CIRCULAR ARRAY PROCESSING FOR SONAR APPLICATIONS

H.D. Griffiths (1), R. Eiges (1) and Prof. D.E.N. Davies (2)

(1) University College London, Department of Electronic & Electrical Engineering, Torrington Place, London WC1E 7JE, UK
(2) University of Technology, Loughborough, Leics LE11 3TU, UK

1. INTRODUCTION

Circular arrays in communications and radar systems at RF and microwave frequencies have been studied over the past three decades. Their particular advantage lies in applications such as beamforming, null-steering or direction finding over the full 360° of azimuth angle and over broad bandwidths, and several practical systems have been built and demonstrated. The application of these techniques to sonar systems appears to have some significant attractions, since the frequencies involved are sufficiently low that all the signal processing may be implemented digitally, avoiding the imperfections in analogue components which ultimately limit the performance of microwave circular arrays.

The purpose of this paper is therefore to consider the use of these techniques in sonar applications. Section 2 provides a brief summary of the theory of circular arrays and their uses. Section 3 describes the basis of a simulation program that has been written to predict the behaviour of a circular array of sonar transducers, then Section 4 presents and discusses some results obtained with the program, and compares them with some experimental measurements presented in a companion paper [1]. Finally, Section 5 draws some conclusions and presents ideas for further work.

2. CIRCULAR ARRAY THEORY

This section provides a brief summary of some of the relevant properties of circular arrays and phase modes. A full account may be found in refs. [2] and [3].

Consider firstly a continuous array (i.e. of very small interelement spacing) of antenna elements or sonar transducers. The excitation of the array can be considered as a periodic function of azimuth angle ϕ, of period 2π, and may be analysed in terms of a Fourier series:

$$F(\phi) = \sum_{m=-N}^{N} C_m \exp(jm\phi) \qquad \ldots (1)$$

Each term of the Fourier series is known as a *phase mode*, and each coefficient C_m represents the complex Fourier coefficient of each spatial harmonic of the array excitation.

When an array is excited by a single mode, the far field radiation pattern $D_m(\phi)$ has a similar phase mode form, but is weighted by a complex coefficient, K_m, known as the phase mode coefficient:

$$D_m(\phi) = C_m j^m J_m(\beta r) \exp(jm\phi) = K_m \exp(jm\phi) \qquad \ldots (2)$$

where $J_m(.)$ is the m^{th} order Bessel function, $\beta = 2\pi/\lambda$ and r is the array radius.

Thus the zero-order phase mode represents an excitation (and a corresponding far-field pattern) that is omnidirectional in amplitude and of constant phase, as a function of ϕ; the m^{th} phase mode represents an excitation that is also omnidirectional in amplitude, but has $2m\pi$ variation of phase shift as a function of azimuth. The $-m^{th}$ phase mode is similar, but the sense of phase variation is opposite.

When a discrete array is used the same theory applies, but the excitation function is effectively sampled at the transducer locations, by a sampling function which we shall call $S(\phi)$. Hence:

$$F_m(\phi) = C_m e^{jm\phi} S(\phi) \qquad \ldots (3)$$

For an array of n omnidirectional transducers:

$$S(\phi) = \sum_{q=-\infty}^{\infty} e^{jnq\phi} = 1 + \sum_{q=1}^{\infty} e^{jnq\phi} + \sum_{q=1}^{\infty} e^{-jnq\phi} \qquad \ldots (4)$$

giving:

$$D_m(\phi) = C_m j^m J_m(\beta r) \exp(jm\phi)$$

$$+ \sum_{q=1}^{\infty} C_m j^{-g} J_{-g}(\beta r) \exp(-jg\phi) + \sum_{q=1}^{\infty} C_m j^h J_h(\beta r) \exp(jh\phi) \qquad \ldots (5)$$

where $g = (nq-m)$ and $h = (nq+m)$.

It can be seen that this corresponds to the same result as for omnidirectional transducers (equation (2)), plus higher order distortion modes, or *spatial ripple*. To keep the level of the spatial ripple low the inter-transducer spacing should be less than half a wavelength.

The only frequency-dependent term in equation (5) is the Bessel function $J_m(\beta r)$. There will clearly be some frequencies at which $J_m(\beta r) = 0$, i.e. at which the far-field amplitude falls to zero. If wideband performance is to be achieved, then either such frequencies must be avoided, or some means must be found to smooth out the zeroes of the Bessel functions. Fortunately, the use of transducers with directional patterns is able to achieve this [4], and it has been found that directional patterns of the form $(1 + \cos\phi)$ give good results.

In an RF or microwave circular array, the discrete Fourier Transform that provides the phase modes may conveniently be realised using a Butler Matrix network. The individual phase mode terms, which are isolated (orthogonal), are then available at the output ports of the Butler Matrix. An n-element circular array yields n phase mode terms, from -n/2 to +n/2, though the -n/2 and +n/2 phase modes are actually identical and emerge from the same port of the matrix. In a sonar circular array the same processing would be achieved by digitising the signals directly at the transducers, followed by a digital DFT.

Circular arrays excited by means of phase modes have been used for the following applications:

2.1 Null steering: If two phase modes whose orders differ by one are equalised in amplitude and added, there will be one direction in which their phase difference is 180°, and hence in which they cancel, forming a null. If a phase shifter is inserted in one of the paths, then the null may be steered in direction, and the null direction is directly proportional to the setting of the phase shifter. Furthermore, if the variation with frequency of the phase mode coefficients K_m can be compensated, then the direction of the null will be invariant with frequency [5]. The presence of the higher order modes (equation (5)) will cause variations in the null direction and depth, but provided they are small the effect will be insignificant.

2.2 Wideband pattern synthesis: Phase modes can be used for pattern synthesis, in exactly the same way as for a linear array [6]. The main differences are: (i) phase modes are omnidirectional in azimuth, so the 'elements' are also omnidirectional. The radiation pattern corresponds to the array factor for the linear array; (ii) phase modes are orthogonal functions, so there is no mutual coupling between the 'elements' of this linear array; (iii) the radiation patterns have the same shape (in θ space) as those formed by a linear array (in $u = kd\sin(\theta)$ space) with an interelement spacing of $\lambda/2$; (iv) for a discrete array, exciting a single mode port excites a periodic sequence of modes. For example, if the +1 mode is excited on a four element array, the -7, -3, +5, and +9 modes are also excited. These additional harmonics correspond to additional linear array 'elements'.

If the variation with frequency of the phase mode coefficients K_m can be compensated, then the patterns formed will be independent of frequency. Figure 1 shows an example of an instantaneously broadband beam pattern formed in this way [7], with < -20dB sidelobes, measured over the frequency band 8-12 GHz.

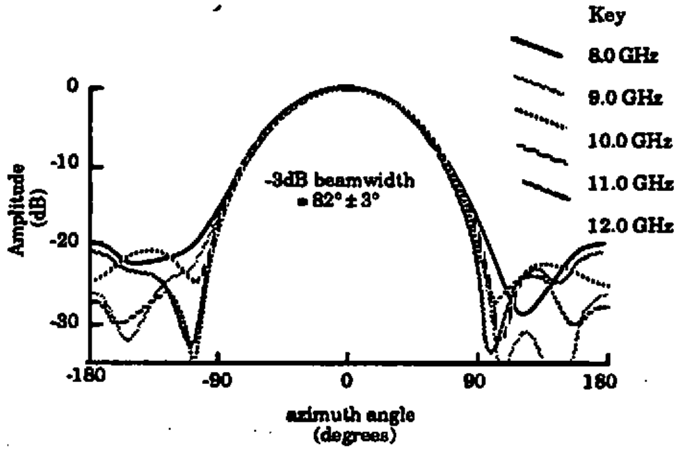

Figure 1. Example of instantaneously broadband beam pattern synthesised over the band 8-12 GHz.

Although this result demonstrates a fairly convincing constant-beamwidth pattern, the sidelobe levels vary significantly with frequency. This is due to the difficulty of realising networks with accurate phase and amplitude responses at microwave frequencies.

2.3 <u>Direction finding</u>: A plane wave incident on the array will excite all phase modes simultaneously. The phase difference between two adjacent phase modes will provide a direct indication of the direction of arrival of the signal. Again, if the variation with frequency of the phase mode coefficients K_m can be compensated, then direction finding can be carried out over a broad bandwidth. As an example, an experimental system based on this principle has been constructed and demonstrated, operating over the band 2-30 MHz [8], and is now commercially in production.

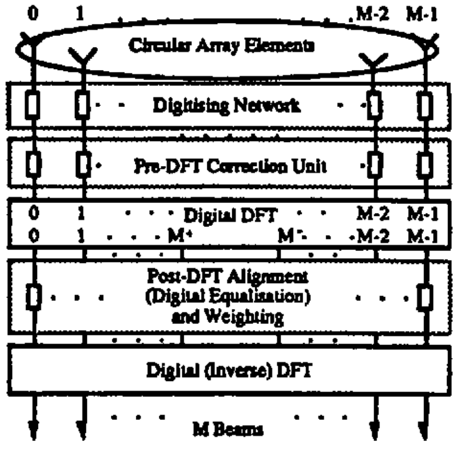

Figure 2. Digital multiple-beamforming processing for circular arrays - schematic diagram

Clearly, all these functions could also have useful applications to sonar systems. A sonar array employing an analogue or indeed digital beamforming architecture could be implemented with a far greater control over its feed network responses which leads to superior performance. Figure 2 shows a schematic diagram of a modally excited circular array which uses digital processing to form a multiple-beam pattern.

The next section describes a computer program to simulate and evaluate the above foregoing theory applied to a circular array of sonar transducers.

3. SIMULATION PROGRAM

The computer program is a user-friendly menu-driven PC package with built-in 'help' screens available at most menu levels. It may be used to assess the effectiveness of various schemes for the calibration and alignment of a circular array of discrete radiators, analyse under various configurations and error models the performance of directly or modally formed far-field beams and simulate phase comparison DF based on either omni-directional or directional phase mode patterns.

The input to the program input is entered either manually or loaded from a pre-saved data file, and includes: the array geometry (number of elements, array radius), element pattern definition (uniform, parametrically synthesised, randomised or loaded from a measured data file), error model (amplitude and phase fluctuations, element pattern rotation as well as phase centre displacements), weighting input (amplitude and phase excitation taper for directly or modally formed beams), correction input (narrowband pre-DFT/post-DFT compensation algorithms applied to specified phase modes) and ranges for relevant parameters (frequency, time, phase modes and angular direction).

Output may be graphically displayed on user-defined scales (and dumped onto a graphics printer), listed (and printed) in table form or saved as two ASCII data files, one of which allows the output data to be read and displayed by external graphics packages (such as CricketGraph and DeltaGraph for the Apple Macintosh) while the other is an input/output data file that can be read and displayed by the simulation program itself.

A number of simulated results for a 7-element sonar array, for which equivalent practical results are given in [1], are presented and discussed in the following section.

4. RESULTS AND DISCUSSION

The measured element patterns at 30 kHz of this array, with a soft cylindrical baffle core, were modelled as:

$$F_e(\phi) = 0.612 + 0.384 \cos(\phi+\Delta\phi) + 0.004 \cos 2(\phi+\Delta\phi) \qquad \ldots (6)$$

where for each element, $\Delta\phi$ is a constant rotational shift which is modelled as a uniformly distributed random angular error of ±15°. In addition, uniformly distributed gain and phase fluctuations of ±1 dB and ±10° respectively are assumed between channels. The array radius is 25.4 mm and the velocity of sound in water is taken as 1500 ms^{-1}. This means that at 30 kHz, the frequency at which all simulations were carried out, the radius is equal to 0.508λ and the inter-element spacing is 0.456λ.

Figure 3 displays the nominal amplitude patterns of phase modes -1, 0 and 1. The response is nominal in the sense that the electrical (gain and phase) errors as well as the rotational angular error have all been set to zero. The modes have, however, been aligned by applying a (post-DFT) least-squares correction algorithm based on sampling each phase mode at 14 observation angles. A small ripple of ±0.1 dB is observed for mode 0 whereas modes -1 and 1 show a somewhat larger fluctuation of ±0.5 dB.

Figure 4a shows the effect of introducing the above electrical and rotational errors on the amplitude patterns of the 3 phase modes. No attempt was made to compensate for these errors but, as before, post-DFT alignment has been applied. As can be seen, the ripple in amplitude of all modes has deteriorated to a level of (depending on the specific ensemble of errors) ±2.0 dB to ±2.5 dB.

The effect of applying a (pre-DFT) correction algorithm to try and compensate the set of phase modes for the electrical and rotational errors is demonstrated in Figure 4b, where the level of amplitude ripple is shown to have been reduced to less than ±1 dB for mode 0 and ±1 dB – ±1.5 dB for modes -1 and 1. In fact it may be shown that in the absence of rotational errors, the array may be fully compensated and the amplitude ripple reduced to those obtained in Figure 3.

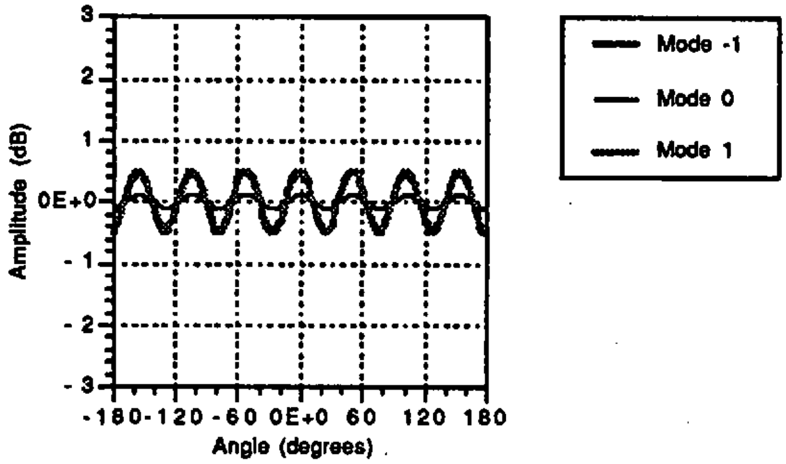

Figure 3. Amplitude characteristics of modes 0 and ±1 for a nominally excited 7-element array (radius = 0.508λ, element pattern = 0.612 + 0.384 cos ϕ + 0.004 cos 2ϕ).

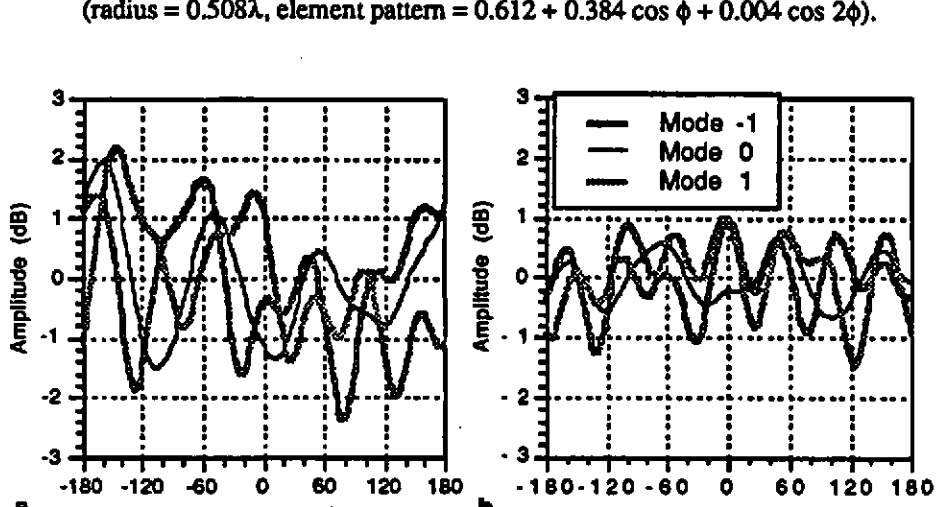

Figure 4. Amplitude characteristics of modes 0 and ±1 for a 7-element array under excitation errors of: ±1dB in amplitude, ±10° in phase and ±15° in rotation, after:
a. post-DFT alignment only b. pre-DFT + post-DFT correction

Perhaps more relevant than the amplitude patterns is the phase response of phase modes. The simulated phase patterns of the 3 modes are shown in Figures 5, 6 and 7 for nominal excitation, electrical and rotational error condition and for the case of pre-DFT compensation of errors respectively, and as before, all patterns are aligned in the least-squares sense. The phase errors which are displayed alongside the phase plots refer to the phase deviation of each aligned phase mode pattern from the ideal $e^{-jm\phi}$ response.

It can be seen that the introduction of electrical and rotational errors increases the phase error of the array phase modes from a level of ±2° for the zero mode and ±4° for modes -1 and 1 to a level of ±10° to ±15° when pre-DFT correction is not applied and ±5° to ±10° when pre-DFT compensation is attempted. Here too it may be shown that in the absence of rotational errors, the array may be fully compensated and the phase ripple reduced to those shown in Figure 5.

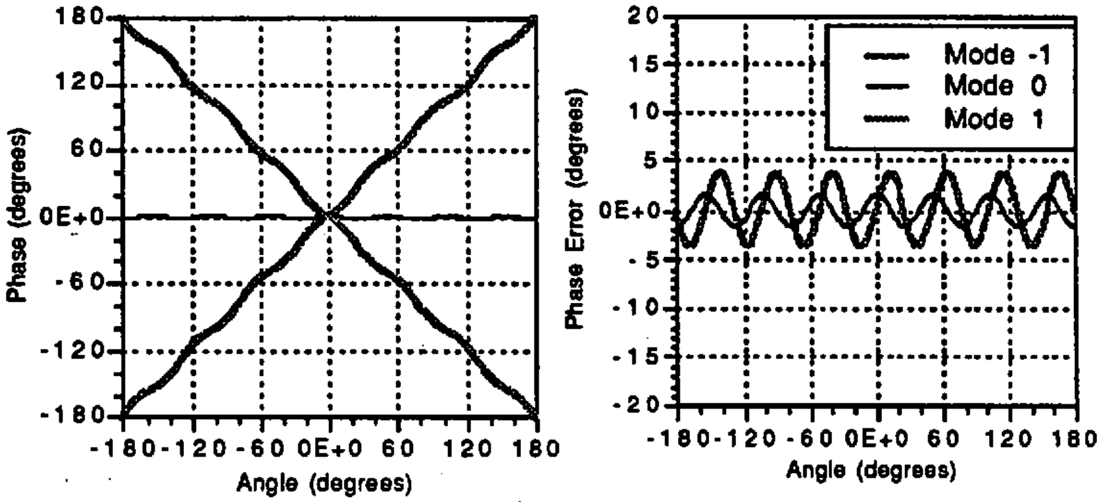

Figure 5. Phase and phase error patterns of modes 0 and ±1 for a nominally excited 7-element array.

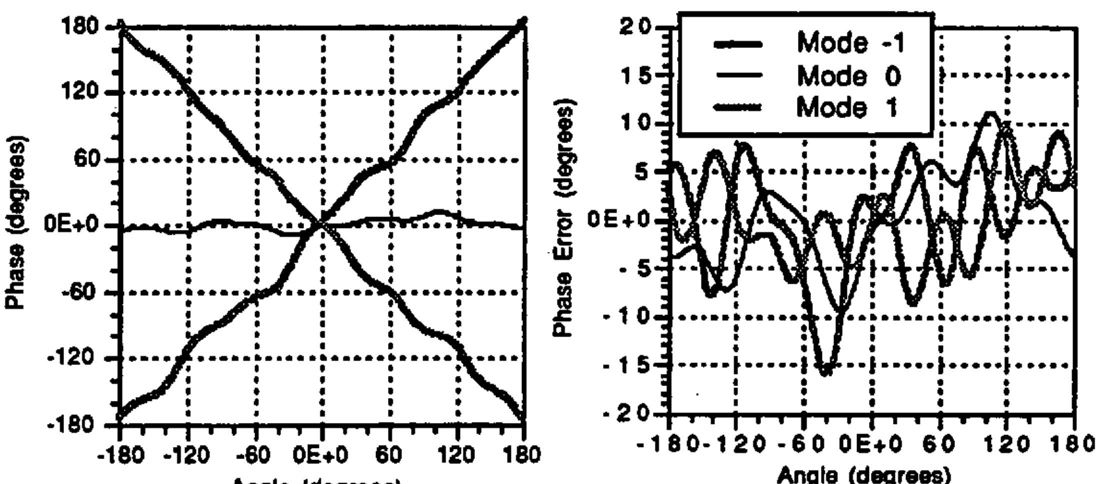

Figure 6. Phase and phase error patterns of modes 0 and ±1 for a 7-element array under excitation errors of: ±1dB in amplitude, ±10° in phase and ±15° in rotation, after post-DFT alignment only.

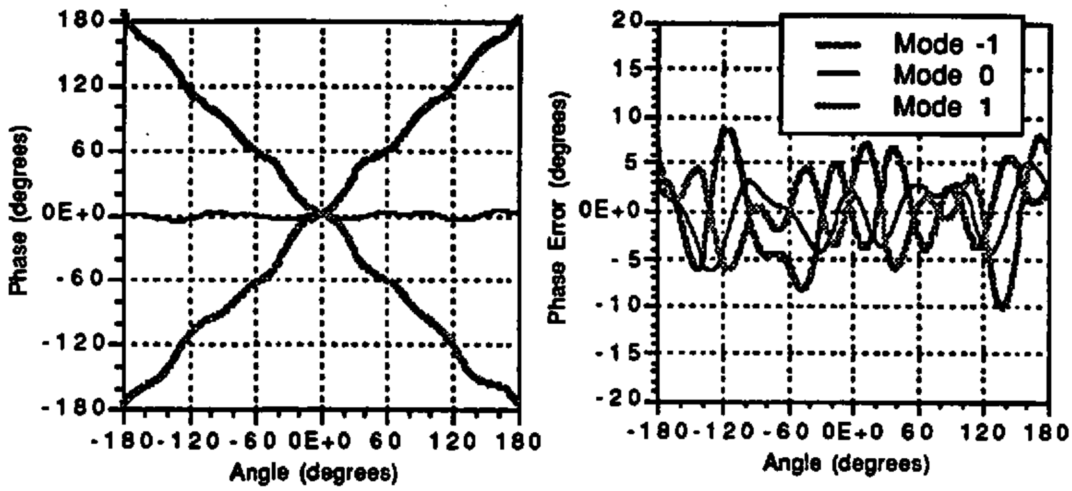

Figure 7. Phase and phase error patterns of modes 0 and ±1 for a 7-element array under excitation errors of: ±1dB in amplitude, ±10° in phase and ±15° in rotation, after pre-DFT correction and post-DFT alignment.

Finally, Figure 8 presents a multibeam patterns formed by the application of a 7-point (inverse) DFT on the set of 5 nominal phase modes 0, ±1, ±2 which are pre-attenuated as follows:

Mode	-2	-1	0	1	2
Weights	-10.5 dB	-2.3 dB	0 dB	-2.3 dB	-10.5 dB

These patterns will be degraded by errors, but correction algorithms may be applied to yield, for the above excitation and rotational errors, very reasonable beam patterns with peak sidelobe level better than -20 dB.

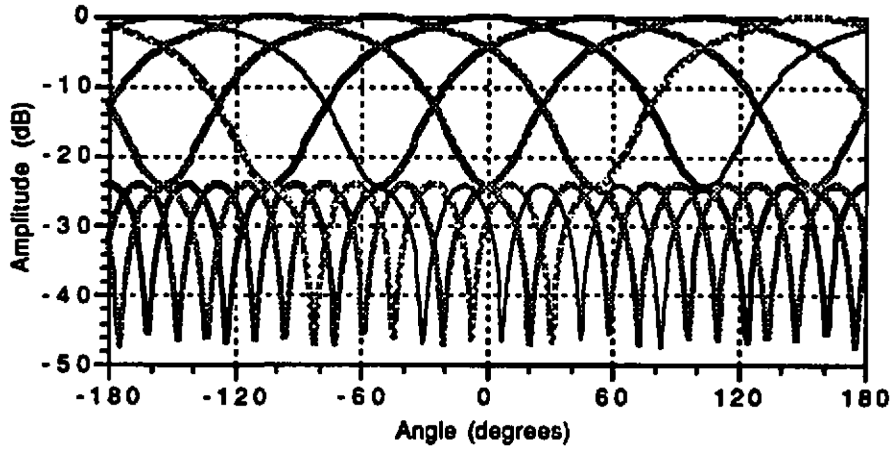

Figure 8. Multibeam pattern formed by inverse DFT of 5 weighted phase modes from a 7-element array (weights: -10.5 dB, -2.3 dB, 0 dB, -2.3 dB, -10.5 dB).

By selecting a different set of phase modes, each of the resulting directional beams may be given a phase gradient thus forming what may be described as directional phase modes.

These results show how it is possible to use the simulation program to predict the form of the phase modes and the patterns formed from them, to define the signal processing operations required, and to assess the effect of errors on the system performance.

5. CONCLUSIONS

The application of circular array techniques to sonar systems is potentially very appealing. The low frequencies involved enable such tasks as null steering, multiple-beamforming and direction finding to be realised digitally, thereby improving system performance and allowing more complex processing together with elaborate alignment routines. Owing to the low velocity of sound, the size of such an array will not be prohibitive: a sonar array designed to operate at an upper frequency of 30 kHz would have dimensions similar to those of a corresponding microwave array designed for an upper frequency of 6 GHz. One other interesting point to note is that the sonar array is expected to provide better elevation coverage as it is free from the problem of elevation-dependent element polarisation.

Future study on the subject of circular arrays in the sonar context will concentrate on the digital implementation of classical methods over wide bandwidths and extended elevation coverages, and will also address the application of modern superresolution techniques, where schemes based on mode space will be compared to the more traditional element space or beam space methods.

6. REFERENCES

[1] Griffiths, J.W.R., Curtis, T.E., Wood, W.J. and Qian, Q., 'An experimental sonar system using a circular array'; these proceedings.

[2] Davies, D.E.N., 'Circular arrays', Chapter 12 in *The Handbook of Antenna Design, Vol.II*, A.W. Rudge, K. Milne, A.D. Olver and P.Knight eds, Peter Peregrinus, Stevenage, 1983.

[3] Davies, D.E.N., 'Circular arrays: their properties and potential applications'; Invited Review Paper, Proc. 2nd IEE Intl. Conference on Antennas and Propagation, York, IEE Conf. Publ. No. 195 Part I, pp1-10, April 1981.

[4] Rahim, T. and Davies, D.E.N., 'Effect of directional elements on the directional response of circular antenna arrays'; Proc. IEE Pt.H, 1982, Vol.129, No.1, pp18-22.

[5] Griffiths, H.D., Karavassilis, N., Jones, M.R., and Davies, D.E.N., 'Broadband nulls from a circular array'; Proc. 4th IEE Intl. Conference on Antennas and Propagation, Warwick, 1985; IEE Conf. Publ. No.248, pp304-306.

[6] Davies, D.E.N, 'A transformation between the phasing techniques required for linear and circular aerial arrays'; Proc. IEE, 1965, Vol.112, No.11, pp2041-2045.

[7] Jones, M.R. and Griffiths, H.D., 'Broadband pattern synthesis from a circular array'; Proc. 6th IEE Intl. Conference on Antennas and Propagation, Warwick, IEE Conf. Publ. No. 301 Part I, pp55-59, April 1989.

[8] Cvetkovic, M., Davies, D.E.N., Griffiths, H.D. and Collins, B.S., 'An HF direction-finding and null steering system employing a four-element circular array'; Proc. 4th IEE Intl. Conference on HF Radio Systems and Techniques, London; IEE Conf. Publ. No.284, pp221-225, April 1988.