

# A LINE-SPECTRUM ACTIVE NOISE CONTROL SYSTEM WITH MULTI-FREQUENCY ESTIMATION BASED ON PARALLEL ADAPTIVE NOTCH FILTER

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Active noise control (ANC) shows excellent performance for low-frequency periodic noise reduction rather than traditional passive techniques. In some line-spectrum ANC (LANC) cases, non-acoustic reference signals are used to remove the secondary feedback. However, this method often leads to frequency mismatch especially for those sources with time-varying frequencies. In order to obtain accurate reference signal, a line-spectrum active noise control system with frequency estimation is adopted in this work. To be more specific, a parallel adaptive notch filter (PANF) algorithm is proposed to estimate the frequencies while the reference signals are generated in the form of sinusoidal. The proposed frequency estimation algorithm can track multiple independent frequencies at the same time with high accuracy and fast convergence as well as triangular cascaded ANF algorithms while the computation complexity has been reduced to linear cascaded ANF's level. Moreover, results of extensive computer simulations and pipe structure ANC experiments are given and analyzed.

Keywords: line-spectrum ANC, frequency estimation, ANF, cascaded ANF, PANF

#### 1. Introduction

Nowadays, active noise control methods have been widely investigated and are coming into use for its excellent low-frequency properties [1-4]. As a special narrowband case, line spectrum ANC system plays an important role in mechanical noise elimination involving engine, fan system and other circumstances. For tonal noises, reference signals are often obtained in non-acoustic ways such as revolving speed, vibration signal and even a generated sinusoidal one [5, 6]. Since the secondary feedback could be removed, these LANC systems with non-acoustic reference signals work well when the primary source is a stable one with fixed frequencies. In contrast, for unstable sources especially those with time-varying frequencies, frequency mismatch (FM) often results in poor control performance [7].

To solve the problem caused by FM, several improved ANC systems with frequency estimation algorithms have been proposed in [8-11]. Those frequency estimation algorithms based on frequency spectrums (e.g. MVDR iteration frequency estimation algorithm [10]) converge fast, but they suffer from high computation complexity and time delay for their dependence on autocorrelation matrix (or its inverse matrix). In [8], a linear cascaded adaptive notch filter (ANF) system has been proposed to estimate the harmonic frequency online and generate the reference signal. Further, this technique has been applied to control the primary source with several independent time-varying frequencies [9]. However, it has been shown that the result of cascaded ANF algorithm with a linear structure is biased [12], which dramatically increases the mean-square-error (MSE) of ANC system. To track multiple frequencies accurately, a triangular cascaded ANF algorithm [13] is widely used. However, the complicated triangular structure leads to higher computation burden than a linear one.

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In this work, we propose a line-spectrum ANC system with a parallel-structure ANF (PANF) algorithm to acquire the unbiased estimation of frequencies. The PANF algorithm separates multiple frequencies by constructing several estimated signals of primary source, and each of them contains one target frequency. Then the adaptive filter process of the ANC system is proposed as conventional FxLMS algorithm while the reference signal is generated as a sinusoidal one, and the MSE of ANC system is reduced as the FM has been minimized. Simulations and experiments are carried out to validate its performance.

# 2. Line-spectrum ANC system with linear cascaded ANF

One type of conventional line-spectrum ANC (LANC) system can be illustrated in Fig.1 [1, 6-7]. In this work, we assume that the primary noise signal has K frequencies and its form can be expressed as:

$$d(n) = \sum_{i=1}^{K} \left[ a_{di} \cos(\omega_{di} n) + b_{di} \sin(\omega_{di} n) \right] + v_d(n)$$
(1)

where  $\omega_{di}$  is the i-th component frequency while  $a_{di}$  and  $b_{di}$  are the amplitudes of cos- and sin- term of each frequency, and the term  $v_d(n)$  is a white Gaussian noise with zero-mean and variance  $\sigma_v^2$ . The reference signal x(n) can be written in the following form:

$$x(n) = \sum_{i=1}^{K} [x_{ai}(n) + x_{bi}(n)]$$
 (2)

$$x_{ai}(n) = \cos(\omega_{di}n)$$
  

$$x_{bi}(n) = \sin(\omega_{di}n)$$
(3)

where  $\cos(\omega_{di}n)$  and  $\sin(\omega_{di}n)$  can be easily generated as the frequency  $\omega_{di}$  is known. The output can be expressed as

$$y(n) = \sum_{i=1}^{K} [y_{ai}(n) + y_{bi}(n)]$$
 (4)

where  $y_{ai}$  and  $y_{bi}$  is updated using the FxLMS algorithm

$$y_{ri}(n) = w_{ri}(n) \cdot x_{ri}(n) \tag{5}$$

$$W_{ri}(n) = W_{ri}(n-1) + 2\mu \cdot e(n) \cdot u_{ri}(n)$$
(6)

where  $w_{ri}(n)$  is the filter weight for each frequency and e(n) is the error signal obtained by the error sensor. In order to simplify the expression, r is used to represent a or b. In this work, we assume that the secondary path  $H_i(n)$  is independent for each frequency channel and secondary multi-tone signals linearly superpose at the error point. To be more specific,  $u_{ri}(n)$  stands for the reference signal  $x_{ri}(n)$  filtered by the estimation of secondary path  $\hat{H}_i(n)$  while  $y_i'(n)$  is the output of each signal passed through the secondary path  $H_i(n)$ . Then the physical progress of active noise reduction at the error point can be written as

$$y_S'(n) = \sum_{i=1}^K y_i'(n)$$
 (7)

$$e(n) = d(n) - y'(n) \tag{8}$$

where the real secondary source signal  $y'_s(n)$  is the sum of each  $y'_i(n)$ . The estimation of secondary path parameters is calculated by the secondary path modeling method. Thanks to avoiding the effect of secondary feedback, this type conventional line-spectrum ANC system enjoys good performance if the primary source signal is a stable one with fixed frequencies and the parameters of secondary path can be estimated accurately.

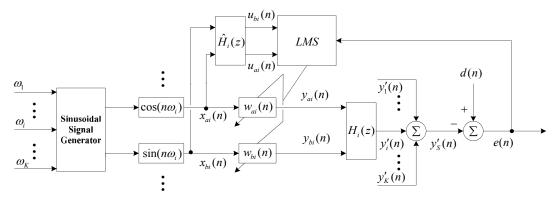


Figure 1: Conventional line-spectrum control system.

To deal with the primary noise source with time-varying frequencies, line-spectrum ANC systems with a frequency estimation or corrector algorithm have been investigated. One common technology is the cascaded ANF system with a linear structure. As shown in Fig.2, the primary signal  $\hat{d}(n)$  is estimated as the superposition of the estimated secondary signal  $\hat{y}'_{s}(n)$  and the error.

$$\hat{d}(n) = \hat{y}'_{S}(n) + e(n) = \sum_{i=1}^{K} \hat{y}'_{i}(n) + e(n)$$
(9)

Further, a linear cascaded ANF algorithm is used to estimate and track the frequencies of the primary signal and the reference signal is generated in the sinusoidal form just as the conventional LANC system.

The proposed ANF algorithm has a second-order notch filter with direct form [14],

$$H_N(z) = \frac{N(z)}{D(z)} = \frac{1 + k_0 z^{-1} + z^{-2}}{1 + \alpha k_0 z^{-1} + \alpha^2 z^{-2}}$$
(10)

where D(z) is an all-pole section while N(z) is an all-zero one, the parameter  $\alpha$  is the pole zero contraction factor which is positive but smaller than unit, and  $k_0$  is a variable to estimate the target frequency  $\omega$ . The adaptive iteration process is given as the dashed box in Fig.3. First, the target frequency of input of ANF is enhanced by D(z), and we define the result is  $\chi(n)$ . After that, the frequency estimation process is conducted as follow(the detailed algorithm is given in [12])

$$D(n) = \lambda D(n-1) + (1-\lambda)\chi^{2}(n-1)$$
(11)

$$C(n) = \lambda C(n-1) + (1-\lambda)\chi(n-1)[\chi(n) + \chi(n-2)]$$
(12)

$$C(n) = \lambda C(n-1) + (1-\lambda)\chi(n-1)[\chi(n) + \chi(n-2)]$$

$$\breve{k}_0(n) = -\frac{C(n)}{D(n)}$$
(13)

where D(n) and C(n) are intermediate variables both with given original values. The parameter  $\lambda$  is a forgetting factor between 0 and 1, and  $k_0(n)$  is the original estimation of  $k_0(n)$ . Then the smooth value  $k_0(n)$  and the estimation of frequency can be obtained as

$$k_0(n) = \gamma k_0(n-1) + (1-\gamma) \check{k}_0(n) \tag{14}$$

$$\hat{\omega} = \arccos[-k_0(n)/2] \tag{15}$$

where  $\hat{\omega}$  is the estimation of target frequency and  $\gamma$  is a smoothing factor between 0 to 1. Finally,  $\gamma(n)$  is filtered by N(z) to remove the sinusoidal signal with target frequency  $\hat{\omega}$ .

To track multiple independent frequencies at the same time, a series of ANFs are cascaded one by one in the linear structure [14] as illustrated in Fig.3. More precisely, the result of the i-th ANF is used as the input of (i+1)-th ANF.

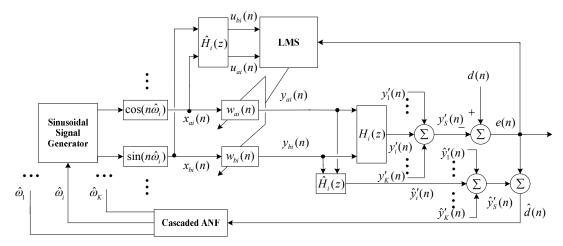


Figure 2: Line-spectrum control system with a cascaded ANF algorithm.

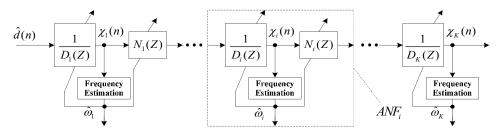


Figure 3: Linear cascaded ANF algorithm.

# 3. New LANC system with a PANF algorithm

Although the linear cascaded ANF algorithm is simple to understand and conduct, it has been proven that the estimation of frequencies is biased [12]. For those previous ANFs, tracking one certain frequency in the multi-frequency signal means poor performance because of the influence from other frequencies, and the estimated result is close to one of the real frequencies but biased. As a result, frequency mismatch degrades the performance of noise reduction severely, especially for systems under small step-size or inaccurate secondary-path modelling [7]. In order to obtain the unbiased estimation of multiple frequencies, a complicated triangular structure cascaded ANF algorithm has to be used, which results in tremendous computational burden [13].

In this work, we propose a parallel structure ANF algorithm to track multiple frequencies at the same time. As shown in Fig.4 and Fig.5, rather than one single estimated primary signal we reconstruct K desire signals  $\hat{d}_1 \dots \hat{d}_K$  each with an estimated secondary signal  $\hat{y}'_i$  and track multiple frequencies by parallel ANFs. To avoid that multiple ANFs converge at the same frequency, the error signal is filtered by cascaded notch filters to remove the frequency estimated by previous ANFs.

$$\hat{d}_{i}(n) = \hat{y}'_{i}(n) + e_{i-1}(n) \tag{16}$$

Thanks to the advantage of independently tracking, each ANF can estimate one certain frequency without other's influence. This method is, to some extent, similar to triangular cascaded ANF algorithm but simplified. As a result, the proposed algorithm obtains unbiased estimation with a little increase of computation complexity compared to the linear cascaded ANF algorithm. There are (2K-1) pole sections D(z) and (K-1) zeros sections N(z) in the proposed algorithm. These values are K and K-1 in linear cascaded ANF algorithm. By contrast, triangular cascaded ANF algorithm has (K(K+3)/2-1) pole sections D(z) and (K(K+1)/2-1) zeros sections N(z) [12]. Obviously, the computational complexity of the proposed algorithm is in the level of K order which is same to linear cascaded ANF algorithm while triangular cascaded algorithm in square of K order.

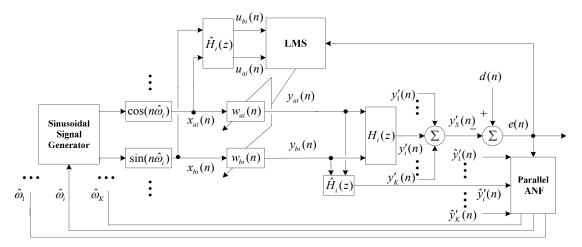


Figure 4: Proposed line-spectrum control system.

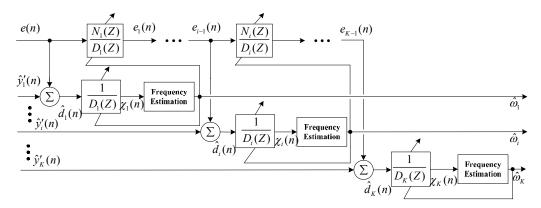


Figure 5: PANF algorithm.

#### 4. Simulations

In this section, computer simulations are presented to demonstrate the performance of the proposed new LANC system with the PANF algorithm. A comparison is conducted between the proposed system and other LANC systems with conventional frequency estimation algorithms, namely, linear cascaded ANF and triangular cascaded ANF, and the results are given in Fig.6. In these simulations, the sampling frequency  $f_s$  is fixed at 5 kHz. The primary source signal is generated as a sinusoidal one (SNR=20) with four independent time-varying frequencies. The change rate of the frequency of four tones is between -4Hz/s and 4Hz/s, and sudden changes happen in some tones.

The secondary path modeling is assumed to be perfect while the additive noise is white and Gaussian. The pole zero contraction factor  $\alpha$ , forgetting factor  $\lambda$  and smoothing factor  $\gamma$  are set at 0.99 while step-size  $\mu$  is 0.05 in each LANC system. In Fig.6.(d), the proposed system demonstrates nice effectiveness and robustness as well as fast convergence. The reduction of multi-tone under the proposed LANC system is over 40dB in steady state, which is equivalent to the system with triangular cascaded ANF algorithm shown in Fig.6.(c). Although the convergence of LANC system with a linear ANF is as fast as the former two systems, Fig.6.(b) shows that the performance of the system with linear cascaded ANF algorithm is limited and the sinusoidal reduction is only around 20 dB in some cases.

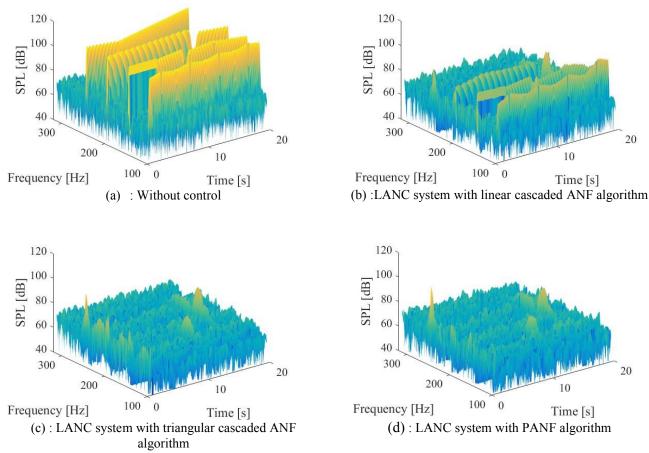


Figure 6: Performance of four-tone tracking and control in simulation

# 5. Experiments

In this section, the experimental results in a pipe system are given in detail to compare the performance of LANC system with the linear ANF algorithm and the proposed LANC system. As shown in Fig.7, the circle pipe system(radius 50mm, length 1500mm) with an active-passive muffler(inner radius 50mm, outer radius 135mm, length 500mm) is the same one as our previous work[4]. The primary multi-tone acoustic signal is generated by a loudspeaker at the front-end of the pipe system. The primary signal is two tones with close time-varying frequencies in the first experiment and three tones with time-varying frequencies in the second. The error signal is acquired in the form of sound pressure by a B&K microphone (4189-a-021) located at the after-end of the pipe. The work of online signal processing and data recording is performed by a controller with a development board Omap-1137 (tms320c 6747) inside. To generate secondary acoustic signal, a sound source is set at the pipe orifice just near the error sensor. The secondary path modeling is performed in the off-line method and the sampling frequency  $f_s$  is set at 5 kHz. The parameters of ANF are  $\alpha = 0.99$ ,  $\lambda = 0.997$ ,  $\gamma = 0.9$  and the step size of LMS algorithm is  $\mu = 0.05$  for both two systems.

In the first experiment, the primary signal is given as a sinusoidal one with two close frequencies (the minimal gap is just around 29Hz) both of which are time-varying. The performance of proposed system and the LANC system with linear cascaded ANF algorithm is compared in time-frequency domain. Figure 8 (a) demonstrates that the sound pressure level (SPL) of each tone is about 101dB before control. As shown in Fig. 8 (b), the SPL reduction of the system with linear cascaded ANF algorithm is over 30dB in one tone while hardly any in the other. It can be seen that

FxLMS algorithm converges in only one tone but fails in the other (due to FM and imperfect secondary path modeling). In contrast, the proposed LANC system shows better reduction

performance in both two tones (over 30dB) in Fig.8 (c).



Figure 7: ANC experiment system

The second experiment is a comparison between two NANC systems in the control performance of three tones with time-varying frequencies. From Fig.9, the intervals of these tones in frequency domain are about 100Hz and each tone has a SPL of 101dB just the same as tones in the former experiment. The frequency changes are continuous in most situations except an abrupt jump from 190Hz to 210Hz in one tone at 9s. Figure 9 (b) illustrates the performance of LANC system with linear cascaded ANF algorithm. Specifically, one tone is reduced around 30dB in SPL, the other two, in contrast, have only 10~20dB SPL reduction. By comparison, the reduction of all three tones in the proposed system is around 30dB. The unsteady state caused by the abrupt jump at 9s has little influence on both two algorithms and both of them can reduce the SPL effectively in less than 0.5s.

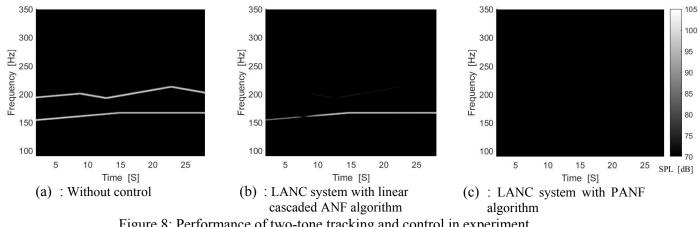


Figure 8: Performance of two-tone tracking and control in experiment

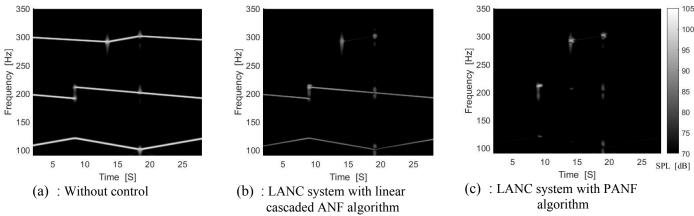


Figure 9: Performance of three-tone tracking and control in experiment

#### 6. Results

This work introduces a new line-spectrum active noise control system with a frequency estimation algorithm based on parallel ANF. The PANF algorithm reconstructs multiple estimated primary signals and tracks multiple frequencies at the same time. Computer simulations and two comparison experiments in a pipe system are given. From the results of simulations and experiments, the PANF algorithm demonstrates fast convergence as well as effective frequency tracking in both steady and unsteady state. In addition, the proposed LANC system shows better SPL reduction performance than the system with linear cascaded ANF in multi-tone control especially when some frequencies are close. From the analysis of computation complexity, it is clear that PANF algorithm is similar to linear cascaded ANF algorithm in aspect of computation burden, which is much lower than triangular cascaded ANF algorithm.

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