A REVIEW OF THE METHODS FOR SOLVING THE ILL-CONDITIONED PROBLEM IN FREQUENCY DOMAIN IDENTIFICATION OF LOAD

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Compared with the time-domain identification method of load force, the frequency domain method has high recognition accuracy and wide application. Due to the large condition number at resonance frequency points, the problem of large inversion error of the frequency response function matrix becomes a bottleneck problem that restricts the development of load identification.The traditional frequency domain identification method are the inverse method, the least square method and the modal analysis method. To solve the ill-conditioned problem at resonant frequency, Tikhonov regularization, TSVD, TTLS, ridge parameter method, iterative regularization and so on are emerging.After analyzing the research progress and characteristics of those methods, it is pointed out that the hybrid intelligent algorithm based on the regularization idea will become a new research hotspot, and the regularization matrix, the singular threshold and the parameter selection will be the key to breaking the bottleneck problem in load recognition.

1. Introduction

In recent years, with the development of science and technology and the complex intelligence of modern engineering structure, people pay more attention on the load in the structure of the system, the researchers also gradually strengthen the research of load identification.In the field of engineering, such as spaceflight, aviation, building structure, transportation, windbreak and disaster resistance, which are characterized by dynamic load, the load identification provide some important design parameters and theoretical and technical support .Inside the dynamic load has great influence on the structure, and it is destructive and unpredictable, such as the aerodynamic load of the propeller in the flight and the impact load of the ship, the dynamic load identification is a key issue.

Dynamic load identification began in the late 1970s, due to the study of helicopter flight propeller spindle force and other military applications [1], because the complex structure of load are often difficult to measure, such as high-speed trains caused the additional impact on the axle due to rail joints, offshore platforms and other large buildings affected by wind and waves, etc,people

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developed a variety of load identification methods[2-3].To sum up,it can be divided into two categories: time domain identification technology and frequency domain identification technology.

Compared with time domain load identification technology, frequency domain method dynamic calibration is simple, has high recognition accuracy and more widely used. However, due to the morbidity of frequency response function at the resonance point, the large error of the inverse process leads to the recognition result affected, in order to improve the accuracy of load identification,many methods for solving ill posed equations have been put forward in recent year . In this paper, we mainly summarize the methods of solving the problem of disease identification in recent years, including inverse frequency response method, Tikhonov regularization method, ridge estimation method, truncated singular value decomposition method, total least squares method, iterative regularization method and modal analysis method, which can be reference to related research and engineering application.

2. Present algorithm

2.1 FRF inverse method

Direct inverse method is a recognition method to establish the relationship between dynamic load and structural vibration response, and then through the direct inverse operation to determine the dynamic load. In the frequency domain identification method, the relationship between the dynamic load and the vibration response of a linear time-invariant system can be expressed as

$$H(w)F(w) = X(w) \tag{1}$$

Where $H(w)$ is the frequency response function, $F(w)$ is the dynamic load, and $X(w)$ is the vibration response.

According to equation (1) ,the dynamic load of the structure can be obtained as

$$F(w) = H(w)^{+} X(w) \tag{2}$$

In the formula, the superscript "+" denotes the generalized inverse operation of the matrix.

In 1979, Bartlett earliest use of frequency domain direct inverse method to identify dynamic load of helicopter paddle center under the main harmonic frequency .Subsequently, in 1985, Okubo [4]used direct inverse method to study the identification of excitation force of machine tool and automobile engine, and analyzed the influence of noise on the recognition of excitation force.In 1996, John [5] used singular value decomposition to summarize the inverse method has better recognition effect when load position is known. In 2002, Tian Yan [6] using the improved frequency response function matrix inverse method, that is, combine the excitation point and the response point , the frequency response function from the rectangular matrix can be a simple square form, and then calculate the average incentive force (arithmetic average and conditional weighted average),the experimental results show that the two algorithms are superior to the traditional inverse method.

However, when there is a response error Δy , it causes the corresponding variable Δf , the load identification equation is $y + \Delta y = H(f + \Delta f)$;the resulting error is $\|\Delta f\|/f \le \|H\|\|H^{-1}\|\|\Delta y\|/y$, error is magnified by $\|H\|\|H^{-1}\|$;

Similarly, there is a matrix error ΔH , which can cause the error proportional to $\|H\|\|H^{-1}\|$.

In particular, at the structural resonant frequency point, the number of conditions is very large, small disturbance lead the large error of load recognition.

2.2 Tikhonov regularization method

The regularization method was first proposed by the former Soviet Union A.N. Tikhonov [7]and his working group in the 1960s, which is an effective way to solve the ill posed problem. Its idea is that by applying constraints, the equation has a proper solution closing to the original solution of the original ill-posed equation.Criteria for judging are:

$$\|Ax-b\|_2^2+\alpha\|Lx\|_2^2=\min \tag{3}$$

The corresponding solution is

$$x=\left(A^TA+\alpha L^TL\right)^{-1}A^Tb \tag{4}$$

Where L is a regularization matrix and α is a regularization parameter. The selection of α is crucial, and its selection is divided into a priori selection and a posteriori selection. The priori selection is easy to analyze the theory, but the actual calculation is more difficult, therefore, the posterior selection strategy is more extensive. In the case where the observation error is known, the regularization parameter is generally determined by the deviation principle. When the observation error is unknown, generalized cross validation criteria (GCV) and L-curve method are generally used.Beside, there are some new regularization methods that have emerged, such as variance component estimation, U-curve, etc.

2.2.1 Morozov deviation principle

In practical engineering, the level of error δ in response is predictable in many cases, so the Morozov deviation principle can be used.This was developed by Morozov in 1966, $\|y-y_\delta\|_2\le\delta$.

Using Morozov deviation principle [8] to select regularization parameters ,means selecting $\alpha=\alpha(\delta)$,then

$$A^TAx_\delta+\alpha x_\delta=A^Ty_\delta \tag{5}$$

make the solution of Eq. (5) satisfy the deviation Eq. (6)

$$\|Ax_\delta-y_\delta\|=\delta \tag{6}$$

It's solved by Newton iterative method,get

$$F(\alpha)=\|Ax_\delta-y_\delta\|^2-\delta^2 \tag{7}$$

then,get

$$F'(\alpha)=2\alpha\left\|A\frac{dx_\delta}{d\alpha}\right\|^2+2\alpha^2\left\|\frac{dx_\delta}{d\alpha}\right\|^2 \tag{8}$$

The iteration format is

$$\alpha_{k+1}=\alpha_k-\frac{F(\alpha_k)}{F'(\alpha_k)} \tag{9}$$

1) Give the initial regularization parameter first, $\alpha_0>0$,make k=0;

2) Solve Eq.(5), get x_δ;

3) Derive the Eq.(5), get $dx_\delta/d\alpha$;

4) Calculate the Eq.(7) and Eq.(8), get $F(\alpha_k)$ and $F'(\alpha_k)$;

5) According to the Eq.(9), When $|\alpha_{k+1}-\alpha_k|$ less than the specified error,the calculation is terminated;or k=k+1,jump to 2).

Although the Newton method has a second order convergence rate for a single root, it is relatively difficult to estimate the initial value because it is locally convergent and the iterative process is computationally large. If the initial value is not properly selected, iterative divergence may occur.

In 2013, Wang Zewen [9] used Morozov deviation principle to study the linear model function method of regularization parameter selection. In order to overcome the local convergence of the basic algorithm, a new linear model function relaxation algorithm is proposed.

2.2.2 The generalized cross validation (GCV) method

The Generalized Cross Validation (GCV) method was proposed by Graven [10] in 1979, as a cross validation method to promote the general model. This method has been widely used in the selection of model adjustment parameters.

When using the GCV method to select the ridge parameters, you need to solve the minimum value of the GCV function

$$GCV(\alpha) = \frac{\frac{1}{n}\left\| I - H(\alpha)L \right\|^2}{\left[\frac{1}{n} tr\left(I - H(\alpha)\right)\right]^2} \tag{10}$$

Where $H(\alpha) = A\left(A^T A + \alpha I\right)^{-1} A^T$,n is the number of observations and tr () is the trace of the matrix.

In the calculation of GCV, there is no information related to disturbance variables,so GCV method can get the optimal ridge parameters in theory,but sometimes the GCV function changes too gentle, it is difficult to locate the minimum value.

In 2011,in order to avoid the correction effect of regularization parameters on the reliable components of the downward extension process, Deng Kailiang [11] used the modified generalized interactive confirmation method (GCV) to determine the truncation parameters and regularization parameters, and proposed two parameter Tikhonov regularization method.

2.2.3 L-curve method

The L-curve criteria were first proposed by PC Hansen [12] in 1993,it gives a comparison between the residual norm $\left\| Hf_{reg} - y \right\|_2$ (abscissa) and the solution norm $\left\| f_{reg} \right\|_2$ (ordinate).It is shown that the regularized parameter corresponding to the inflection point of the L curve is considered to be the optimal solution when the L curve is concave and the solution of the problem satisfies the discrete Picard condition .At this point, the curvature of the curve is the largest, through the curvature formula, α is calculate.

L-curve method is simple, easy to implement and can get a more accurate solution, but L-curve method is too dependent on the results of the curve fitting accuracy, sometimes the solution may not converge.

2.2.4 U-curve method

U-curve method is different from L-curve method, U-curve method can get $U(\alpha)$-α curve according to the definition of the $U(\alpha)$ function, one of the largest curvatures on the left side of the curve is the ridge parameter determined by the U curve method.

The U-curve function is defined as

$$U(\alpha) = \frac{1}{x(\alpha)} + \frac{1}{y(\alpha)} = \frac{1}{\left\| Hf_{reg} - L \right\|^2} + \frac{1}{\left\| f_{reg} \right\|^2} \tag{11}$$

Among them, $\left\| Hf_{reg} - y \right\|^2 = \sum_{i=1}^{r} \frac{\alpha^4 f_i^2}{\left(\lambda_i^2 + \alpha^2\right)}$; $\left\| f_{reg} \right\|^2 = \sum_{i=1}^{r} \frac{\lambda_i^2 f_i^2}{\left(\lambda_i^2 + \alpha^2\right)}$; $f = U^T L$;U is the left matrix which matrix H is decomposed by singular value;

D Krawczyk-Stańdo [13] proposed the U-curve method in 2007 and pointed out that the function has the following properties:the U-curve is strictly decreasing in the interval $\left(0, \lambda_k^{\frac{2}{3}}\right)$,In the interval $\left(\lambda_1^{\frac{2}{3}}, +\infty\right)$ within the strict increase, there is a local minimum in the interval $\left(\lambda_k^{\frac{2}{3}}, \lambda_1^{\frac{2}{3}}\right)$.($\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_k > 0$)

The U curve method has the advantages of tight theory and accurate positioning.Compared with the L curve method, the U curve method has the advantages of small computation and no curve fitting; compared with the GCV method, the curve will not be divergence and unable to determine the ridge parameter, so it is worth further study and popularization.

2.3 Ridge estimation method

The ridge regression method proposed by Hoerl [14] in 1970 is an improvement to the least squares estimation, which gives up the unbiasedness of the least squares method,to get more stable,reliable and realistic regression coefficient at the expense of reducing the accuracy of regression .

From the way of expression, it is a special form of Tikhonov regularization, taking its stable functional as a unit matrix, adding the same small constant to the main diagonal elements of the law equation coefficient matrix to reduce the morbidity of matrix.

2.4 Truncated singular value decomposition

The Truncated Singular Value Method (TSVD) was first proposed by Hansen [15] in 1987 and further elaborated in 1990, the basic idea is to remove the smaller singular values to reduce the morbidity of the coefficient matrix. After the emergence of the TSVD regularization method, many scholars have made further research on this basis, and proposed a kind of regularization method related to SVD decomposition, such as truncated generalized singular value decomposition (TGSVD), modified SVD method and so on.

In 2013, Liu Shaolin [16] proposed a modified singular value decomposition for the large error and less amount of information when the signal-to-noise ratio (SNR) is low. The simulation results show that the actual received signal source can be estimated accurately in a large range, and expand the scope of the original algorithm.

In 2014, Guo Rong [17] proposed a method of load identification using Tikhonov regularization and singular value decomposition, and gave the method flow, and obtained that the condition number is more than 1000, the use of regularization effect is better. Simulation and experimental results show that this method can improve the accuracy of structural load recognition .

2.5 Total least squares method

In 1980, Golub [18] proposed the famous SVD algorithm, and introduced the concept of "total least squares" for the first time. For the linear system of equations Ax = L, the basic idea of ordinary least squares is to get the optimal parameters under the constraint of residual sum and minimal criterion. But for

$$(A + E_A)x = L + E_L \tag{12}$$

the overall least squares method takes the observed error and the systematic error into account , and evaluates it by singular value decomposition.

Schaffrin [19]did a lot of work: researched the overall least squares problem with linear constraints in 2005 and studied the total least squares algorithm with weighted case in 2008.

In 2013, Wang Qisheng [20] proposed a linear iterative algorithm based on the overall least squares. Using the Lagrangian principle, the process is simple and easy to implement and numerical experiments show their effectiveness.

In 2016, Gu Yongwei [21] analyzed the root of the disequilibrium matrix, and found that there is a complex linear relationship between some data columns, and pointed out that the parameters corresponding to the disturbed data list are the parameters that are affected by the error by TLS estimation, proposed a weighted regularization on the series of disturbances, and through numerical analysis ,proved its effectiveness compared to the normalized regularization and TLS.

2.6 Iterative regularization method

The direct regularization method is effective in solving small and medium-sized linear discrete ill-posed problems, but for large-scale problems, the solution speed will slow down, so some new iterative regularization algorithms are proposed. The coefficient matrix of the iterative regularization method need not be explicitly expressed in the form of a matrix, but only in the form of a matrix vector product, which greatly reduces the computational complexity and accelerates the speed of the operation.

In general, we divide the iterative regularization method into two categories: classical stationary iteration method and Krylov subspace iterative method.Where the Landweber-Fridman is represented by a smooth iteration method, and the Krylov subspace method includes: GMRES iteration method, Arnoldi iteration method, CGLS iterative method and so on.

2.6.1 Landweber Regularization Iteration Method

Consider the general equation $Ax = y$, according to the regularized solution $x_\delta = \left(\alpha I + A^T A\right)^{-1} A^T y$, Take the Landweber iteration method [22] and get

$$x_k = \left(I - \alpha A^T A\right) x_{k-1} + \alpha A^T y \tag{13}$$

among them, $0 < \alpha \le 1/\|A\|^2$ is the relaxation factor, x_0 is the initial value.

The more iterations of the Landweber iteration method, the higher the precision, but the less iterations of the Landweber iteration method,the more stable, so choose an appropriate regularization parameter α is necessary.The Landweber regularization iterative method is reliable and can obtain the stable approximation solution of the demand. However, as the noise level increases, the recognition error increases, so the signal to noise ratio should be improved.

In 2013, Chang Xiaotong [23], according to the failure of conventional least squares method, used the Landweber iterative regularization method to solve the ill-posed problem of load identification in the process of load identification . And the simulation experiment is carried out on a bridge model, and the simulation results show that the proposed recognition method is effective and the stable approximation solution satisfying the engineering requirements.

2.6.2 GMRES regularization method

In 1986, Saad [24] proposed the generalized minimal residual method (GMRES), which is one of the most efficient methods for solving asymmetric matrix problems based on the Arnoldi process. Calvetti [33] first proposed to apply the GMRES method to solve discrete ill-posed problems.

The idea of the GMRES method is to find a vector in the Krylov subspace z_n, to minimize it

$$\min\left\|b - A\left(x_0 + z_n\right)\right\| = \min\left\|r_0 - Az_n\right\| \tag{14}$$

among them, $z_n = V_n y_n$, $V_n = \left(v_1, v_2, \cdots v_n\right)$,the column is a set of orthogonal vectors of the Krylov subspace formed by the Arnoldi process, so $\min\left\|b - A\left(x_0 + z_n\right)\right\| = \min\left\|r_0 - V_{n+1} \hat{H}_n y_n\right\|$, $\hat{H}_n$ is the Hessenberg matrix on $(n+1)\times n$.

The GMRES method is to solve $x_m = x_0 + V_m y_m$ .Algorithmic process:

(1) Give x_0 ,calculated $r_0 = b - Ax_0$;

(2) Arnoldi orthogonalization, until get a satisfactory value V_n;

(3) Seek the least squares, $\|r_n\| = \min\left\|\|r_0\|e_1 - \hat{H}_n y_n\right\|$, got y_n;

(4) Solve $x_m = x_0 + V_m y_m$ .If $\|r_n\| = \|b - Ax_n\| \le \delta$, export x_n ;or ,jump to (2).

In fact, with the increase of m, the singular value of H_m will gradually approximate the singular value of matrix A. If A is a morbid matrix with large number of conditions, then high-dimensional H_m is also morbid matrix, so it needs to be regularized to get High precision solution.

In 2014, Qiu Jing [25] analysed the RRGMRES method in the iterative regularization method, he proposed the method of determining the RRGMRES regularization parameter triangular condition number L-curve method,obtained a method of determining the regularization parameters of the RRGMRES method.And through a large number of classic numerical tests and comparison, its effectiveness was proved .

2.7 Modal analysis

By the vibration theory [26], for the linear time-invariant system, any response to the system can be expressed as a linear combination of modal responses for each order, response to any point

$$x_l(w) = \varphi_{l1} q_1(w) + \varphi_{l2} q_2(w) + \cdots + \varphi_{lN} q_N(w) = \sum_{r=1}^{N} \varphi_{lr} q_r(w) \tag{15}$$

among them, $q_r(w)$ the rth order modal coordinates, φ_{lr} is the l-point, the r-order mode of the mode coefficient.

For the forced vibration of a multiple degree of freedom proportional damping system, by using modal superposition method,any point at i response, j incentives, there are

$$X_i = \sum_{r=1}^{N} \frac{\varphi_{ir}\varphi_{jr}F_j}{-w^2 m_r + jwc_r + k_r} \tag{16}$$

Using frequency domain coordinate transformation method, load spectrum is got by

$$\{F\} = [\phi]^{-T} diag[k_i - w^2 m_i + jwc_i][\phi]^{-1}\{X\} \tag{17}$$

For modal coordinate transformation method, the natural frequency of the system, modal damping and modal formation must be known to identify the load characteristics.

3. Development trendency

Through the comparison of several different pathological methods, it can be found that the hybrid intelligent algorithm based on the regularization idea is a new research hotspot.The traditional Tikhonov regularization method, mathematically speaking, can obtain the regularization of the optimal convergence order if the regularization parameter is chosen according to a suitable regularization parameter selection principle. However, in the calculation of practical engineering problems, it is difficult to determine an appropriate value for the regularization parameter from the mathematical theory because the prior information is insufficient and the error in the measurement is unknown. The regularization matrix is also an abstract function.Therefore, it is necessary to make further research on the regularization parameter and the regularization matrix. In addition, the singular value decomposition technique can improve the error, and the coherence function can be used as the threshold to control the ill-conditioned matrix. However, there is no mature method to select the threshold value, and the recognition accuracy needs to be further improved.

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