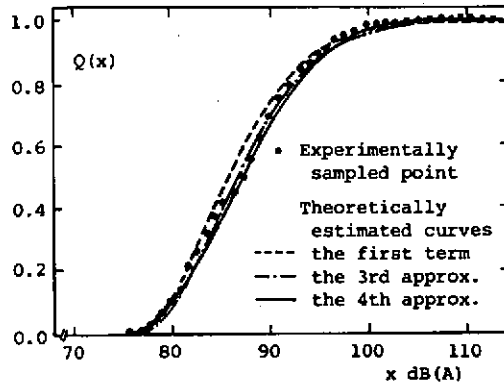

Fig. 1 A comparison between theory and experiment for the cumulative peak distribution, $Q(x)$ ($\triangleq \int_\mu^x P(\xi)d\xi$), of actual traffic noise.

CONCLUDING REMARKS

In the present paper, especially from the practical viewpoint, a general expression of the peak distribution for the arbitrary random process fluctuating only over a positive region has been discussed based on the expected number of level-crossings. Consequently, the general expression of peak distribution is given in Eq. (9) with first and higher order correlations between instantaneous amplitude and its velocity. We would like to express our cordial thanks to Mr. M. Takakuwa for his helpful advice.

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