

# MODELLING METHOD AND VIBRATION REDUCTION OPTIMIZATION FOR PERIODIC MASS-SPRING AUXETIC CELLULAR MOUNT

Haoxing Qin, Deqing Yang

*Collaborative Innovation Center for Advanced Ship and Deep-Sea Exploration, Shanghai Jiao Tong University, Shanghai, China*

*email: qinhaoxing@163.com*

Periodic structure performs well in vibration reduction, especially for solving structure vibration problems in low-frequency range. In this paper, the periodic mass-spring structure is filled into auxetic cellular mount, which mount has good vibration reduction performance in the range of middle and high frequency, therefore a novel periodic mass-spring auxetic effect cellular honeycomb mount is designed. Then, two methods of finite element modelling for this novel mount are discussed, whose finite element models are constructed to simulate this novel mount by plate element and beam element, respectively. Creating the parameters of the periodic mass-spring structure as optimization variables, two models with optimal vibration reduction performance are calculated by optimization analysis, and the better modelling method of this novel mount is determined by comparing the vibration reduction performance of the two models. Finally, band gap characteristics of the optimized novel mount is calculated by dynamic frequency response. Numerical optimization results show that the novel periodic mass-spring auxetic cellular mount has good performance for low-frequency vibration reduction.

Keywords: periodic mass-spring; vibration reduction; auxetic; cellular mount; optimization

---

## 1. Introduction

As the natural frequency of modern large ship is low, and the peak value of dynamic load of ship equipment is mainly concentrated in low-frequency range, it will cause the resonance and amplitude of vibration response. Mount is an important structure to support ship power device, and it can be used to reduce the vibration through structure design, and the common methods of vibration reduction include isolation, absorption and damping technology and so on [1].

Honeycomb material has the characteristics of high porosity and low density, and with a wide range of applications in the structure's lightweight and vibration reduction noise [2]. Banerjee studied the free vibration characteristics of a honeycomb structure without excitation using an equivalent continuum model [3]. Hayes used the micropolar theory to study the dynamic deformation of the honeycomb structure of the hexagonal honeycomb cell under vertical harmonic excitation [4]. Yang proposed an auxetic cellular mount, which has the good performance of vibration reduction and shock resistance [5].

In view of the periodic mass-spring structure can be used to suppress the phenomenon of low-frequency vibration, a novel periodic mass-spring auxetic cellular mount is proposed by combining the periodic mass-spring with auxetic cellular mount together. Penny simulated the localized resonant bandgap characteristics in periodic plates and verified the results in experiments [6]. Diaz studied the vibration bandgap of periodic Timoshenko rods in periodic mass-spring [7]. For the low-frequency vibration reduction of periodic mass-spring, Liu proposed the concept of local resonant band gap of periodic mass-spring, which laid a theoretical foundation for the application of periodic mass-spring

structure in low-frequency vibration and noise reduction [8]. Wen applied the local resonant bandgap properties of periodic mass-spring in the field of mechanical vibration control, that is, spring-mass oscillator is used to further improve the local resonant bandgap of the simplified model [9].

In this paper, the periodic mass-spring auxetic cellular mount is proposed, and the size parameters of phononic crystal are optimized to obtain the structure with good vibration reduction performance in the range of low-frequency. And finite element analysis results are calculated by the Hyperworks software.

## 2. Structural design of periodic mass-spring auxetic cellular mount

Periodic mass-spring structure is consisted of three parts: matrix, elastomer and scatterer. Based on the structure of auxetic cellular mount, the periodic mass-spring auxetic cellular mount is designed by embedding periodic mass-spring into cellular cell. Auxetic cellular cell is used as a matrix, and each monocellular is filled with rubber (as a scatterer) and plumbum (as an elastomer).

In this paper, the periodic mass-spring auxetic cellular vibration reduction system, as shown in Figure 2, is consisted of two parts: auxetic cellular mount and steel grillage. The size definition of periodic mass-spring auxetic cellular cell is shown in Figure 1,  $B$  is the width of the cell,  $H$  is the height of the cell,  $\theta = 38.2^\circ$  is the concave angle of the cell,  $b$  is the width of the scatterer,  $d$  is the height of scatterer, and  $h$  is the thickness of elastomer, the thickness of both the mount's plate and steel grillage is 6 mm, the thickness of the cellular is 1mm.

The grillage is reinforced by  $20 \times 3$  angle steel and  $50 \times 50 \times 5 \times 7$  T-stub steel, the material is isotropic and the properties of density, Young's modulus, and Poisson's ratio are  $\rho_s = 7800 \text{ kg/m}^3$ ,  $E = 200 \text{ GPa}$ , and  $\nu_s = 0.27$ , respectively. The rubber's elastic modulus is  $E_r = 1 \text{ MPa}$ , density is  $\rho_s = 1300 \text{ kg/m}^3$ . The plumbum's elastic modulus is  $E_l = 40.8 \text{ GPa}$ , density is  $\rho_l = 11600 \text{ kg/m}^3$ .

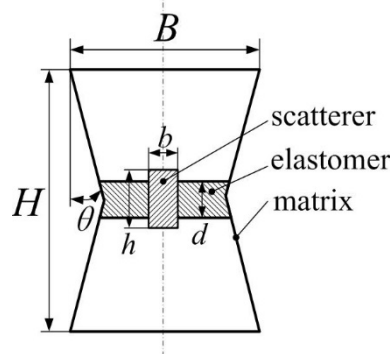


Figure 1 Cell structure of periodic mass-spring auxetic cellular mount

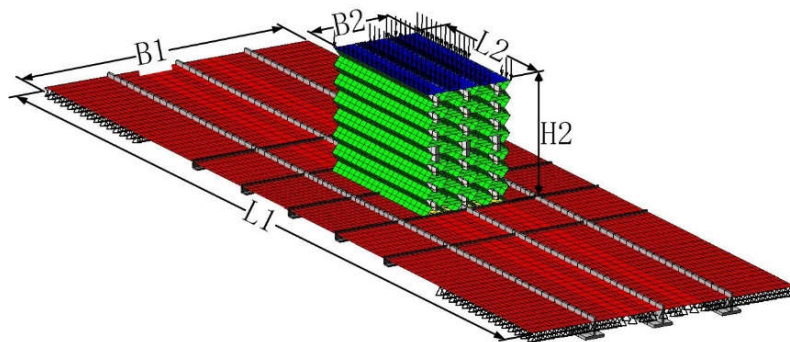


Figure 2 Periodic mass-spring auxetic cellular mount vibration reduction system, where  $L1=2000\text{mm}$ ,  $B1=710\text{mm}$ ,  $L2=400\text{mm}$ ,  $B2=224\text{mm}$ ,  $H2=300\text{mm}$

The weight of the machinery device is 500kg, which is applied at the upper plate of the mount. The vertical harmonic excitation force is applied at the upper plate of the mount, the magnitude of

force is 1N, and the frequency range is 10~500Hz, grillage is simply supported on two edges. The finite element model of the periodic mass-spring auxetic cellular mount is shown in Figure 2.

### 3. Measuring method for vibration reduction system

The concept of VLD (vibration level difference) is introduced to measure the dynamics response of periodic mass-spring auxetic cellular mount. Six measuring points are arranged under the steel grillage, as shown in Figure 3.

Decibels (dB) is often used to measure vibration response in engineering. In the whole frequency range (10 ~ 500Hz), the total VLD of  $p$ -th measuring point can be expressed as follows:

$$L_p^{all} = 10 \lg \left( \sum_{i=1}^N 10^{0.1 L_p^i} \right) \quad (\text{dB}). \quad (1)$$

where,  $L_p^i = 20 * \lg(a_p^i / 10^{-6})$  is the total VLD of  $i$ -th frequency point,  $N$  is the total number of frequency points in the calculated frequency band.

If there are  $M$  measuring points, then the formula of mean VLD is

$$\bar{L} = 10 \lg \left( \frac{1}{M} \sum_{p=1}^M 10^{0.1 L_p^{all}} \right) \quad (\text{dB}). \quad (2)$$

where,  $\bar{L}$  is the mean VLD of  $M$  measuring points,  $L_p^{all}$  is the total VLD of  $p$ -th measuring point.

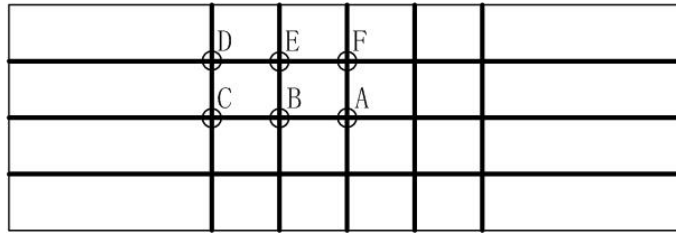


Figure 3 Layout of measuring points in the steel grillage.

### 4. Modelling of finite element and vibration reduction optimization

Optimization method is used to obtain the optimal parameters of periodic mass-spring, optimization design variables of the periodic mass-spring are given as follows:  $b$  is the width of the scatterer,  $h$  is the thickness of the scatterer,  $d$  is the thickness of the elastomer. As the vibration reduction performance is influenced by the periodic mass-spring parameters of each cell, therefore the design variables of the periodic mass-spring are distributed from top to bottom in order,  $b_1, \dots, b_7$  are the width variables of the scatterer,  $h_1, \dots, h_7$  are the thickness variables of the scatterer,  $d_1, \dots, d_7$  are the width variables of elastomer. The upper limit of the maximum equivalent stress constraint is used to ensure the requirement of strength, and the maximum mean VLD of the measuring points is taken as the objective function.

Vibration reduction optimization model of this mount is

$$\begin{cases} \text{Find} & \mathbf{b} = \{b_1, \dots, b_7\}^T; \mathbf{h} = \{h_1, \dots, h_7\}^T; \mathbf{d} = \{d_1, \dots, d_7\}^T \\ \text{Max} & \bar{L} \\ \text{s.t.} & \sigma \leq \sigma_{\max}, \underline{b}_j \leq b_j \leq \bar{b}_j; \underline{h}_j \leq h_j \leq \bar{h}_j; \underline{d}_j \leq d_j \leq \bar{d}_j \quad (j=1, 2, \dots, 7) \end{cases} \quad (3)$$

where, scatterer's width within the range of  $3 \text{ mm} \leq b_1, \dots, b_7 \leq 10 \text{ mm}$ , scatterer's thickness within the range of  $10 \text{ mm} \leq h_1, \dots, h_7 \leq 40 \text{ mm}$ , elastomer's thickness within the range of  $5 \text{ mm} \leq d_1, \dots, d_7 \leq 30 \text{ mm}$ , the upper limit of maximum equivalent stress of mount is  $\sigma_{\max} = 150 \text{ MPa}$ .

In this paper, two finite element models of dynamic analysis are studied. Finite element analysis are calculated by Hyperworks software, modal analysis and frequency response analysis are also calculated in this software. Correspondingly, vibration reduction performance of the two finite element models are calculated, respectively.

#### 4.1 Method I

ModelII is constructed by the simplified modelling method I, which ignores the influence of width of each scatterer in the periodic mass-spring, and the Standard Quad4 element type is used to simulate the scatterer, auxetic cellular cell and elastomer in the finite element model. As the width values of scatterer are fixed as  $b_1, \dots, b_7 = 8.3 \text{ mm}$ , that is, the variables of Model-I are only the scatterer's thickness and the elastomer's thickness.

Optimization is calculated by using the modelling method I, iteration curves of the design variables (the thickness of the scatterer and the thickness of the elastomer) are shown in Figure 4 and Figure 5, respectively. The curve of the objective is shown in Figure 6, the mass curve of vibration reduction system is shown in Figure 7.

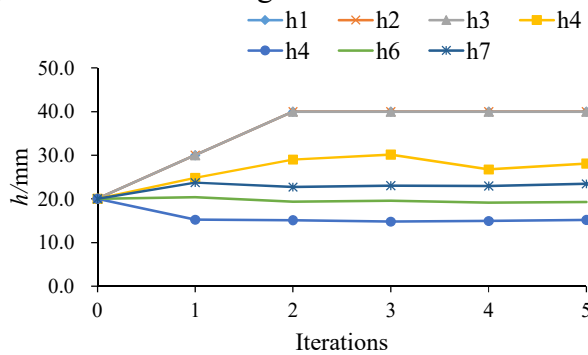


Figure 4 Iteration curve of scatterer thickness.

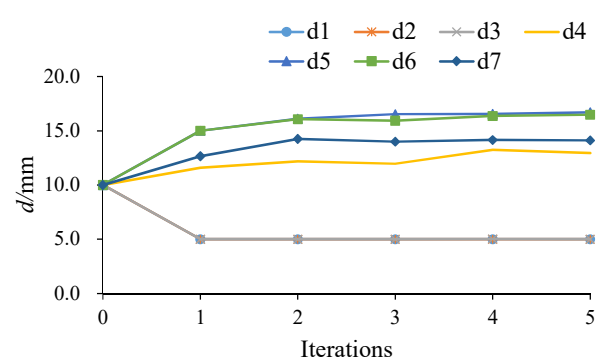


Figure 5 Iteration curve of elastomer thickness.

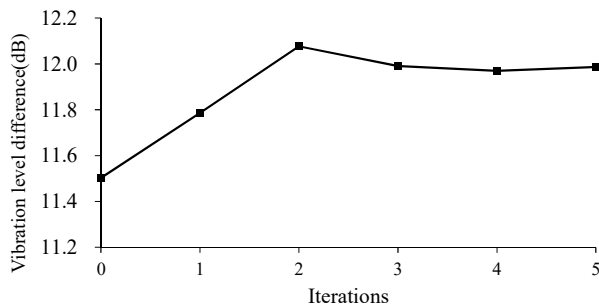


Figure 6 Iteration curve of objective.

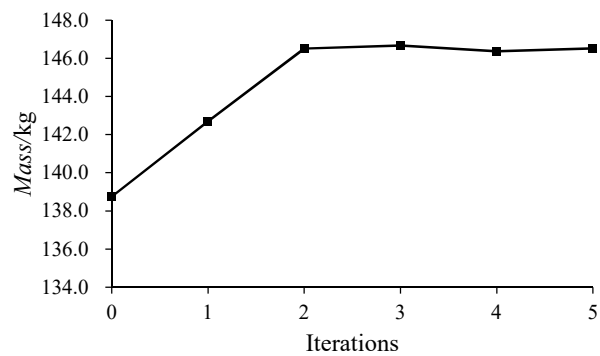


Figure 7 Iteration curve of mass.

After extracted the optimization results of this periodic mass-spring auxetic cellular mount, modal analysis is calculated, the first order vertical vibration nature frequency of this reduction system is 25.12Hz. The analysis results, as in Figure 4 and Figure 5, show that the size parameters of each periodic mass-spring are quite different, imply that the vibration reduction performance can be better realized by setting the different design variables of each cell. The optimized variables' value are shown in Table.1. The mass of the optimized mount is 21.77kg which is higher than the auxetic cellular system without adding periodic mass-springs, and the mean VLD of measuring points reaches to 11.90dB in the range of 10~500Hz. In the low frequency range of 10 ~ 100Hz, the mean VLD is 12.27dB, indicating that the novel mount has a good vibration reduction performance in the range of low-frequency.

Tab.1 Optimal value of design variables (mm)

Initial value of design variables: $h_1=h_2=\dots=h_7=20$ mm, $d_1=d_2=\dots=d_7=10$ mm						
$h1$	$h2$	$h3$	$h4$	$h5$	$h6$	$h7$
40.0	40.0	40.0	28.1	15.1	19.3	23.4
$d1$	$d2$	$d3$	$d4$	$d5$	$d6$	$d7$
5.0	5.0	5.0	12.9	16.7	16.5	14.1

## 4.2 Method II

The finite element model II is constructed with modelling method II, and the design variables in modelling method II concluded three types,  $b_1, \dots, b_7$  is scatterer's width and  $h_1, \dots, h_7$  is scatterer's thickness, and  $d_1, \dots, d_7$  is elastomer's thickness. The finite element model of the scatterer can be constructed by the CBARL finite element, and the element type of the elastomer is constructed by the Standard Quad4 element type.

Optimization is calculated by the modelling method II, the results are as follows. The iteration curves of the design variables (the width and thickness of the scatterer, the thickness of the elastic body) are shown in Figure 8, Figure 9 and Figure 10, respectively. The variation curve of the objective is shown in Figure 11. The mass curve of vibration reduction system is shown in Figure 12.

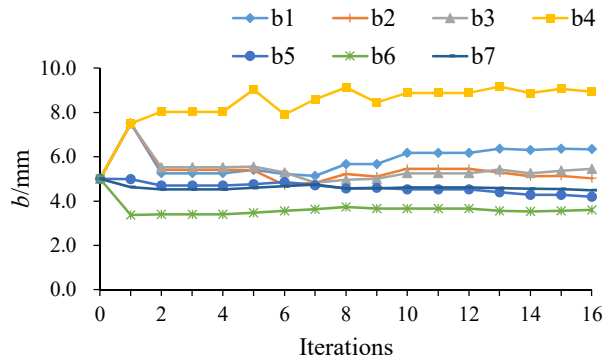


Figure 8 Iteration curves of scatterer's width.

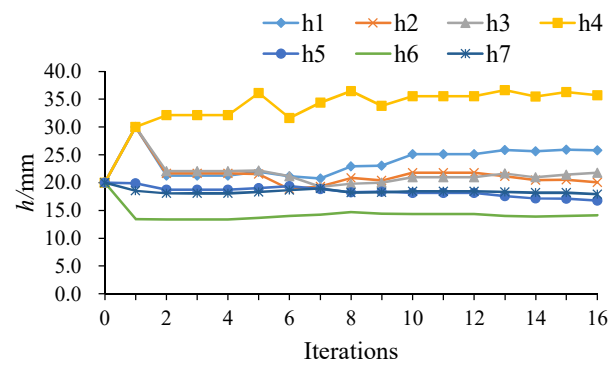


Figure 9 Iteration curves of scatterer's thickness.

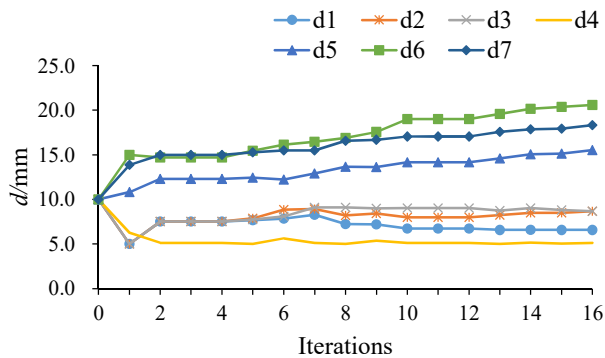


Figure 10 Iteration curves of elastomer's thickness.

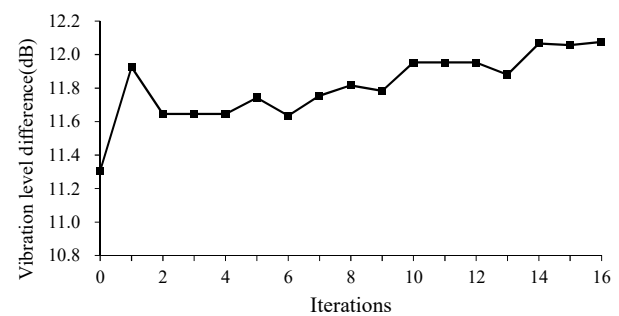


Figure 11 Iteration curve of objective.

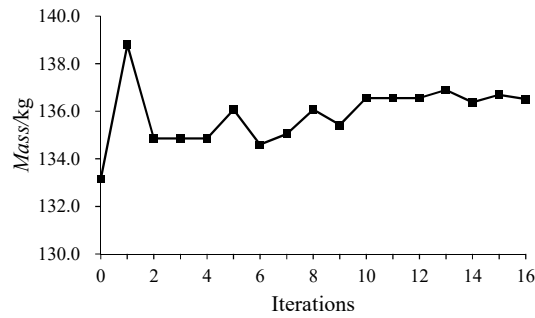


Figure 12 Iteration curve of mass.

Tab.2 Optimal value of design variables (mm)

<i>b1</i>	<i>b2</i>	<i>b3</i>	<i>b4</i>	<i>b5</i>	<i>b6</i>	<i>b7</i>
6.3	5.0	5.5	8.9	4.2	3.6	4.5
<i>h1</i>	<i>h2</i>	<i>h3</i>	<i>h4</i>	<i>h5</i>	<i>h6</i>	<i>h7</i>
25.8	20.1	21.7	35.7	16.8	14.2	17.9
<i>d1</i>	<i>d2</i>	<i>d3</i>	<i>d4</i>	<i>d5</i>	<i>d6</i>	<i>d7</i>
6.6	8.6	8.7	5.1	15.5	20.6	18.3

After the optimization, the first order vertical vibration nature frequency of this novel mount is 24.04Hz. The analysis results in Figure 8, Figure 9 and Figure 10 show that the elastomer's width value in model II is very different from the fixed width value in the model I. One can conclude that the width of elastomer have great influence to the vibration reduction performance.

The optimized design variables are shown in Table 2. The mass of optimized mount system is 11.25kg higher than the auxetic cellular system without adding periodic mass-spring, the mean VLD of measuring points in the range of 10~500Hz is 12.08dB, and the mean VLD is 12.15dB in the low-frequency range of 10 ~ 100Hz.

### 4.3 Models comparison

Under the same initial conditions, optimization results show that the vibration reduction performance of the two modelling methods is similar. However, the weight is much lighter by using the modelling method II, which means the modelling method II can optimized the vibration reduction performance more effectively.

### 4.4 Analysis of band gap characteristic for periodic mass-spring auxetic cellular mount

After the optimization of periodic mass-spring auxetic cellular mount, acceleration frequency response is reanalyzed to plot the acceleration frequency response curve of the exciting point and the measuring points.

When the structural damping is considered, the acceleration frequency response curves are shown in Fig 13, which shows that the existence of structural damping can be used to wider the bandgap. The first wide band gap frequency range is 10 ~ 165Hz, the second bandgap range is 185 ~ 500Hz. The frequency response curves of acceleration show that the novel mount has a wide band gap in low-frequency, which can effectively suppress the low-frequency vibration problem.



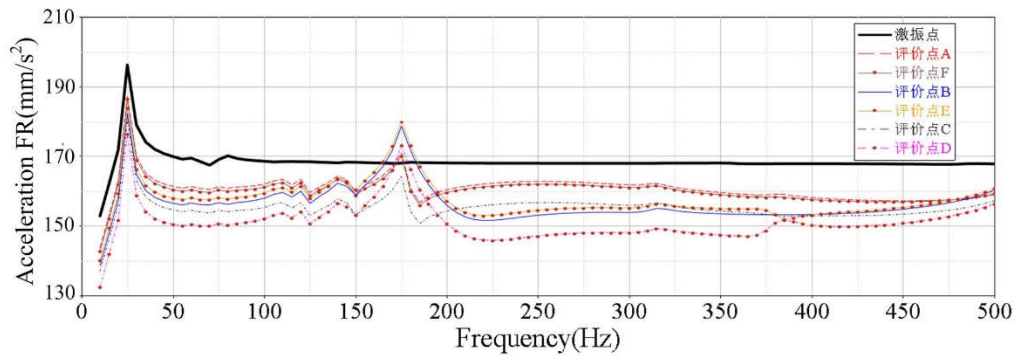


Figure 13 Acceleration frequency response curve with structural damping.

## 5. Conclusion

A novel periodic mass-spring auxetic cellular mount is designed by fill periodic mass-spring into conventional honeycomb structures, and the comparison of the two finite element models shows that the optimization effect of model II is better. The research of this paper provides a design method for vibration reduction.

## Acknowledgement

The support for this work, provided by the National Natural Science Foundation of China under Grant 51479115 is gratefully acknowledged.

## REFERENCES

- 1 Lv L H, Yang D Q. Study on Vibration Reduction Design of Steel-Composite Materials Hybrid Mounting for Ships [J]. Journal of Shanghai Jiaotong University. 2012, 46(8): 1196-1202.
- 2 Victor S, Tanchum W. On the feasibility of introducing auxetic behavior into thin-walled structures [J]. Acta Materialia. 2008, 57(1): 125-135.
- 3 Banerjee S, Bhaskar A. Free vibration of cellular structures using continuum modes [J]. Journal of Sound and Vibration, 2005, 287 (1-2): 77-100.
- 4 Hayes A M, Wang A J, Dempsey B M, *et al.* Mechanics of linear cellular alloys [J]. Mechanics of Materials, 2004, 36(8), 691-713.
- 5 Zhang X W, Yang D Q. A novel marine impact resistance and vibration isolation cellular base [J]. Journal of Vibration and Shock. 2015, 10: 40-45.
- 6 Pennec Y, Djafari-Rouhani B, Larabi H. *et al.* Low frequency gaps in a phononic crystal constituted of cylindrical dots deposited on a thin homogeneous plate [J]. Physical Review B, 2008, 78: 104105.
- 7 Diaz-de-Anda A, Pimentel A, Flores J, *et al.* Locally periodic Timoshenko rod: experiment and theory [J]. Journal of the Acoustical Society of America, 2005, 117(5): 2814-2819.
- 8 Liu Z, Zhang X, Mao Y, *et al.* Locally resonant sonic materials [J]. Science, 2000, 289(5485): 1734-1736.
- 9 Xiao Y, Mace B R, Wen J H, *et al.* Formation and coupling of band gaps in a locally resonant elastic system comprising a string with attached resonators [J]. Physics Letters A. 2011, 375(12): 1485-1491.