24th INTERNATIONAL CONGRESS ON SOUND AND VIBRATION
23–27 July 2017, London

ACOUSTIC CLOAK IN THE PRESENCE OF COMPRESSIBLE NON-UNIFORM MEAN FLOW

Hyeonbin Ryoo and Wonju Jeon

Department of Mechanical Engineering, Korea Advance Institute of Science and Technology (KAIST), 34141, Daejeon, Republic of Korea
email: rhb9306@kaist.ac.kr

For a decade, the acoustic cloak has attracted considerable research interests. Meanwhile, the effect of mean flow on the scattering pattern from acoustic cloak was rarely explored. As reported by previous investigations on the acoustic cloak in flow field, the existing cloak designs fail to preserve its original properties to make an object acoustically invisible. Recently, the authors studied that the failure reason of acoustic cloak is mainly due to effects of compressibility in fluid and non-uniformity of flow. Thus, to improve the performance of conventional acoustic cloak, we focused on how to suppress non-zero gradient of flow velocity. When we performed the numerical simulations for various thicknesses of acoustic cloak, the thinner acoustic cloaks had better performance than the thicker ones. This work is expected to provide an insight toward a design of perfect acoustic cloak in moving medium.
Keywords: acoustic cloak, non-uniform mean flow, effect of compressibility

1. Introduction

Over the past decade, research on acoustic cloak to make an object acoustically invisible has attracted a great deal of interest from engineers and scientists [1-4]. Although the acoustic cloak has been studied rapidly by using an analogy with optical cloak [5,6], the research on the acoustic cloak has a clear limit to be applied to practical application due to the differences of physical properties between acoustic and optical waves. Unlike the optical cloak, there can be a medium convection around the acoustic cloak. Thus, it is obvious that the existing acoustic cloak designs that do not consider the background flow will fail to hide an object when it is placed in a moving medium.

Recently, only a few researchers have considered the background flow effect around the acoustic cloak [2-4]. In 2013, Garcia-Meca et al. [2] studied analogue transformation formalism to design a ground acoustic cloak in a uniform flow field. In 2014, Huang et al. [3] proposed an analytic framework considering the uniform flow effect on the left side of the convective wave equation and considering the remaining non-uniform flow effect on the right side. However, there is no precedent research on acoustic cloak taking into account of the density inhomogeneity due to compressible flow around the cloaked objects.

Thus, in this paper, we discuss the effect of non-uniformity of flow and compressibility in fluid separately on scattering pattern of acoustic cloak by proposing a theoretical framework. Then, we improve the cloaking performance of existing convective cloaks [3] by making a thinner acoustic cloak or modifying the material properties of cloak.

In Section 2, we propose a theoretical framework for taking into account of the density inhomogeneity and velocity gradient around a cloaked object. In Section 3, numerical method is briefly stated to analyse the scattering patterns of acoustic cloak in moving medium by using the proposed framework. In Section 4, numerical results of scattering patterns are illustrated and discussed. In Section 5, the conclusion of this work will be briefly stated.

2. Theoretical Framework

In this section, we propose a theoretical framework to investigate density inhomogeneity and velocity gradient around the cloaked object. By dividing the equivalent source term into two parts, we discuss the physical meanings of each source term. When the background flow is considered as inviscid flow, the convective wave equation can be derived as follows:

$$\left(\frac{D_0^2}{D_0 t^2} - c_0^2\nabla^2\right)p' = S_{eq}(\mathbf{x},t) = S_{comp}(\mathbf{x},t) + S_{non}(\mathbf{x},t), \tag{1}$$

where
$$\begin{aligned} S_{comp}(\mathbf{x},t) = &-\rho_0 c_0^2\left[\mathbf{u}'\cdot\nabla + \frac{D_0}{D_0 t}\left(\frac{\gamma p'}{\rho_0 c_0^2}\right) + \frac{\gamma p'}{\rho_0 c_0^2}\frac{D_0}{D_0 t}\right](\nabla\cdot\mathbf{u}_0) \\ &-\rho_0 c_0^2\frac{D_0}{D_0 t}\left[\frac{1}{\gamma\rho_0}(\mathbf{u}'\cdot\nabla)\rho_0 + \frac{1}{\gamma c_0^2}(\mathbf{u}'\cdot\nabla)c_0^2\right] \\ &-c_0^2\frac{D_0 p'}{D_0 t}(\mathbf{u_0}\cdot\nabla)\left(\frac{1}{c_0^2}\right) - \rho_0\left[\frac{D_0 p'}{D_0 t}(\mathbf{u_0}\cdot\nabla) + \rho_0 c_0^2\nabla p'\cdot\nabla\right]\left(\frac{1}{\rho_0}\right), \end{aligned} \tag{2}$$

$$S_{non}(\mathbf{x},t) = \nabla\cdot\left[2(\mathbf{u}'\cdot\nabla)\mathbf{u}_0 + \frac{\rho'}{\rho_0}(\mathbf{u}_0\cdot\nabla)\mathbf{u}_0\right], \tag{3}$$

where $D_0/D_0 t$ denotes a total derivative defined by $\partial/\partial t + \mathbf{u_0}\cdot\nabla$, ρ is the density of the fluid, $\mathbf{u}$ is the particle velocity, t is the time, p is the pressure, c_0 is the speed of sound in air, and γ is ratio of specific heats. The physical variables, ρ, $\mathbf{u}$ and p, are divided into the fluctuating variables denoted by $(\cdot)'$ and background variables denoted by $(\cdot)_0$. In Eq. (1), the the non-uniform mean flow velocity is coupled with the differential operator. $S_{eq}(\mathbf{x},t)$ indicates the equivalent source terms due to the coupled effect of background flow with incident wave. Such mathematically complicated terms are divided into two parts with their own physical meanings: $S_{comp}(\mathbf{x},t)$ for compressibility in fluid (See Eq. (2)) and $S_{non}(\mathbf{x},t)$ for non-uniformity of flow (See Eq. (3)).

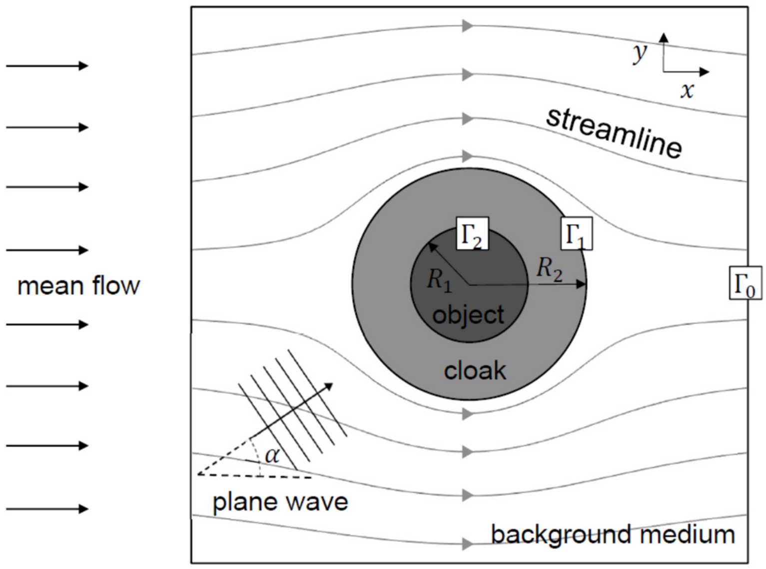

Figure 1: A schematic of acoustic cloak in potential flow field.

3. Numerical Method

In this section, by performing numerical simulations with finite element method (FEM), we analyse the scattering patterns of acoustic cloaks impinged by a plane wave in flow field. As shown in Fig. 1, a cylindrical object with radius $R_1(=1[m])$ is covered by convective cloak depicted as an annulus in the region of $R_1 < r < R_2(=2[m])$. The governing equation solved in numerical simulation is Eq. (1) and the boundary conditions are imposed as follows: the continuity condition of acoustic pressure p' and particle velocity $\mathbf{u}'$ on Γ_1, the acoustically rigid boundary condition on Γ_2, and artificially damping boundary condition on Γ_0 for avoiding unphysical reflections. To compare the performance of newly designed cloak, we used the previous convective cloak proposed in Ref. [3], whose material properties are written by

$$\frac{\rho^r(r)}{\rho_0} = \frac{r}{r-R_1}\frac{1}{1+M\,cos\alpha}, \qquad \frac{\rho^\theta(r)}{\rho_0} = \frac{r-R_1}{r}\frac{1}{1+M\,cos\alpha}, \qquad (4)$$
$$\frac{\kappa(r)}{\kappa_0} = \left(\frac{R_2-R_1}{R_2}\right)^2 \frac{r}{r-R_1}(1+Mcos\alpha),$$

where M is the Mach number of background flow, α is the angle between the incident wave and background flow, κ is the bulk modulus, and the superscripts of $(\cdot)^r$ and $(\cdot)^\theta$ represent the anisotropic properties of the acoustic cloak in r and θ directions, respectively. To evaluate the cloaking performance of previous and present convective cloaks, the directivity patterns of acoustic pressure are calculated at $r = 10R_1$ by using equation (5),

$$e(r,\theta) = 10\,log_{10}\frac{p'(r,\theta)^2}{p'^{\,2}_{inc}} \quad \text{at } r = 10R_1 \qquad (5)$$

where p'_{inc} is the amplitude of impinged acoustic wave. A large deviation from 0 dB represents more unwanted scattering occurred.

4. Results and Discussions

In this section, we analyse the numerical results of scattering patterns of convective cloaks in moving medium. Figure 2 shows contour plots and a directivity plot of acoustic pressure around the previous convective cloak covering a cylindrical object in the flow field with 0 and 0.2, where $kR_1 = 3$ and $\alpha = 0°$.

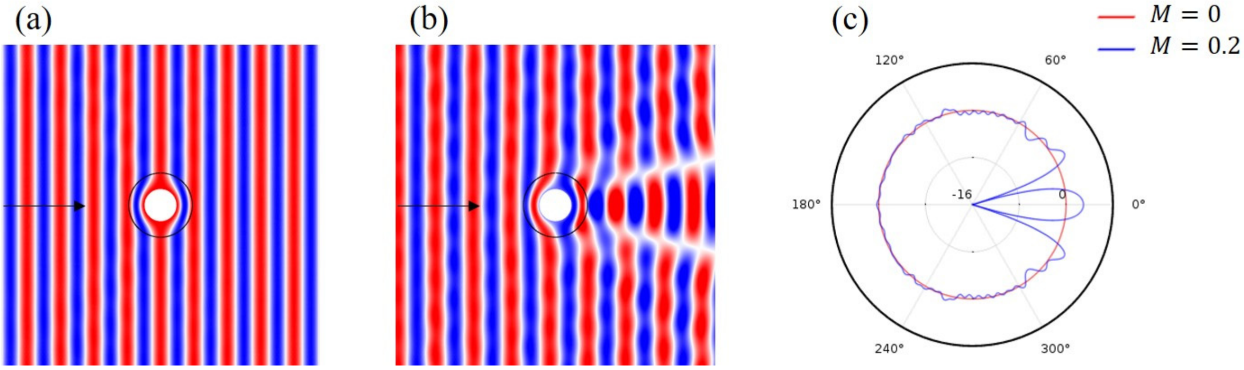

Figure 2: Scattering patterns of previous convective cloak in (a) stationary and (b) moving medium. (c) Comparison of directivity patterns.

As shown in Fig. 2(a), the acoustic cloak can hide the object almost perfectly. However, as shown in Fig. 2(b), it can be seen that the acoustic cloak loses its unique property in presence of background flow. In addition, as shown in Fig. 2(c), the failure of conventional acoustic cloak in flow can be clearly seen in the geometrical shadow zone (forward scattering region).

The unwanted scattering from an acoustic cloak in the presence of flow is mainly due to the velocity gradient and density inhomogeneity around the cloak. Therefore, an easy way to reduce the scattering is to change the thickness of the cloak. The effect of thickness of the cloak is important in presence of flow while it is not in absence of flow. In a moving medium of $M = 0.2$, we performed a parametric study on the scattering patterns by changing the thickness of acoustic cloak. As shown in Figs. 3(a) and 3(b), the convective cloak with reduced thickness shows better performance. In addition, the thinner cloak is, less scattering occurs in geometrical shadow zone as shown in Fig. 3(c).

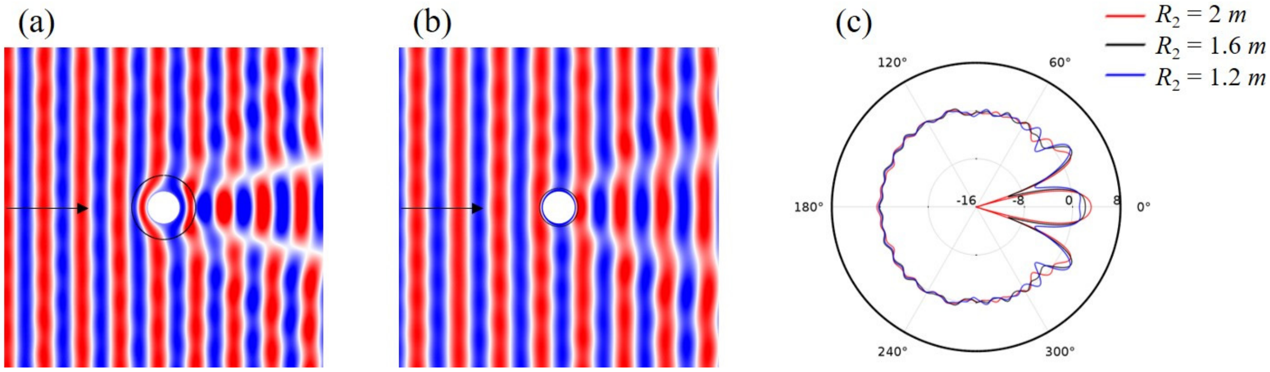

Figure 3: Scattering patterns of convective cloaks in moving medium with $M = 0.2$ where (a) $R_2 = 2R_1$ and (b) $R_2 = 1.2\ R_1$. (c) Comparison of directivity patterns for cloak thicknesses of $R_2 = 2R_1$, $1.6R_1$, $1.2R_1$.

5. Conclusion

We proposed a theoretical framework to physically understand the scattering pattern of acoustic cloak in moving medium. In addition, the failure of previous convective cloak [3] near the shadow zone is numerically observed with this formulation. When we performed the numerical simulations for various thicknesses of acoustic cloak, the thinner acoustic cloaks had better performance than the thicker ones. Further, to reduce the unwanted scattering more than the cloak with simply reduced thickness, we can modulate the material properties of the convective cloak. We expect that this work can provide an insight toward a design of perfect acoustic cloak in moving medium.

REFERENCES

1 Cummer, S. A. and Schurig, D., One path to acoustic cloaking, *New Journal of Physics*, **9** (3), 45, (2007).

2 Garcia Meca, C., Carloni, S., Barcel, C., Jannes, G., Sanchez-Dehesa, J. and Martinez A., Analogue transformations in physics and their application to acoustics, *Scientific Reports*, **3**, (2013).

3 Huang, X., Zhong, S. and Stalnov, O., Analysis of scattering from an acoustic cloak in a moving fluid, *The Journal of Acoustical Soceity of America*, **135** (5), 2571-2580, (2014).

4 Iemma, U., Theoretical and numerical modelling of acoustic metamaterials for aeroacoustic applications, *Aerospace*, **3** (2), 15 (2016).

5 Pendry, J. B., Schurig, D. and Smith, D. R., Controlling electromagnetic fields, *Science*, **312** (5781), 1780-1782, (2006).

6 Leonhardt, U., Optical Conformal Mapping, *Science*, **312** (5781), 1777-1780, (2006).