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THE EFFECT OF VIBRATION LOCALIZATION ON THE PERFORMANCE OF A VISCOUS DAMPER

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The presence of irregularity in a periodic structure results in the vibration localization which exhibits locally large amplitude. The combination of the vibration localization and a viscous damper can be one way to realize a fast damping of free vibration by intentionally creating a vibration mode which is strongly localized at the damper position. This paper investigates the behaviour of a uniform tensioned string which is coupled to ground through homogeneously distributed stiffness and to a viscous damper at its centre, examining the effect of vibration localization on the damping performance. Localization is induced by a concentrated mass that is attached to the string at the same point as the viscous damper. In order to evaluate the rate of vibration decay, the characteristic equation is derived and the eigenvalue is computed by the Galerkin Method in which the mode of the string is represented by the superposition of a half sine wave and shape functions whose slope is discontinuous at the position of the added mass. The results show that the added concentrated mass induces vibration mode which is strongly localized at the damper position, demonstrating improvement in damping performance with the increasing concentrated mass.
Keywords: localization, damping, free vibration

1. Introduction

The structure consisting of almost identical subsystems exhibits a localized vibration mode depending on the magnitude of the disorder present in the system [1]. The vibration localization is known to result in drastic change in vibration mode in many structural systems [1-6], but its effect on the damping characteristics of the system is not clear. The aim of this paper is to examine the effect of vibration localization on the damping performance of free vibration.

Hodges [2] investigated wave propagation in a vibrating string that is coupled to a number of constraints such as mass-spring systems placed at random intervals, and showed that wave transmissions and reflections at the constraints lead to vibration localization when the wavelength is much smaller than the fluctuation of the constraint interval. The effect of vibration localization on energy transmission has been examined by the analysis of a chain of pendula coupled by springs and been confirmed by a corresponding experimental system which comprises a tensioned string having small weights at slightly irregular intervals, concluding that vibration localization can give a significant decrease in transmitted energy [3]. Parameter criterion for predicting the occurrence of strong localization in coupled pendula has been derived by Pierre and Dowell [1] by the modified perturbation method in which coupling strength as well as parameter variation is treated as a perturbation. It should be noted that this perturbation approach provided an important physical account of vibration localization: Localized modes can be regarded as perturbed modes of an uncoupled disordered system where only one subsystem vibrates at its own natural frequency.

Localization of coupled beams has also been investigated [4-6]. Cha and Pierre [4] investigated the effects of coupling strength, coupling position, mode number and disorder strength on vibration localization which occurs in a large number of cantilever beams arranged in parallel and coupled by linear springs. They revealed that effect of coupling becomes smaller as mode number is higher and coupling position is closer to vibration node, resulting in strong localization. Pierre et al. [5] examined vibration modes of a uniform pinned-pinned beam connected to ground at an intermediate point by a torsional spring, regarding it as a system constructed from two almost identical beams that are coupled in series. They confirmed that the vibration mode is strongly localized when the stiffness of the spring is large and beam lengths are slightly different. Vibration mode of beams coupled in series by torsional spring was calculated by Lust et al. [6], showing strong localization disappears in a certain range of the coupling parameter.

In this paper, the effect of vibration localization on the damping performance is investigated. The Galerkin method is applied to the free vibration analysis of a uniform tensioned string which is coupled to ground by a viscous damper as well as homogeneously distributed stiffness. The rate of the vibration decay is examined by evaluating the eigenvalue for the first mode of the system.

2. Analysis

2.1 Analytical model

A schematic of the system under consideration is shown in Fig. 1. A uniform string of length l, mass per unit length ρ and tension T is coupled to ground by distributed stiffness of spring constant per unit length k_d. The string is placed along the x-axis and its deflection at a point x and a time t is represented by $u = u(x,t)$. To investigate the effects of mode localization on the damping performance, an added concentrated mass Δm and damper c_P are attached to the string at the centre.

By considering forces acting on a small element of the string as well as forces acting from the added concentrated mass and damper at $x = l/2$, the equation of motion of this system is derived as

$$\rho \frac{\partial^2 u}{\partial t^2} - T\frac{\partial^2 u}{\partial x^2} + k_d u + \left(\Delta m \frac{\partial^2 u}{\partial t^2} + c_P \frac{\partial u}{\partial t} \right) \delta\left(x - \frac{l}{2} \right) = 0, \tag{1}$$

where $\delta(\cdot)$ is Dirac's delta. This equation can be written as

$$\frac{\partial^2 \overline{u}}{\partial \overline{t}^2} - \frac{1}{\pi^2}\frac{\partial^2 \overline{u}}{\partial \overline{x}^2} + \kappa^2 \overline{u} + \left(\mu \frac{\partial^2 \overline{u}}{\partial \overline{t}^2} + \zeta_P \overline{\omega}_\kappa \frac{\partial \overline{u}}{\partial \overline{t}} \right) \delta\left(\overline{x} - \frac{1}{2} \right) = 0, \tag{2}$$

where following variables and parameters are introduced:

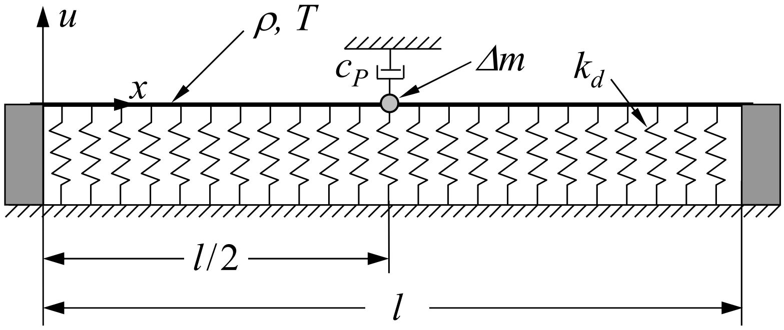


Figure 1: Analytical model.

$$\left.\begin{aligned} &\bar{t} = \omega_0 t,\ \bar{u} = \frac{u}{l},\ \bar{x} = \frac{x}{l},\ \mu = \frac{\Delta m}{\rho l},\ \zeta_P = \frac{c_P}{\rho l \omega_\kappa} \\ &\omega_\kappa = \sqrt{1+\kappa^2}\omega_0,\ \omega_0 = \frac{\pi c_0}{l},\ \kappa = \frac{l}{\pi}\sqrt{\frac{k_d}{T}},\ c_0 = \sqrt{\frac{T}{\rho}},\ \bar{\omega}_\kappa = \frac{\omega_\kappa}{\omega_0} \end{aligned}\right\} \tag{3}$$

Here, μ is the mass ratio of the concentrated mass to the mass of the string, ζ_P is a parameter indicating the strength of the viscous damper, ω_κ and ω_0 are the first angular natural frequencies of the string when $\Delta m = 0$ and $\Delta m = k_d = 0$, respectively, κ is a parameter indicating the strength of the distributed stiffness. It should be noted that $\bar{t}$ is dimensionless time normalized by ω_0 whose value is independent of Δm and k_d, which means scale of the dimensionless time $\bar{t}$ is influenced by neither of μ and κ.

In addition, the boundary conditions on $\bar{u} = \bar{u}(\bar{x}, \bar{t})$ are given by

$$\bar{u}(0,\bar{t}) = \bar{u}(1,\bar{t}) = 0. \tag{4}$$

2.2 Application of the Galerkin method

The Galerkin method is used to obtain $\bar{u} = \bar{u}(\bar{x}, \bar{t})$ satisfying Eqs. (2) and (4). Assuming the solution of the form

$$\bar{u}(\bar{x},\bar{t}) = \bar{U}(\bar{x})e^{\lambda \bar{t}} \tag{5}$$

and substituting into Eq. (2) yield the following characteristic equation

$$(\lambda^2 + \kappa^2)\bar{U}(\bar{x}) - \frac{1}{\pi^2}\bar{U}''(\bar{x}) + (\mu\lambda^2 + \zeta_P \bar{\omega}_\kappa \lambda)\bar{U}(\bar{x})\delta(\bar{x} - 1/2) = 0. \tag{6}$$

Here, λ is an eigenvalue whose real part indicates the rate of free vibration decay per dimensionless unit time.

When at least one of the concentrated mass or the damper is attached, the slope of the string becomes discontinuous at $\bar{x} = 1/2$ since the coefficient of Delta function in Eq. (6) has a nonzero value. Considering this discontinuity, solutions of the form

$$\bar{U}(\bar{x}) = c_1\bar{U}_1(\bar{x}) + \sum_{m=1}^{n}\{c_{2m}\bar{U}_{2m}(\bar{x}) + c_{2m+1}\bar{U}_{2m+1}(\bar{x})\} \tag{7}$$

are assumed in the Galerkin Method. Here, $\bar{U}_1(\bar{x})$, $\bar{U}_{2m}(\bar{x})$ and $\bar{U}_{2m+1}(\bar{x})$ are the following shape functions:

$$\bar{U}_1(\bar{x}) = \sin \pi\bar{x}, \tag{8}$$

$$\bar{U}_{2m}(\bar{x}) = \begin{cases} \sin(2m\pi\bar{x}) & (\bar{x} \le 1/2) \\ 0 & (\bar{x} > 1/2), \end{cases} \tag{9}$$

$$\bar{U}_{2m+1}(\bar{x}) = \begin{cases} 0 & (\bar{x} \le 1/2) \\ -\sin(2m\pi\bar{x}) & (\bar{x} > 1/2), \end{cases} \tag{10}$$

where, $m = 1, 2, \cdots, n-1, n$. Figure 2 illustrates the shape of these functions. As shown in the figure, slope of the curves in $\bar{U}_{2m}(\bar{x})$ and $\bar{U}_{2m+1}(\bar{x})$ is discontinuous at $\bar{x} = 1/2$, which allows precise representation of the localized mode and consequently accurate calculation of the eigenvalue λ. Furthermore, $\bar{U}_1(\bar{x})$ in Eq. (7) is necessary to allow the string to have a nonzero amplitude at $\bar{x} = 1/2$,

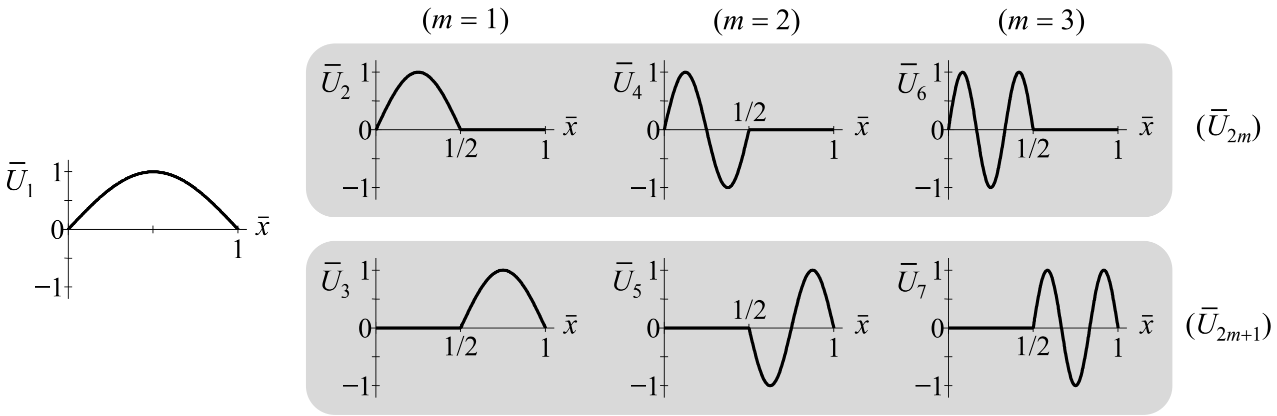


Figure 2: Functions $\bar{U}_p(\bar{x})$ $(p = 1, 2, ,\cdots, 2n+1)$.

since both $\bar{U}_{2m}(\bar{x})$ and $\bar{U}_{2m+1}(\bar{x})$ are zero at $\bar{x} = 1/2$. $\bar{U}(\bar{x})$ given by Eq. (7) satisfies the boundary conditions in Eq. (4), since $\bar{U}_1(\bar{x})$, $\bar{U}_{2m}(\bar{x})$ and $\bar{U}_{2m+1}(\bar{x})$ are all zero at $\bar{x} = 0$ and $\bar{x} = 1$.

Eigenvalue λ and coefficients $c_1, c_2, \cdots, c_{2n+1}$ are determined from following $2n+1$ equations:

$$\int_0^1 \left[(\lambda^2 + \kappa^2)\bar{U}(\bar{x}) - \frac{1}{\pi^2}\bar{U}''(\bar{x}) + (\mu\lambda^2 + \zeta_P \bar{\omega}_\kappa \lambda)\bar{U}(\bar{x})\delta(\bar{x} - 1/2) \right] \bar{U}_p(\bar{x}) d\bar{x} = 0$$

$$(p = 1, 2, \cdots, 2n+1). \qquad (11)$$

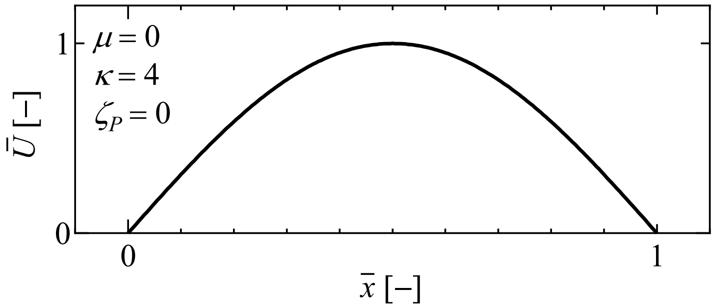
Then substituting the values of λ and $c_1, c_2, c_3, \cdots, c_{2n+1}$ into Eqs. (5) and (7) yields the string deflection $\bar{u}(\bar{x}, \bar{t})$.

The larger the value of n in the Galerkin Method, the more accurate the calculated results become. In this paper $n = 50$ is chosen by trial and error. All calculations in the following chapter are made for $\kappa = 4$ which is randomly chosen, and the values of μ and ζ_P are shown together with each calculated results.

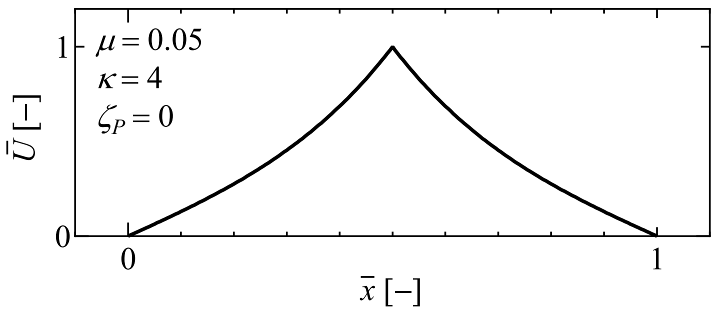
3. Results

Effects of a concentrated mass on the mode shape in undamped free vibration is shown in Fig. 3. The figures illustrate the first mode shape of the string (a) without a concentrated mass and (b) with a concentrated mass of $\mu = 0.05$. We see that the vibration mode is a half sine wave in Fig. 3(a) which is the first mode shape of the string under the condition of $\mu = \zeta_P = 0$ where neither of the concentrated mass nor the damper is attached. In contrast to this, the vibration mode in Fig. 3(b) is strongly localized at $\bar{x} = 1/2$ where the concentrated mass is attached, showing a dramatic change from the mode shape in Fig. 3(a). As described in Section 2.2, the slope of the string becomes discontinuous due to the existence of Delta function when at least one of μ or ζ_P is not zero. It is seen in Fig. 3(b) that the Galerkin Method formulated by Eqs. (6)-(11) represents well this discontinuity at $\bar{x} = 1/2$.

For reference, Figs. 4(a) and 4(b) illustrate the first mode shapes of the string in the condition when distributed stiffness is removed from Figs. 3(a) and 3(b), respectively. We see that the vibration mode is not strongly localized in the absence of distributed stiffness, even in the case of $\mu = 0.05$. It should be noted that the slope of the string is discontinuous at $\bar{x} = 1/2$ in Fig. 4(b) since μ has nonzero value, but this discontinuity is so small that it produces only a minor change in the mode shape from that shown in Fig. 4(a).

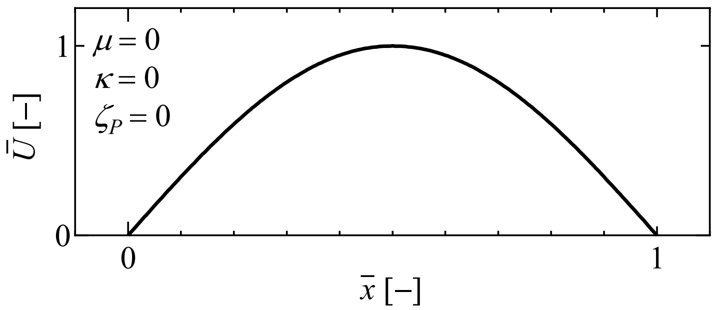


(a) String without a concentrated mass.

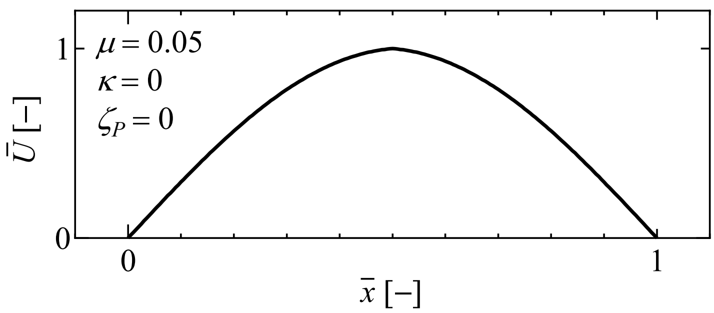


(b) String with a concentrated mass of $\mu = 0.05$ at its centre.

Figure 3: Mode of the string in case where distributed stiffness is present.



(a) String without a concentrated mass



(b) String with a concentrated mass of $\mu = 0.05$ at its centre.

Figure 4: Mode of the string in case where distributed stiffness is absent.

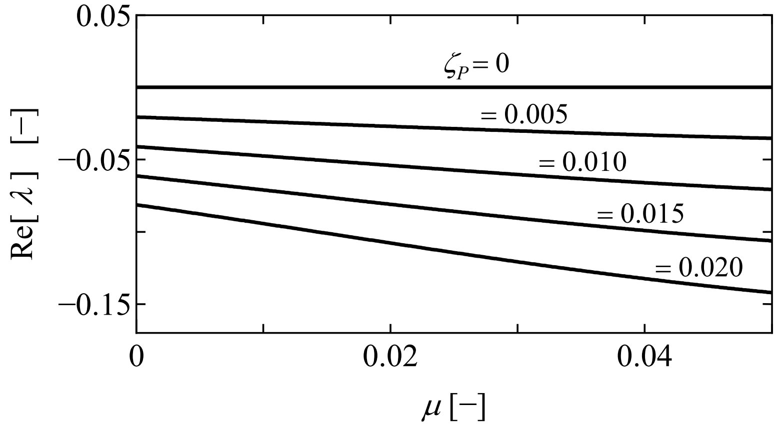


Figure 5: Effect of a concentrated mass and damper on the damping performance for the first mode.

Table 1: Improvement factor $\mathrm{Re}[\lambda]/\mathrm{Re}[\lambda\,|_{\mu=0}]$ for various values of μ and ζ_P.

Non dimensional concentrated mass μ	Damping strength ζ_P			
	0.005	0.010	0.015	0.020
0.00	1.00	1.00	1.00	1.00
0.02	1.31	1.32	1.32	1.33
0.04	1.60	1.61	1.62	1.63

Figure 5 shows the effect of the concentrated mass μ on the rate of free vibration decay for several values of damper strength ζ_P. The ordinate $\mathrm{Re}[\lambda]$ of the figure represents the real part of the eigenvalue λ for the first mode. In the case of $\zeta_P = 0$, the value of $\mathrm{Re}[\lambda]$ is zero independently of μ since in this case there is not any damping component in the system. When ζ_P has a positive value, the value of $\mathrm{Re}[\lambda]$ decreases with increasing μ, which means that the occurrence of mode localization is effective for improving the performance of the viscous damper.

As for the effect of the damper strength, increasing ζ_P decreases $\mathrm{Re}[\lambda]$ in Fig. 5, intensifying the decreasing trend of $\mathrm{Re}[\lambda]$ with increasing μ. To examine the effect of ζ_P more closely, the ratio of $\mathrm{Re}[\lambda]$ at $\mu \neq 0$ to that at $\mu = 0$ was calculated as an improvement factor for damping performance and shown in Table 1. It is seen that the dependence of the improvement factor on ζ_P is small whereas the concentrated mass μ has a great influence. The 4 percent of the concentrated mass improves the damping rate of free vibration by a factor of approximately 1.6.

4. Conclusion

The effect of vibration localization on the damping performance has been theoretically investigated using a uniform tensioned string with an added concentrated mass and a viscous damper at its centre. The Galerkin Method has been formulated to calculate accurately the localized mode shape as well as the eigenvalue of the system whose real part indicates the damping rate of free vibration.

The results have demonstrated that the vibration mode is strongly localized by attaching the concentrated mass when the string is coupled to ground through distributed stiffness. In contrast, the vibration mode is not localized when at least one of the concentrated mass or the distributed stiffness is absent.

The calculation of the eigenvalue for the first mode has also been conducted in a certain range of the concentrated mass and for several values of the damping strength. It has been clarified that the value of the real part of the eigenvalue decreases with the increase of the concentrated mass, which means the occurrence of the mode localization improves the performance of the viscous damper.

The ratio of damping rate between the cases with and without the concentrated mass has been defined as an improvement factor for damping performance. The results show that the concentrated mass has a great influence on the improvement factor, indicating the 4 percent of the concentrated mass improves the damping rate of free vibration by a factor of approximately 1.6.

Acknowledgments

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