A LOW-FREQUENCY METASURFACE ABSORBER BASED ON HELMHOLTZ RESONATORS

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An acoustic metasurface (AMS), a planar version of acoustic metamaterial (AMM), has attracted a great deal of interest in recent years. Alongside the theoretical and experimental investigations of various new physical phenomena, such as cloaking, negative refraction, hyper-lensing, zero mass/stiffness, focusing, collimating, and so on, thin and lightweight structures that can absorb airborne sound over a low frequency and wideband range are strongly desired. The present paper designs an acoustic metasurface absorber (AMSA) based on Helmholtz resonators (HRs) to acquire low frequency absorption, in which the lateral surface dimensions are used to reduce the overall thickness of the AMSA. The effect of the coupling of different HRs on the absorption is further investigated. To broaden the bandwidth, some sound absorbing materials are added to the design, such as foam in the cavity of the Helmholtz resonator. Finally, the structural parameters of the Helmholtz resonators are optimized by a differential evolution (DE) algorithm to enhance the absorption from 200 Hz to 1000Hz.
Keywords: Metasurface absorber, Helmholtz resonator, Differential evolution

1. Introduction

Conventional acoustic absorbers (various porous/fiber materials and wedges) for airborne sound usually comprise of structures with a thickness comparable to the wavelength, which results in major obstacles in bulky treatments if low frequency range is to be targeted. This is usually an impractical schema for low-frequency sound. A perfect acoustic absorber with subwavelength thickness at low frequency is always a challenge due to the weak energy loss and interaction between the waves and materials dictated by the linear dynamics of acoustic systems at low sound pressure levels. The conventional micro-performed panel (MPP) is a good candidate as low frequency acoustic absorbers, however an air cavity is needed, then the total thickness of the whole structure is usually not sufficently sub-wavelength[1].

Over the past decade there has been a great amount of research effort devoted to the topic of acoustic metamaterials (AMMs). Metamaterials are man-made macroscopic composites with optimized periodic (not must be) structure, designed to have properties not found in nature[2]. The acoustic metasurface (AMS), the planar version of AMM, is an emerging concept that has attracted a great deal of interest in recent years [3]. One of the key features of metasurfaces is their subwavelength thickness, which greatly facilitates their integration into compacted systems. Generally, there are two types of AMMs or AMSs, i.e., passive [3] and active ones [4]. In AMMs or AMSs, the effective density and bulk modulus of the unit cell are design parameters. By tailoring these properties, the AMMs or AMSs exhibit extraordinary effects such as reflection control, cloaking, negative refraction, hyper-lensing, zero mass/stiffness, focusing, collimating, low frequency sound isolating, and so on. For detail overview one can refer to refs [2, 5].

Beyond all those new physical phenomena, nearly total absorption of sound by tailored AMSs is receiving tremendous interest due to its potential practical applications. A variety of designs based on AMSs have been proposed and implemented, however a thin and lightweight absorber that is both easily installed and capable to absorb sound over a wide frequency from low frequency range still to be developed [6, 7]. To achieve this, two complex problems need to be solved: 1) Matching the material impedance to background medium while preventing back-reflected waves, 2) reducing the geometric dimensions of the structure while increasing the density of states at low frequencies and providing sufficient material losses to guarantee attenuation.

This paper aims to design an AMSA based on Helmholtz resonators (HRs) to acquire lower frequency absorption, in which the lateral dimensions[8, 9] are used to reduce its overall thickness. The mechanism of the absorption is further investigated. To broaden the bandwidth, some sound absorbing materials are added, such as foam in the cavity of the HR. Finally, the structural parameters of the HRs and the foam arrangement are optimized by a differential evolution (DE)[10] algorithm to enhance the absorption in the range between 200 Hz and 1000Hz.

2. Model and theory

A basic unit of two-dimensional (2D) metasurface configuration is shown in Fig. 1. The two different HRs are arranged along the x direction with period a (lattice constant in the x direction). The whole structure thickness is H. The structure extends infinitely along the z-axis. The cavity in left HR has width (length) w_1 and height b_1, and the width and depth of the neck are d_1 and t_1 respectively, and w_2, b_2, d_2 and t_2 present the corresponding parameters of the right HR. A plane longitudinal harmonic wave is incident from the air half space below with incident angle θ.

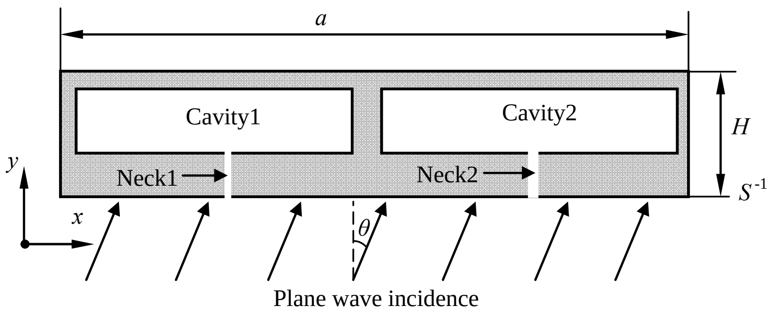

Figure 1: Structure of the unit cell in the metasurface.

The incident plane wave with angular frequency ω is described by the following equation

$$p_{in}(x,y) = p_0 e^{j(k_x x + k_y y)}, \tag{1}$$

where $k_x = k\sin\theta$, $k_y = k\cos\theta$ are the wave numbers in the x and y directions respectively, θ is the angle between the incident wave and y axis. The wave number k is given by $k = \omega/c$, where c is sound speed in air. According to Bloch theorem, the discrete translational symmetry with period a in the x direction implies the physical field $\chi(x,y)$ (denoting the pressure, velocity) satisfy

$$\chi(x+a,y) = \chi(x,y)e^{jak_x}. \tag{2}$$

In the domain below the s^- surface, the pressure reflection coefficient is given by the ratio between the pressure $p_r(x, y)$ in the reflected wave and that in the incident wave

$$r = p_r / p_{in} \ . \tag{3}$$

According to the energy conservation, the absorption coefficient for the rigidly backed structure is

$$\alpha = 1 - |r|^2 \ . \tag{4}$$

The wave simulation is based on the finite-element method implemented in commercial software COMSOL Multiphysics. In the simulations, the Floquet periodic boundary conditions have been applied on both sides of the unit.

3. Results and discussions

3.1 The coupling effect

Firstly, we compute the acoustic coefficients of the model identical to that described by J. Li[3]. Fig. 2 shows the acoustic coefficients for the structure in which both necks are located at the symmetry centres of the HRs. The same parameters as in Li [3] are used in simulations, the air density is 1.25 kgm^{-3} and sound speed 343(1+0.01i) ms^{-1}. The parameters describing dimensions of the absorber are listed in the first row of Table 1 (original). One can readily see that the near-total absorption appears at 1311 Hz, accordingly the reflection dip is clearly observed. The peak frequency is little less than that of Li, which is mainly induced by the different mesh size. Since the distance between the two resonators are much smaller than the wavelength, there will be a strong coupling between them. The coupling of these two resonators forms a combined resonant mode where both of them vibrate significantly with a phase difference of 180^o. At the resonance peak, the real part of the metasurface impedance normalized to that of air is equal to 1, and the imaginary part of relative impedance is zero, both indicate the impedance matching of the metasurface and the background air[3]. For this reason, the metasurface becomes “dark” to the incident wave at the frequency of absorption peak.

Table 1: Parameters of dimensions (in unit of mm) of unit cell in the metasurface

	a	H	w_1, w_2	b_1, b_2	d_1	d_2	t_1, t_2
Original	58	40	20	20	2.3	3	6
Prolonged	200	22	95	14	2.3	3	6

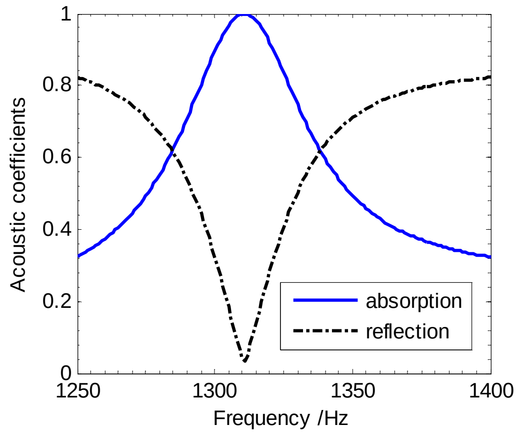

Figure 2: The acoustic coefficients of the model of J. Li.

In order to reveal the formation of the total absorption by the coupling effects induced by the two coupled HRs, Fig. 3 further compares the absorption of different absorbers composed of the 2.3mm, 3.0mm width identical neck HRs and the coupled (different slit widths) HRs respectively. One can see that both non coupled HRs show weak absorption peaks with amplitude slightly above 0.2, which is much lower than that of the coupled one. The absorption peak moves to higher frequency if the width of the neck of the non coupled HR is increased. The pressure amplitude inside the resonators is much larger than the incident wave pressure. Since energy absorption is proportional to the square of the pressure amplitude, such a strong vibration can efficiently dissipate the energy even with a small loss factor in the air, with most of energy absorption located at the neck region and happening due to viscous friction.

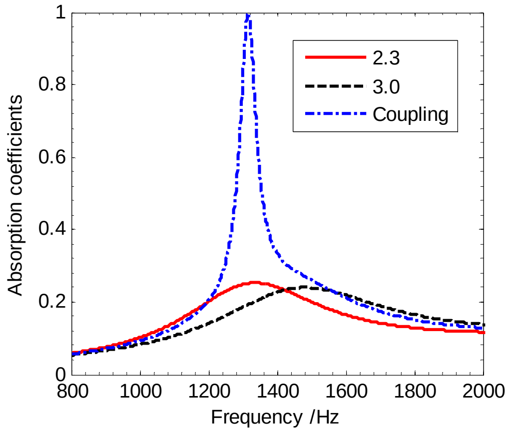

Figure 3: The acoustic absorptions of different AMSAs.

Although the above AMS shows low frequency absorption, the ratio of the overall thickness to the wavelength at resonance is larger than 1/10, which means that the sound absorber does not have very thin subwavelength thickness. In the following sections, a new design of the structure is investigated by using the hybrid resonant mode to make a thinner absorber, in which the idea of coiled coplanar HRs[8, 9] is adopted.

3.2 Side neck and lateral extension of cavity

Generally, one can obtain the lower frequency absorption peak by prolonging the length of the cavity. Fig. 4 shows the variation of the absorption coefficients by locating the neck at the distance of 1mm from the left wall of the respective cavity. In Fig. 4, the red solid line presents the original design. When the neck is located at the left end, indicated by the black dashed line, one can see that the absorption peak moves to lower frequency at 1266Hz. Then the strategy of using the lateral dimension (as examples of coiled coplanar air chambers[8, 9]) to prolong the wave propagation length is applied to reduce the overall thickness of the metasurface to 2.2 cm. All the detail parameters of the structure are listed in Table 1 in the row labelled as "prolonged". Now the elongated cavity has 3.3 times volume of the original one. The neck is also located at the distance 1mm on the right from the left wall of the cavity. One can see from Fig. 4 that the elongated cavity has a much lower frequency absorption peak at 570Hz (blue dash dotted line). Here the ratio of the overall thickness to the wavelength at resonance is about 0.037, which is much less than that of Li. One can conclude that the lateral extension of the cavity can be used to decrease the thickness of the structure, and is an efficient way for subwavelength absorption design.

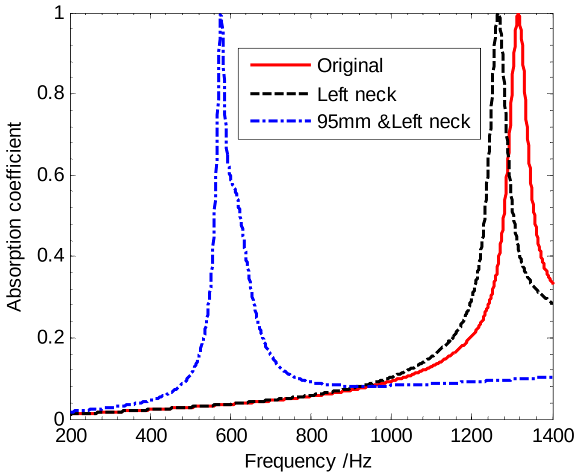

Figure 4: Comparison of absorption coefficients of different AMSAs.

3.3 The effects of the foam in the cavity

In order to widen the absorption band, adding porous foam in the cavity may be an easy means. A simple arrangement of the foam is shown in Fig. 5. Here the thickness of the left foam is denoted by H_1, and the thickness of the backed cavity is L_1. The right foam has the corresponding parameters H_2 and L_2.

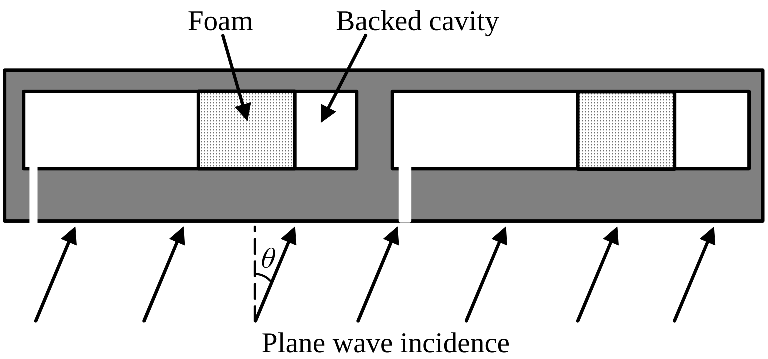

Figure 5: Structure of the unit cell in the AMSA with foam slabs.

Fig. 6 compares the absorption plots of the above elongated HRs with and without 40mm thick foam. Both the foam blocks are backed with 10mm length cavity. In the computation the Johnson-Champoux-Allard (JCA) model with a rigid frame for foam is employed. This is an equivalent fluid model for a rigid frame porous material, the parameters are listed in Table 2. From Fig. 6 one can see that the bandwidth is almost three times of the original one. Here the bandwidth is marked by cross points of the plot and horizontal black line of 0.5. For example, f_1 and f_2 in Fig. 6 denote the absorption bandwidth of AMSA with the foam.

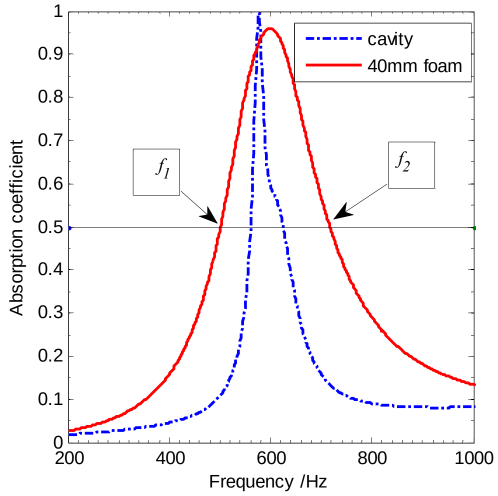

Figure 6: Comparison of absorption coefficients of the AMSA with and without foam.

Table 2: Parameters of the used foam.

Porosity	Flow resistivity ($Pa \cdot s \cdot m^{-2}$)	Tortuosity	Viscous characteristic length (μm)	Thermal characteristic length (μm)
0.995	10.5×10^3	1.0059	240	470

3.4 The absorption optimization

There are more than ten parameters that determine the acoustic absorption. The coupled resonance between the HRs, then the resonance induced by the back cavity of the foam, and the air friction in the neck and the foam all enhance the damping of the energy. It is difficult to find an desired absorption without an optimization scheme. Here a differential evolution algorithm[10] is used to find a optimization of absorption. The mean value of amplitude of reflection (r) in the design frequency range is used as an objective function, i.e.,

$$F = \text{mean}\left(\log_{10}\left(abs\left(r\left(f,\Omega\right)\right)\right)\right) \tag{5}$$

where Ω represents the set of design variables, f is the absorption frequency, the frequency range is [200, 1000Hz]. As an example, seven design variables and the corresponding search space are listed in Table 3. The unlisted parameters are kept as the same as those used in the prolonged HRs in Table 1. The whole absorber thickness is kept 22mm. Table 3 shows the optimized values of the design variables.

Table 3: Range and Result of optimized parameters of dimensions (mm) of unit cell

	H_1	L_1	w_2	b_2	d_2	H_2	L_2
Range	[20, 60]	[5, 30]	[70, 90]	[10, 15]	[2, 6]	[20, 50]	[5, 20]
Result	53.1	5.3	89.5	14.6	3.9	48.8	19.3

Fig. 7 presents the variation of absorption coefficient of the AMSA with and without optimization. One can see that the optimal absorption above 0.8 in the design frequency range is extended beyond the bandwidth of the original bandwidth without optimization (the same as that in Fig. 6).

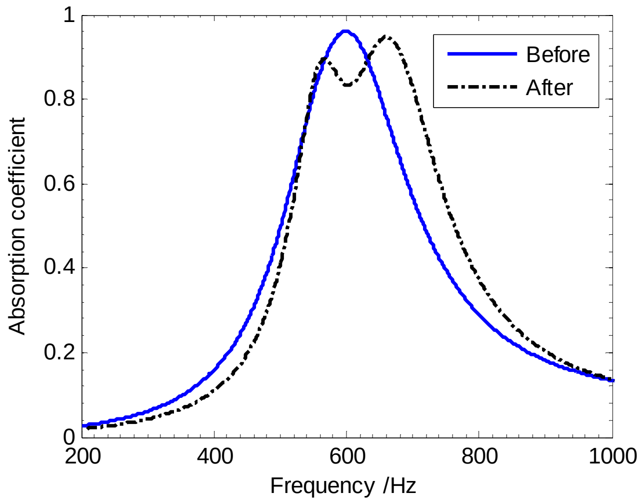

Figure 7: Variation of absorption coefficient of the AMSA with and without optimization.

4. Conclusions

An AMSA based on coupled HRs is designed to acquire low frequency absorption, in which a strategy using the lateral dimension is applied to reduce the overall thickness of the AMSA. The overall thickness is about half of that in published work, while at the same time the working frequency is much lower. The absorption mechanism enhanced by coupling effect is further revealed. The absorption bandwidth can be broadened by adding some sound absorbing materials, such as foam in the cavity of the HR. Finally, some parameters of the AMSA are optimized by a differential evolution (DE) algorithm to gain a better absorption from 200 Hz to 1000Hz. The results show that the absorption bandwidth is indeed extended. Future work can be extended in the following aspects to gain better absorption:

1) Some parameters were optimized as an example in present paper. The full optimization including the structure and the material (such as the foam or other damping materials), even its arrangement, such as groove structure, can also be further investigated.

2) Large necks or holes (above 1mm) cannot offer large friction, which means the efficiency of the absorption in the present AMSA can be low, as showed in Fig. 3. The neck or hole of submillimeter size (less 1mm) and their coupling should be further investigated.

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