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EXPERIMENTAL VIBRATIONS OF AN ANNULUS - AN HARMONIC DRUM.

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1. INTRODUCTION

Following Gottlieb's analysis of the harmonic properties of an ideal annular membrane [1], and the effect of an air cavity [2] on the eigenfrequencies, Gottlieb and Aebischer have systematically investigated the vibration frequencies of a baffled circular annular membrane with an attached gas cavity vibrating in a surrounding fluid [3-5] using analytical and computational techniques. Most recently [6] they have shown that the effect of the surrounding fluid in raising the ratios of the overtones to the fundamental frequency can compensate the cavity effect which lowers these ratios. The result is a series of overtones that are virtually harmonic. Computational investigations yielded twenty specific sets of membrane and cavity parameters which could potentially yield practically realizable annular kettledrums with excellent musical qualities.

The present report describes a slight extension of those computations in the context of an attempt at the practical realisation of an annular membrane exhibiting harmonic overtones.

2. DESIGN PARAMETERS

A detailed discussion of the relevant analysis is given in [6]. The annular membrane is defined by a radial coordinate r where a $\leq r \leq$ b, the ratio of b/a being Γ. The membrane has an areal density σ and tension T (with free wave speed $c = (T/\sigma)^{½}$). It is assumed to be set in an infinite coplanar rigid baffle, the central excluded region also being rigidly baffled in a coplanar manner. On one side is a semi-infinite compressible fluid with density ρ_o and sound speed C_o. Fixed to the outer membrane rim on the other side is a rigid cavity of volume V_1, containing a compressible gas with pressure P_1, density ρ_1, and ratio of specific heats, γ_1. The sound speed in this gas in $c_1 = (\gamma_1 P_1/\rho_1)^{½}$.

Only circularly symmetric vibrations were considered by the previous authors in their analysis [6], with the adiabatic assumption being made for the gas in the cavity and retaining only radiation masses for the external fluid, following Lax [7]. The parameters which govern the behaviour of the membrane are four:

$$\alpha = c/c_0$$

$$\beta = \rho_0 a/\sigma$$

$$\zeta = (\rho_1 c_1^2 4\pi a^3)/\rho_0 c_0^2 V_1 \qquad (1)$$

and $\Gamma = b/a$

Aebischer and Gottlieb [6] calculated the values of the first six eigenfrequencies for a grid of values given by:

Γ = 1.25, 1.50, 2.0 $\qquad$ α = 0.025 (x 2) 0.2
ζ = 0.1,0.2,0.5,1,0,2.0,3.0,4.0 $\qquad$ β = 0.0625 (x 2) 2.0

They formed 20 possible physical drum designs with excellent harmonic qualities and gave a scaling procedure which permits the construction of a drum with specified harmonic qualities for any given frequency, from 85Hz to 1370Hz. In most cases (for the areal density assumed) there was a choice of at least three possible designs differing in the diameter and height of the drum, the width of the annular membrane, its tension, and in the resulting harmonic qualities.

3. FURTHER COMPUTATIONS.

3.1 Calculation of the eigenfrequencies.
Some simplification of equations (1) can be achieved [6] if the gas inside and outside the cavity is air. If the volume behind the membrane is bounded by coaxial cylinders of height H, the cavity parameter, ζ, becomes:

$$\zeta = 4\pi a^3/V_1 = 4a/H(\Gamma^2 - 1).$$

However the dimensionless parameters given in equations (1) not only affect the eigenfrequencies in a complex way, but also require prior calculation from the physical parameters chosen in practice. In order to facilitate the design process, a menu driven programme was written which took physical values for the input parameters. The values of Γ were restricted to those given above since at present impedance tables are only available for these values [6]. The other parameters input were the areal density of the membrane ,σ, the inner radius, a, the height, H, of the cylindrical volume behind the membrane, and the tension, T, of the membrane. The output of the programme was (for the first six symmetrical modes): the eigenfrequencies of the modes, the overtone ratios (to F1, the fundamental), the deviation of the overtone ratios from harmonicity in cents, and the (60dB) decay times of the modes.

Using design number 16 [6], with σ = 0.26kg m^{-2} appropriate for Mylar, it was found that variations of the parameters α, β and ζ of ± 5% could produce variations in the fundamental frequency F1 of up to 10%, but left the harmonicity of the overtones essentially unaltered. This reinforced the need for a programme which was based on physical parameter values, and easily used, to facilitate the comparison of a set of measured frequencies with theoretical predictions.

Table 1 summarizes the way in which the physical input parameters H, T, a, σ have been found to influence the eigenfrequencies:

Table 1: Variation of eigenfrequencies with input parameter changes.

↑ indicates an increase, ↓ a decrease, = indicates no change.
A → B indicates 'If A, then B'.

H↑ → F_1↓ ; $F_{N>1}$ = ; C↑
H↓ → F_1↑ ; $F_{N>1}$ = ; C↓

T↑ → F_N↑ ; C=
T↓ → F_N↓ ; C=

a↑ → F_N↓ ; C↓
a↓ → F_N↑ ; C↑

σ↑ → F_N↓ ; C=
σ↓ → F_N↑ ; C=

F_1 is the fundamental frequency, F_N the frequency of the Nth symmetrical mode, and C is the deviation (in cents) of F_N/F_1, from harmonicity.

3.2 Possibilities for tuning.

The predictions of Gottlieb and Aebischer [5] indicated that the fundamental frequency of annular drums is small, and they suggested that because of the resulting short decay time (τ_{60}), this mode need not necessarily be in harmony with the overtones to give the ear the impression of an harmonic tone. The cents value, C, could then be computed relative to the second mode rather than to the fundamental [5]. Using a synthetic generation of the appropriate harmonics, it was found (using 2 subjects only) that for a 200Hz tone, the (-60dB) decay time needed to be 0.2sec or less for the anharmonicity of the fundamental to have no perceived influence on the apparent harmonicity of the (first five) overtones. The fundamental could be as far as 180 cents from perfect harmonicity before its effect was noticeable. This suggests that the drums could be tuned over a mode range simply by adjusting the tension (without simultaneously having to change the volume of the rear cavity). This permits the designs of reference [6] to be grouped as shown in Table 2.

Table 2 Grouping of design numbers from [6] if fundamental ignored
[n] = Group n

Γ = 1.25	Γ = 1.5	Γ = 2.0
[1] 1,3,6	[4] 9,10,13	[7] 17,19
[2] 2,4,7	[5] 11,14	[8] 18,20
[3] 5,8	[6] 12,15	

For each group some average value of the appropriate heights would be used in practical construction. Groups with three elements should be tunable over 2 octaves, and groups with two elements over an octave.

In practice, as will be seen later, the fundamental was found to have a much longer decay time than expected (typically 0.9sec) so this simplification may not be realisable in practice. However the subject of the decay times has not been extensively analysed to date, and some experimental adjustments are indicated later which may ameliorate this situation.

4. DESIGN STRATEGY.

The starting point for a practical design is the membrane material, and the determination of its areal density, σ. Mylar is readily available, but the disagreement between experiment and theory of 6% for the symmetrical modes reported by Christian et al [8] raised questions about its suitability for the annular drum. In particular it appears to have much greater resistance to bending and shearing than calfskin (although it is, of course, potentially more uniform). Finally calfskin with an areal density $\sigma = 0.489 \pm 0.019 kgm^{-2}$ was used.

With the idea of stretching the membrane across two concentric cylinders, the choice of design was next limited by the maximum diameter of the outer cylinder. For aluminium tubing of 25mm wall thickness, the largest stock size (and thus the largest diameter economically available) was 300mm. Using the programme described above, the inner diameter of this cylinder was turned to 257.40 ± 0.02mm. The corresponding outer diameter of the inner cylinder was 171.60 ± 0.02mm. To fit design no. 15, the height was then chosen as 35.4cm. The values of the other parameters mentioned above, for this design, are: $\Gamma = 1.5$, $\alpha = 0.12$, $\beta = 0.210$, $\zeta = 0.654$. The exact value of α depends on the tension T. A tension of about $830Nm^{-1}$ was anticipated, but required measurement - see below.

5. CONSTRUCTION OF EXPERIMENTAL DRUM.

The experimental drum is shown in Figure 1. The membrane was tensioned on the body of a conventional snare drum. The two aluminium cylinders were fixed to a base plate to which the snare drum body was also fixed. It was thus possible to adjust the evenness of the tension using the usual snare drum fittings. The overall tension on the membrane was achieved by perforating the membrane and pulling it down in the centre (over the inner cylinder) using a screw mechanism adjusted by a graduated knob beneath the base plate. The volume of the cavity (the space between the cylinders) could be adjusted by the introduction of a liquid (water) via a tap and pipe near the base of the outer cylinder. The overall weight of the drum was about 35kg.

6. RESULTS.

Before testing the harmonicity of the drum, it was necessary to adjust and measure the tension. Although one of the authors (HAA) has extended the deflection method [8,9] to be suitable for an annulus using a hooped weight, the defections proved too small for convenient measurement by the travelling microscope available. For this reason the approach was adopted [8] of deriving the tension from a measurement of the frequency of one of the (low order) (0,n) modes.

Furthermore a novel Fourier transform analysis was developed by one of the authors (HAA) to determine the decay times of the different eigenfrequencies, based on the widths of the harmonic peaks. It was found to work well, provided that the time span of the recorded signal was at least four times the length of the longest decay time appearing.

Twelve different tests were made: four nominally the same, the remainder with different tensions, different strikers, and different heights of the cavity. The signals were digitised with a 50kHz sampling rate and record lengths of about 13 seconds were normally used. This gave an accuracy of 0.1Hz in the determination of the eigenfrequencies, less than 10% error in evaluating decay times less than 4 seconds and less than 1dB uncertainty in the amplitudes (for $\tau_{60} \leq 4$ sec.)

HARMONIC ANNULAR DRUM.

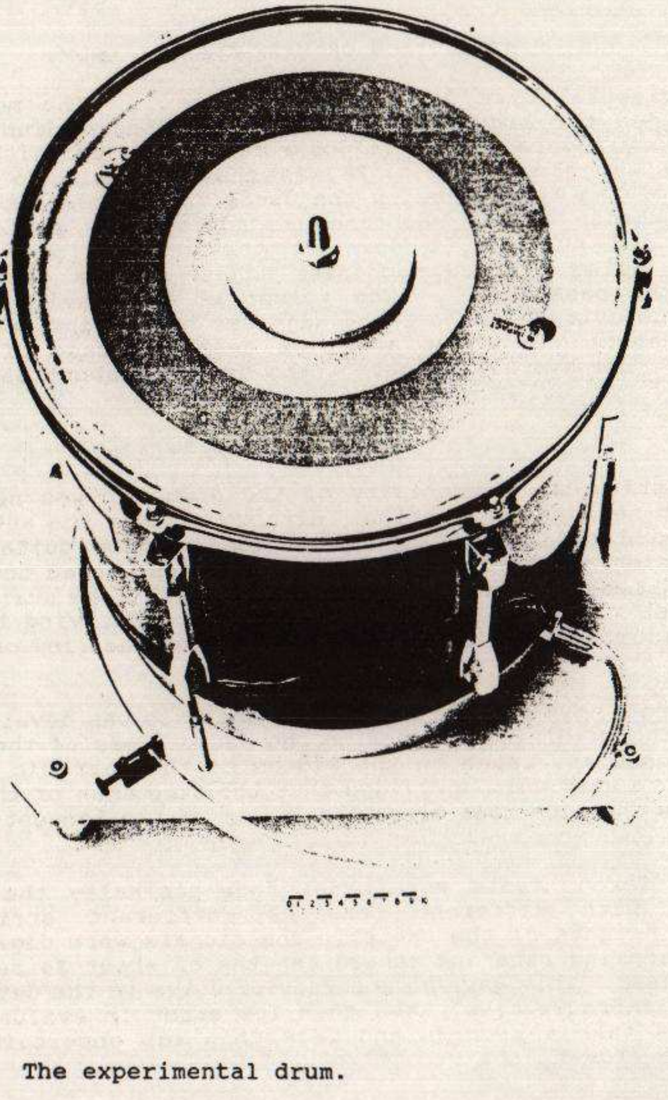

Figure 1: The experimental drum.

It was found that the frequencies measured experimentally agreed very well with the theoretically predicted values. A typical set of results is given in Table 3. Overall almost one third of the frequencies agreed within 0.5%, two thirds within 1.5% and no deviation greater than 3% was found. Four different values were used for the height of the cavity: 35.4cm (no water); 31.7cm, 25.7cm and 18.8cm. When water was to be used, a prior control measurement was made to determine the tension from the frequency of the fundamental. This value of T was then used to predict the fundamental frequency when the water was present. The experimental results agreed with the predictions.

Table 3: A typical set of experimental results, for optimal harmonicity.
$T = 758.8Nm^{-1}$ $H = 35.4cm$

Mode		01	02	03	04	05
Frequency (Hz)	theoretical	455.0	905.7	1366.7	1824.7	2284.6
	experimental	455.0	907.9	1351.3	1833.3	2273.2
Deviation (%)		0	-0.2	+1.1	-0.5	+0.5
F_N/F_1	theoretical	1.0	1.995	3.004	4.010	5.021
	experimental	1.0	1.995	2.97	4.03	4.996
C(cents)	theoretical	0	-8.2	+2.2	+4.5	+7.3
	experimental	0	-4.0	-17.4	+12.6	-1.4
τ_{60} (sec)	theoretical	0.17	8.90	1.09	4.16	1.45
	experimental	0.43	0.19	0.17	0.39	0.28

Table 4 gives a set of results for the shortest value of cavity height, H, used. Halving the volume of the cavity, (almost), raised the frequency of the fundamental by 4.5% from 478.5Hz to 500.0Hz. The higher modes were less affected (theoretically they should be virtually unaltered). The more H was decreased the more the harmonicity of the symmetrical modes was destroyed, as was expected from the theory, (having started from the optimum value of H!). There was some tendency for the measured anharmonicity to exceed that predicted.

Table 4: Results for a non-optimal cavity height.
$T = 834.2 Nm^{-1}$ $H = 18.8cm$. For no water ($H = 35.4cm$),
$F_1 = 478.5Hz$

Mode		01	02	03	04	05
Frequency (Hz)	theoretical	492.9	954.7	1441.1	1923.5	2408.5
	experimental	500.0	932.0	1400.2	1921.1	2395.8
Deviation (%)		-1.4	+2.4	+2.9	+0.1	+0.5
F_N/F_1	theoretical	1.00	1.94	2.92	3.90	4.89
	experimental	1.00	1.86	2.80	3.84	4.79
C(cents)	theoretical	0	-55.6	-44.7	-42.8	-39.9
	experimental	0	-122	-119	-69.8	-73.8
τ_{60}(sec)	theoretical	0.15	6.45	1.05	4.58	1.49
	experimental	0.53	0.35	0.13	0.20	0.06

The decay times measured, with the exception of that of the fundamental, were always much shorter than expected. It may be expected that a larger drum with a wider annulus would have longer decay times. However it is quite possible that, with the present construction, the clamping of the membrane on the rims of the cylinders is not perfect, and energy may drain into the interior and exterior parts of the membrane which are not part of the annulus of interest.

7. CONCLUSION.

The annular drum identified by the theoretical calculations of Gottlieb and Aebischer would appear in practice to have the excellent harmonic properties predicted. The agreement between experiment and theory as far as the eigenfrequencies is concerned testifies to the validity of the basic theory and its potential value in drum design. This all-too-brief investigation has identified a considerable number of theoretical and experimental avenues for exploration, the first of which, it is hoped will be visualization of the mode patterns associated with the individual eigenfrequencies.

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