WAVE PROPAGATION IN TUNABLE FLUID-FILLED BEAMS WITH SPATIALLY VARYING VISCOELASTIC PROPERTY

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This paper investigates structural wave propagation in a fluid-filled beam with spatially varying property of material along the axis of propagation. The beam comprise two elastic faceplates with a central core of tunable fluid such as electro-rheological (ER) or magneto-rheological (MR) fluid. The corresponding wavenumbers depend on the viscoelastic properties of the tunable fluid, which in turn depend on the field applied to the fluid. Consequently, the position-dependent wavenumbers can be controlled by manipulating the local magnitude of field. A generalized wave approach based on reflection, transmission and propagation of waves is proposed for the analysis of such tunable fluid-filled beam. Numerical results are also presented for the spatially varying magneto-rheological fluid-filled beams. An explicit expression is provided for the calculation of propagation constants and thus the complex band structures. Distribution of the applied field is also specially designed for the desired band-gap structure of the whole tunable fluid-filled beam. It is further developed to examine the effects of spatially varying field on the band-gap behaviour, which could be the guidance of designing new tunable meta-structures.
Keywords: Magneto-rheological sandwich beam, Wave based method, Band gap structure

1. Introduction

The dynamic behaviour of a structure are usually described in terms of waves and their propagation, reflection and transmission. This paper is particularly concerned in non-uniform waveguides that have variation in smart structures like magneto-rheological fluid filled beams.

Wave-based methods are developed in order to attempt to enhance the computational efficiency of prediction tools, and therefore extend the applicability of deterministic models to higher frequencies. However, waveguide properties are usually assumed to be homogeneous for the sake of the simplification of computation. Examples include the Wave Based Method(WBM)[1][2][3], based on the indirect Trefftz approach, the spectral element method[4][5], that uses analytical solutions for the wave propagation to assemble dynamic stiffness matrices for waveguides, the Semi-Analytical FE method[6], that uses a FE formulation for the cross section of waveguides and assumes a wave like solution in the direction of propagation of the waves, and the Wave Finite Element(WFE) method, that applies the theory of periodic structures for homogeneous waveguides using a FE model of the cross section.[7]

The theory of vibration bending for viscoelastic layered beam structures was presented by DiTaranto[8], and Mead and Markus[9] calculated the forced vibrations of a three-layer damped sandwich beam system with various boundary conditions. The effectiveness of MR and ER fluid layers in controlling the vibration of various flexible continuous structures has been widely investigated analytically and experimentally in a number of studies during the past decade [10]. Yalcintas and Dai [11] studied the dynamic responses of a simply supported beam comprising a layer of MR and ER fluid. The above studies have considered fully treated beam structures comprising either an ER or a MR fluid layer over the entire length of the beam. It has been suggested that the controllable rheological fluid may also be applied locally or partially in a given structure to achieve relatively higher natural frequencies [12]. Partial fluid treatments are considered to be attractive, particularly for large structures and for realizing more efficient and compact vibration control mechanism [13].

This paper is organized as follows. In section 2, the mathematical model of MR sandwich beam is presented in two methods: Mead and Markus formulation (Abbreviated as: MMF) and finite element formulation (Abbreviated as: FEM). The results calculated in these two methods are compared and these two methods are validated in this way. In section 3, the transmission matrix of partially treated magneto-rheological sandwich beam is presented in order to analyse the pass/stop band structure of MR sandwich beam.

2. Problem statement

2.1 Modelling of MR material

An MR fluid can dramatically change its shear modulus under application of a magnetic field. It is taken as the core material in the considered MR fluid sandwich beam. When a magnetic field is applied through an MR fluid, the magnetic particles align themselves along the lines of magnetic flux. As a result, both the storage modulus G' and loss modulus G'' increase dramatically with increasing magnetic field. The frequency independent MR fluid material as reported in Vibration analysis of sandwich rectangular plates with magneto-rheological elastomer damping treatment [14].

The relation between the storage modulus, loss modulus and the applied magnetic field can be approximated by:

$$G' = -3.3681B^2 + 4.9975\times10^3 B + 0.873\times10^6$$
$$G_2'' = -0.9B^2 + 0.8124\times10^3 B + 0.1855\times10^6 \quad (1)$$

Where B is the magnetic induction. While the storage modulus $G'(B)$ is proportional to average energy stored during a cycle of deformation per unit volume of the material, the loss modulus $G''(B)$ is proportional to the energy dissipated per unit volume of the material over a cycle. Moreover, both the moduli are functions of the magnetic field intensity B. As a result, both storage and loss moduli increase with increasing magnetic field. Consequently the stiffness and damping properties can be controlled using the applied magnetic field. This enables an effective mechanism to suppress the vibration of the structural systems. The properties of shear modulus is shown in Fig.1

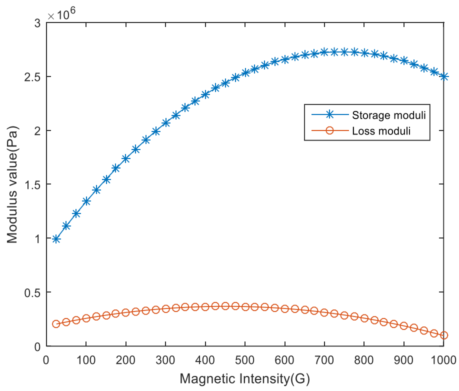

Fig.1: The changing tendency of shear modulus with different magnetic intensity

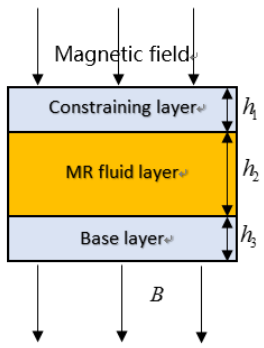

Fig.2: The cross section of MR sandwich beam in magnetic field

2.2 Dynamic model of magneto-rheological sandwich beam

The developed model predicts the transverse vibration response of MR material based non-homogeneous adaptive MR beams. The MR beam was formed of various independent controllable sections, which permit applications of different magnetic field levels at each section. The modelling of the axially non-homogeneous beam was achieved by considering each controllable section first, and then appropriate coupling of adjacent regions resulted with continuous response of the overall beam structure. In this section, the composite adaptive beam model is demonstrated for one controllable section of the beam. In the next section, the application of the model to the overall beam structure will be presented.

This model is based on several assumptions:

(1) Rotary inertia effects are neglected;

(2) No slipping was assumed between the elastic layers and MR layer;

(3) All three layers experience the same transverse displacement;

(4) No normal stress in MR layer and no shear strains in the elastic layers exist.

The cross-sectional configuration of the three-layered beam is shown in Fig.2. The resulting sixth-order transverse vibration equation is given in the form (Mead and Markus 1969) [9] of:

$$\frac{\partial^6 w}{\partial x^6} - g(1+Y)\frac{\partial^4 w}{\partial x^4} + \frac{m(x)}{\overline{\overline{EI}}}\left(\frac{\partial^4 w}{\partial x^2 \partial t^2} - g\frac{\partial^2 w}{\partial t^2}\right) = \frac{-gf(x,t)}{\overline{\overline{EI}}} \tag{2}$$

where $\overline{\overline{EI}}$, m, Y are the total bending stiffness of the beam, the mass per unit length and the geometric parameter. Y is described as

$$Y = \frac{d^2}{\overline{\overline{EI}}}\left(\frac{1}{E_1 h_1} + \frac{1}{E_3 h_3}\right)^{-1} \tag{3}$$

where h_1, h_2 and h_3 represent the height of constraining layer, MR fluid layer and base layer respectively. And the complex shear parameter g is:

$$g = \frac{G^*}{h_2}\left(\frac{1}{E_1 h_1} + \frac{1}{E_3 h_3}\right) = \frac{G_2 + iG_2'}{h_2}\left(\frac{E_1 h_1 E_3 h_3}{E_1 h_1 + E_3 h_3}\right) \tag{4}$$

Consider the free vibration problem. The dispersion equation is given by

$$-k^6 - g(1+Y)k^4 + \frac{\overline{m}}{\overline{\overline{EI}}}k^2\omega^2 + g\frac{\overline{m}}{\overline{\overline{EI}}}\omega^2 = 0 \tag{5}$$

This equation is cubic in k^2 and thus three pairs of positive- and negative-going waves exist, with wavenumbers, which are in general complex.

2.3 Finite element formulation of a multi-layer beam

In finite element analysis (FEM), a standard beam element with two end nodes with three degrees –of-freedom (DOF) for each node is considered. The DOF include the transverse w, axial u and the rotational θ displacements of the beam. The transverse and axial displacements can be expressed in terms of nodal displacement vectors and shape functions, as follows:

$$\begin{aligned}\mathbf{u}(\mathbf{x},\mathbf{t}) &= \mathbf{N_u}(\mathbf{x})\mathbf{d}(\mathbf{t}) \\ \mathbf{w}(\mathbf{x},\mathbf{t}) &= \mathbf{N_w}(\mathbf{x})\mathbf{d}(\mathbf{t})\end{aligned} \tag{6}$$

where $\mathbf{d(t)} = \{u^1, w^1, \theta^1, u^2, w^2, \theta^2\}$, $\mathbf{N_u(x)}$ and $\mathbf{N_w(x)}$ are common linear and cubic polynomial beam shape functions represented as:

$$N_1(x) = 1 - \frac{x}{l_e} \quad N_2(x) = 1 - \frac{3x^2}{l_e^2} + \frac{2x^3}{l_e^3} \quad N_3(x) = x - \frac{2x^2}{l_e} + \frac{x^3}{l_e^2}$$

$$N_4(x)=\frac{x}{l_e}\ N_5(x)=\frac{3x^2}{l_e^2}-\frac{2x^3}{l_e^3}\ N_6(x)=-\frac{x^2}{l_e}+\frac{x^3}{l_e^2} \tag{7}$$

where l_e is the length of the element.

Upon substituting these equations into Lagrange's equations, described as the governing equations of motion for the undamped MR sandwich beam element in the finite element form can be obtained as above.

The discrete dynamic equation of a MR sandwich beam obtained from the FEM model at a frequency ω is given by

$$(\mathbf{K}+j\omega\mathbf{C}-\omega^2\mathbf{M})\mathbf{q}=\mathbf{f} \tag{8}$$

Where $\mathbf{K}$,$\mathbf{M}$ and $\mathbf{C}$ are the stiffness , mass and damping matrices of the MR sandwich beam, respectively, $\mathbf{f}$ is the loading vector and $\mathbf{q}$ is the vector of the degrees of freedom. Introducing the dynamic stiffness matrix $\tilde{\mathbf{D}}=\mathbf{K}+j\omega\mathbf{C}-\omega^2\mathbf{M}$, decomposing into left and right boundaries, and interior degrees of freedom, and assuming that there are no external forces on the interior nodes, results in the following matrix equation:

Then, the transfer matrix can be written in terms of the dynamic stiffness matrix as

$$\mathbf{T}=\begin{bmatrix}-\mathbf{D}_{LR}^{-1}\mathbf{D}_{LL} & \mathbf{D}_{LR}^{-1}\\ -\mathbf{D}_{RL}+\mathbf{D}_{RR}\mathbf{D}_{LR}^{-1}\mathbf{D}_{LL} & -\mathbf{D}_{RR}\mathbf{D}_{LR}^{-1}\end{bmatrix} \tag{9}$$

Free wave propagation is described by the eigen problem

$$\mathbf{T}\begin{bmatrix}\mathbf{q}_L\\ \mathbf{f}_L\end{bmatrix}=\lambda\begin{bmatrix}\mathbf{q}_L\\ \mathbf{f}_L\end{bmatrix} \tag{10}$$

The 6 eigenvalues of equation (5) can be split into two sets of 3 eigenvalues and eigenvectors which are denoted by λ_i and $1/\lambda_i$, with the first set such that $|\lambda_i|\le 1$. In the case $|\lambda_i|=1$, the first set must contain the waves propagating in the positive direction, which are such that $\mathrm{Re}\{j\omega q_L^H f_L\}<0$, then

$$\boldsymbol{\lambda}=e^{-i\mathbf{k}x},\quad \mathbf{k}=\frac{i\,\mathrm{In}\,\boldsymbol{\lambda}}{l_e} \tag{11}$$

Table 1: Comparison of wavenumber of a MR-sandwich beam derived from the MM formulation and the finite-element with the measured frequency (500Hz)

Filed intensity(G)	Mode	Real			-Imag		
		FEM	MMF	deviation	FEM	MMF	deviation
0	1	1.3478	1.0232	0.3246	30.179	28.8907	1.2883
	2	0.3036	0.3015	0.0021	2.9281	2.9286	0.0005
	3	20.0132	20.5718	0.5586	0.7800	0.6859	0.0941
250	1	2.5418	1.8564	0.6854	38.028	34.7234	3.3046
	2	0.3429	0.3354	0.0075	4.2592	4.2535	0.0057
	3	16.5084	17.391	0.8826	0.8736	0.8016	0.072
500	1	2.7925	1.9857	0.8068	42.765	38.1338	4.6312
	2	0.3106	0.2995	0.0111	4.8247	4.8137	0.011
	3	15.1316	16.0827	1.6711	0.7277	0.6788	0.0489
750	1	2.1915	1.5426	0.6489	44.262	39.1833	5.0787
	2	0.2311	0.2216	0.0095	4.9757	4.9620	0.0137
	3	14.793	15.754	0.961	0.5268	0.4927	0.0341

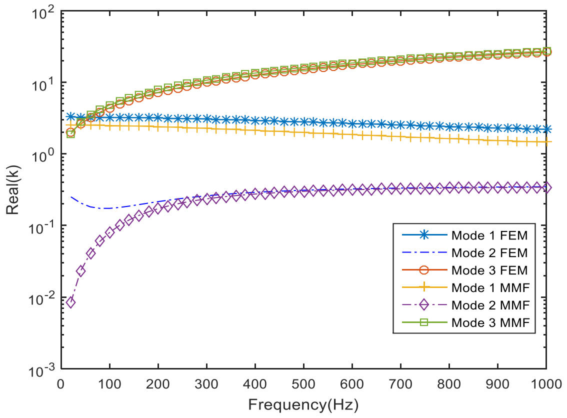

Fig.3: Comparison of the wavenumbers calculated from FEM and MMF at different frequencies (Real part).

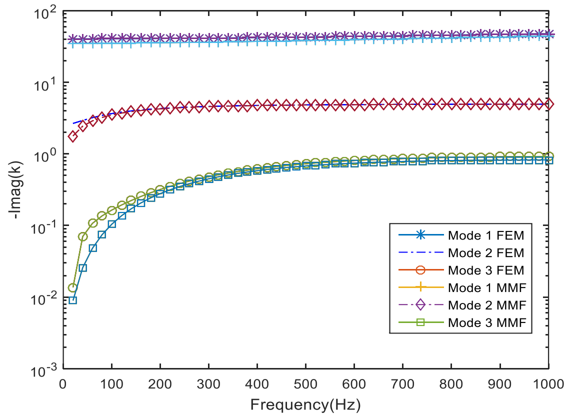

Fig.4: Comparison of the wavenumbers calculated from FEM and MMF at different frequencies (Imag part).

2.4 Comparison of results of two methods

Table 1 shows the real parts and imaginary parts of the wavenumbers calculated by MM formulation and FEM method. The results obtained from these two methods show a great similarity. In Fig.3 and Fig.4, we can see that only the real part of wavenumber for mode 1 shows a little difference. But the error of calculation is absolutely acceptable. For 0-100Hz, the real parts of wavenumber for mode 2 show some difference. It is more likely to attribute to error of the theory of Mead and Markus formulation. To sum up, the results of these two method are reliable.

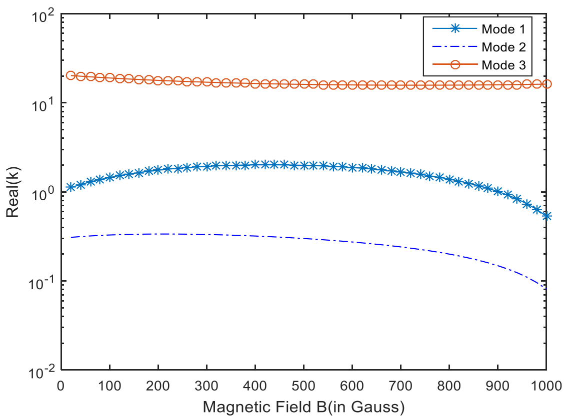

Fig.5: The changing tendency of wavenumbers with different magnetic intensity at 500Hz (Real part).

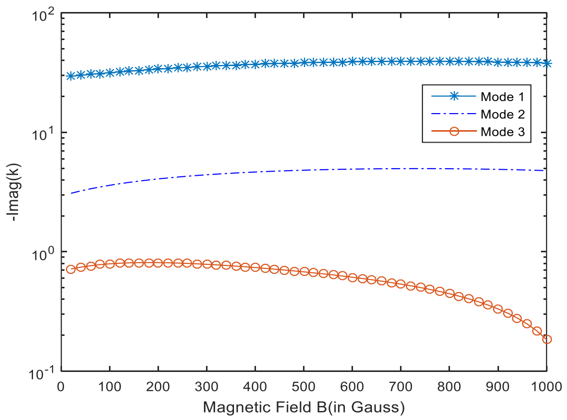

Fig.6: The changing tendency of wavenumbers with different magnetic intensity at 500Hz (Imaginary part).

2.5 Results and discussion

Fig.5 and Fig.6 show the real and imaginary parts of the wavenumber for positive travelling waves at 500Hz, as functions of applied magnetic field. Results at other frequencies are qualitatively similar. The wavenumbers are complex. The real and imaginary parts are related to the phase and attenuation of the wave with distance.

There are three wave modes. For small magnetic fields wave mode 1 behaves like a bending near field, having a relatively large imaginary part. It can propagate with large phase change but it generally propagates little energy. Wave mode 2 behaves like a "push-pull" axial wave motion in the outer layers. It has small phase changes and attenuates rapidly with distance. Wave mode 3 can propagate freely with large phase change and attenuate relatively gradually. It can transport energy relatively freely over large distances. As the magnetic field increases, the imaginary part of wave-number for mode 3 attenuates. It means that, Mode 3 will carry the most energy of all the three modes as the magnetic induction increases.

3. Partially treated MR sandwich beam

3.1 Inhomogeneous MR sandwich beam model

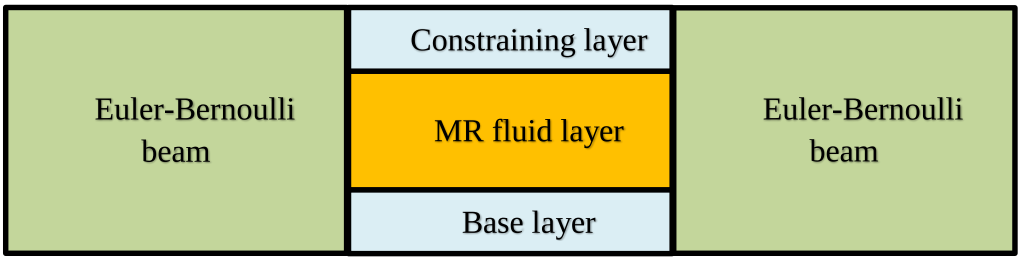

Fig.7: The cross section of inhomogeneous MR sandwich beam

If the MR fluid layer is partially distributed along the whole beam. Then the whole beam can be modelled to be composed of Euler-Bernoulli beam and MR sandwich beam. Then, the whole beam can be divided into three waveguides as Fig.7 shows.

3.2 Wave-based method

Assuming three connected waveguides, the degrees of freedom and internal forces are grouped into vectors as below, where i , j represent the serial number of the interface of these waveguides and a , b represent the left and right waveguides of each interface.

$$\mathbf{W}_{aj} = \mathbf{\Psi}^{+}_{aj}\mathbf{a}^{+}_{j} + \mathbf{\Psi}^{-}_{aj}\mathbf{a}^{-}_{j}, \quad \mathbf{F}_{aj} = \mathbf{\Phi}^{+}_{aj}\mathbf{a}^{+}_{j} + \mathbf{\Phi}^{-}_{aj}\mathbf{a}^{-}_{j}$$
$$\mathbf{W}_{bi} = \mathbf{\Psi}^{+}_{ai}\mathbf{a}^{+}_{i} + \mathbf{\Psi}^{-}_{bi}\mathbf{a}^{-}_{i}, \quad \mathbf{F}_{bi} = \mathbf{\Phi}^{+}_{bi}\mathbf{a}^{+}_{i} + \mathbf{\Phi}^{-}_{bi}\mathbf{a}^{-}_{i} \tag{12}$$

where $\mathbf{\Psi}^{+}_{a}$, $\mathbf{\Psi}^{-}_{a}$, $\mathbf{\Psi}^{+}_{b}$, $\mathbf{\Psi}^{-}_{b}$ are the displacement matrices for the positive and negative going waves at each respective waveguide, and $\mathbf{\Phi}^{-}_{b}$, $\mathbf{\Phi}^{-}_{a}$, $\mathbf{\Phi}^{+}_{b}$, $\mathbf{\Phi}^{-}_{b}$ are the internal forces matrices for the positive and negative going waves at each respective waveguide. Continuity and equilibrium conditions, can be written as

$$\mathbf{C}_{aj}\mathbf{W}_{aj} = \mathbf{C}_{bi}\mathbf{W}_{bi}, \mathbf{E}_{aj}\mathbf{F}_{aj} = \mathbf{E}_{bi}\mathbf{F}_{bi} \tag{13}$$

where $\mathbf{C}_{aj}$ and $\mathbf{C}_{bi}$ are the continuity matrices and $\mathbf{E}_{aj}$ and $\mathbf{E}_{bi}$ are the equilibrium matrices. A scattering matrix $\mathbf{G}_{i}$, relating the waves from the left side of the junction to the waves at its right side, can be defined as

$$\mathbf{a}^{-} = \mathbf{r}^{aa}_{j}\mathbf{a}^{+} + \mathbf{t}^{ba}_{j}\mathbf{b}^{-}$$
$$\mathbf{b}^{+} = \mathbf{t}^{ab}_{j}\mathbf{a}^{+} + \mathbf{r}^{bb}_{j}\mathbf{b}^{-} \tag{14}$$

Applying the continuity and equilibrium conditions, and assuming the matrix inversion exists, thus

$$\mathbf{t}^{ac} = \mathbf{t}_2^{bc}\left(\mathbf{I} - \mathbf{f}_b\mathbf{r}_1^{bb}\mathbf{f}_b\mathbf{r}_2^{bb}\right)^{-1}\mathbf{f}_b\mathbf{t}_1^{ab} \tag{15}$$

$$\mathbf{r}^{aa} = \mathbf{r}_1^{aa} + \mathbf{t}_1^{ba}(\mathbf{I} - \mathbf{f}_b\mathbf{r}_2^{bb}\mathbf{f}_b\mathbf{r}_1^{bb})^{-1}\mathbf{f}_b\mathbf{r}_2^{bb}\mathbf{f}_b\mathbf{t}_1^{ab} \tag{16}$$

Where $\mathbf{f}_b$ represents the propagating matrix of MR sandwich beam, $\mathbf{t}^{ac}$ represents the transmission matrix and $\mathbf{r}^{aa}$ represents the reflection matrix. Since the magnetic field applied to the MR beam determines the wavenumbers and the reflection and transmission matrices at the EB/MR junctions, it is clear that this field can be used to control the vibration energy flow through the insert. Fig.8 shows the transmitted power per unit incident power for various magnetic fields. There is a distinct minimum at a particular frequency, and the stop band is tunable by changing the magnetic field.

3.2.1 Junction between an Euler-Bernoulli beam and a tunable fluid-filled beam

As the above part explained, consider the junction between an Euler-Bernoulli beam and a MR sandwich beam b. From continuity at the junction, the slopes, displacements and the axial deformation on both sides of the junction must equal each other. As a consequence, the continuity matrices are given by

$$C_a = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & \frac{1}{2}d \end{bmatrix}, \quad C_b = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{17}$$

From equilibrium the shear forces and the bending moments on both sides of the junction must also equal each other while the net axial force is zero. Hence,

$$E_a = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad E_b = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \tag{18}$$

3.3 Numerical Example

Consider an MR sandwich beam of length $l_e = 50mm$ inserted into an EB beam, the beam properties being those given in section 3.1. Fig.8 shows the transmitted power per unit incident power τ for various magnetic fields.

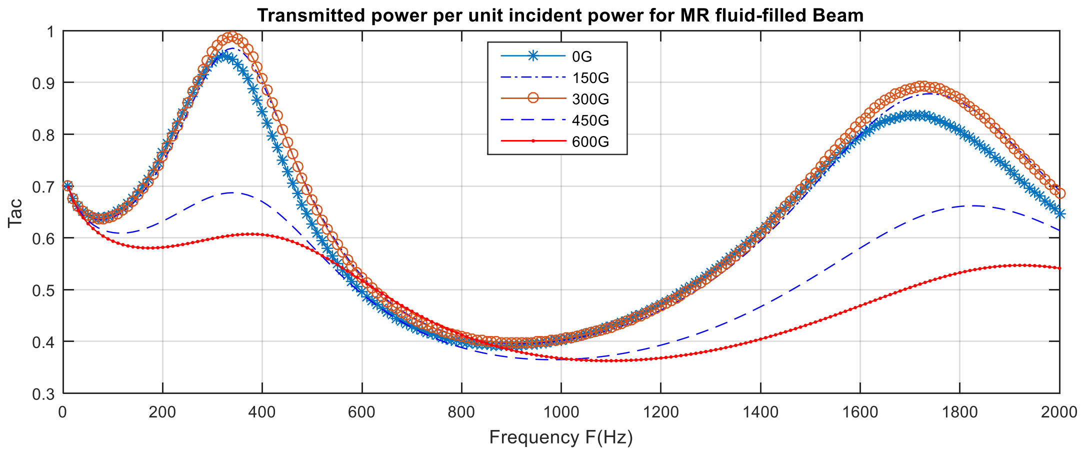

Fig.8:Transmitted power per unit incident power for partially treated MR sandwich beam

3.4 Results and discussion

Fig.8 shows the transmitted power per unit incident power for various magnetic fields. The results are calculated from the equation(15) . Note in Fig.8 that there is a distinct minimum at a particular frequency:800-1000Hz , and that this frequency can be tunable by changing the magnetic field. As the intensity of magnetic field increases from 0G to 600G, the transmitted power per unit incident power at 200-400Hz is attenuated form 1 to 0.6. Then, it is shown that the insert of MR sandwich beam can act as a band-stop filter, can reduce the transmitted poewer in a disired frequency band. It is therefore especially suited to the isolation of structure-borne vibrations in those cases where the incident power is somewhat narrow band. At higher frequencies from 1400 to 2000 Hz, the transmitted power can also be attenuated by enhancing the magnetic field. A distinct pass/stop band structure can also be seen clearly.

4. Conclusion

In this paper, the wavenumber of MR sandwich beam is calculated by two methods: finite element method and MM formulation. The results derived from these two methods are similar to each other. It proves that both methods are reliable. Then, a wave based method is proposed to analyse the wave propagation in the inhomogeneous MR sandwich beam. The transmission matrix of incident power is presented and the transmitted power per unit incident power is calculated by this method. It is shown that MR sandwich beam can be used to act as a pass/stop band structure. And the result is useful for designing new tunable meta-structures.

5. Acknowledgements

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