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SITE AND LABORATORY EXPERIMENTS ON VIBRATION TRANSMISSION

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1. SUMMARY

Research at Cambridge on building vibration includes laboratory measurements of structural damping, measurement and analysis of building vibration and analytical modelling of building and foundation response. An earlier paper [6] described aspects of our analytical approach, while this paper discusses experimental work including an on-going program of measurements at the Gloucester Park development in London. Measurements of the impulse response of piles and the response of the building to railway excitation are compared with theoretical models. Some laboratory experiments have been carried out to investigate damping in reinforced concrete.

2. A SIMPLE MODEL OF A BUILDING

A building is an intricate structure and it is not at present possible to model a building's response to vibration excitation in sufficient detail to give exact answers. This is so for two reasons. Firstly, the dynamic behaviour of building elements themselves is poorly understood (particularly their damping characteristics) and the interaction between elements such as floors and columns is complex. Secondly, vibration at a given point in a building is not usually generated by a single time-varying force acting at a point, but rather by a large number of forces acting both inside the building (for example from air-conditioning plant) and outside (underground railways).
There is a need for simple models of buildings, and in particular for models of buildings constructed on resilient foundations for the isolation of ground vibration. In the 1960's, when buildings in London were first being mounted on springs, the building was represented as a rigid block so that the isolated building could be modelled as a simple single-degree-of-freedom oscillator. It is now well known that this kind of model is inadequate (as shown later in figure 2), but even so its use is still commonplace amongst designers of isolated high-rise buildings. A better model includes some representation of column and pile elasticity, and this is described below.

2.1. Column on pile and pad

A single column resting on a single pile separated by a resilient isolation pad is used to model a building. This model takes no account of vibration that reaches a given point in a building by pathways other than straight up the column as shown in figure 1. By considering building vibration as a random process, it is assumed that the excitation reaching the base of each column is uncorrelated with other sources of vibration input. With this assumption, it is possible to use coherence functions to help distinguish between vibration that is transmitted directly up a column and that which arrives by a circuitous path. In this way, the simple columnar building model described below can be validated experimentally as discussed in sections 3 and 4. The calculation and interpretation of coherence functions is described in section 3 and also in [1] and [5].

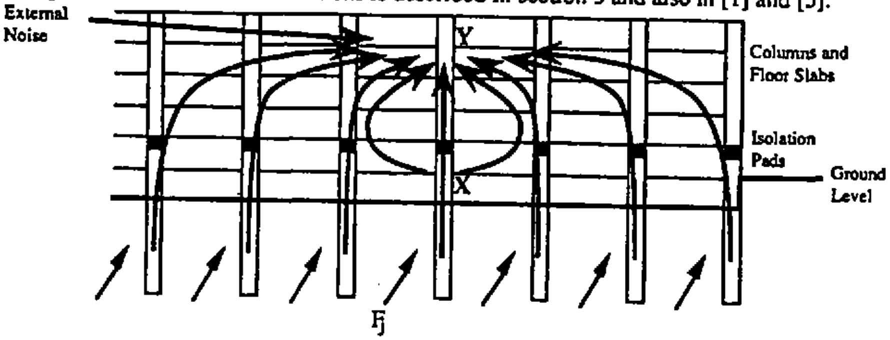

Figure 1 A schematic model of the transmission of vibration in a building. The forces input at the base of each column are assumed to be incoherent. The solid curved arrows indicate vibration transmission paths from the base of each column to measurement points X and Y.

Figure 2a shows a 30-metre column resting on a pile separated by a resilient pad. It is of some interest to a building designer to compare the vibration level at some point up the column with the level of vibration at the pile cap in the absence of the column as this enables predictions of building vibration levels to be made from pile-cap measurements. Figure 2b shows the transmissibility at four points up the column. Here, transmissibility is defined as the ratio of the amplitude response of the column to the vibration amplitude that the pile cap would have in the absence of the column. The differential equation which describes the response of this pile-column system is

$$\frac{\partial^2 y}{\partial t^2} + \frac{\mu}{\rho}\frac{\partial y}{\partial t} - \frac{E}{\rho}\frac{\partial^2 y}{\partial z^2} + \frac{G}{\rho A}\frac{\omega r_0}{c_S} H_1^{(2)}\left(\frac{\omega r_0}{c_S}\right) \Big/ H_0^{(2)}\left(\frac{\omega r_0}{c_S}\right) = 0 \tag{1}$$

where y is the axial displacement of the column. The pile model used is that of Novak [7] (see also [3]) where the soil is considered to be made up of infinitesimally thin horizontal layers which transmit shear waves to infinity and the base of the pile is taken to be anchored in bedrock. The column and pile properties are Young's modulus E, density ρ and viscous damping parameter μ. Additionally for the pile, its radius is r_0, cross sectional area A, soil shear modulus G and soil shear-wave speed c_s. $H_0^{(2)}$ and $H_1^{(2)}$ are Hankel functions of the second kind of order 0 and 1. The properties of pile, pad and column used in figure 2 are as follows:

column	Young's modulus E	10GPa	density	2400kg/m^3
	length	30m	damping μ	51.3kN s m^{-4}
pad	stiffness k	45.2MN/m	damping c	160kN s/m
	natural frequency	9Hz (for column mass of 14×10^{3}kg)		
pile (as for column with)	radius r_0	0.25m	soil shear modulus G	100MPa

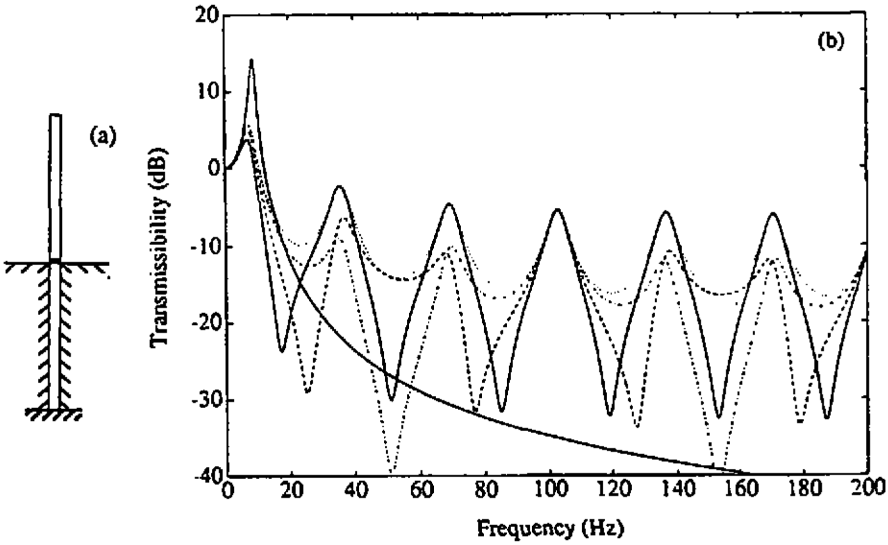

Figure 2 (a) A column resting on a pile separated by a resilient pad. (b) Transmissibility for the column at 0m (solid), 10m (dashed), 20m (chained) and 30m (dotted) up the column. Also shown is the response for the column modelled as a rigid block (solid line).

For the case of a column placed on a pile without an isolation pad, the column significantly influences the response of the pile. For the case shown in figure 2, the resilient pad decouples the column dynamics from the pile at frequencies above about 10Hz and the influence of the pile is not apparent. It is important to ba able to model pile dynamics when considering the transmission of ground vibration through piles into the foundation, and this is the subject of current research.
The rigidity of a pile embedded in the soil is shown in figure 3. For a pile embedded at its base, figure 3a shows the variation with soil shear modulus of the pile-cap receptance (displacement response to a unit harmonic force). The same is shown in figure 3b for a pile that is free at its base. It can be seen that when the shear modulus is very low, the pile behaves as a lightly-damped column, and the two different boundary conditions shift the resonance and anti-resonance peaks as expected for the fixed-free and a free-free boundary conditions. When the soil shear modulus takes a more realistic higher value (greater than 1MPa) the pile is inhibited by the effect of surrounding soil and behaves less like a free column. The boundary condition at the base of the pile is then unimportant for the calculation of pile-cap receptance.

2.2. Variation of response with pad height

Using the model developed above, it is possible to examine the effect of varying the height of the isolation pad above the pile cap. For instance, to minimise vibration on the tenth floor of a given building it may be more sensible to put the isolation pad at third-floor rather than ground-floor

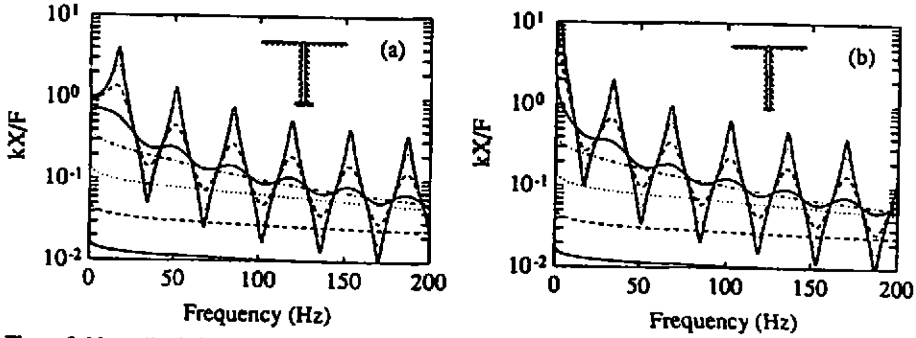

Figure 3 Normalized pile-cap receptance kX/F for a pile with varying soil shear modulus: (a) with the base of the pile anchored to bedrock; (b)with the base of the pile free.

level. The variation of top-of-column response with pad height is shown in figure 4a. The stiffness of the pad at each position is chosen so that the rigid-body isolation frequency is 9Hz and the pad damping is chosen to keep the damping ratio constant. When the pad is close to the top of the column (at 29.75m) the transmissibility is the same as that for a rigid body resting on a 9Hz spring, but its input is the response at the top of a 30m column excited at its base. A three-dimensional plot of the variation of response with pad height is also shown in figure 4.

3. SITE MEASUREMENT OF BUILDING VIBRATION

Measurements of traffic-induced ground vibration at Cambridge were first made at a green field site [4] to test theoretical predictions of the power spectrum of ground vibration in the vicinity of a busy roadway. These predictions may be used to evaluate the input for building models such as those currently under development at Cambridge. The instrumentation used for these site measurements is also used for measurements of vibration in buildings.

3.1. Instrumentation

Instrumentation for measurement of traffic-induced ground vibration - whether inside or outside a building - must include a number of sensitive vibration transducers, with the capability of transmitting signals a long distance, low noise multi-channel signal conditioning and data logging equipment, and mass data storage. The instrumentation used to gather data presented in this paper is given here:

item	number	specification
accelerometers	6	B&K8318
line drive supplies	3	B&K2813
impulse hammer	1	mass 15kg, drop height 2m
force transducer	1	Kistler 971A
accelerometer	1	DJB 302 A/03/W

signal processing	1	CED 1703 16 channel programmable filters and amplifiers
data logger	1	CED 1401 16 channel laboratory interface
computer	1	IBM PC/AT
tape streamer	1	Tecmar QT 60e
miscellaneous		electric generator, 100m multi-core cables, two-way radio

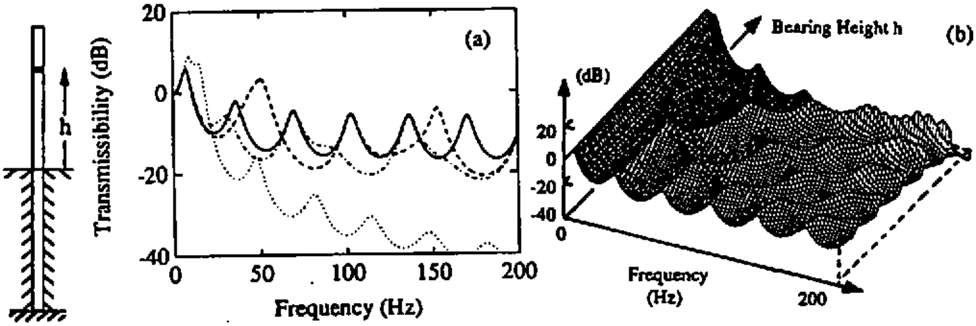

Figure 4 (a) Top-of-column transmissibility with isolation pad at 0m (solid), 10m (dashed), 20m (chained) and 29.75m (dotted) up the column; (b) three dimensional plot of the same.

3.2. Measurement

The object of field tests is to test and improve the simple column model of a building. Impulse tests, which involve applying a known force to the structure - in our case using a 15kg hammer falling from 2 metres - are used to compare the actual response of a structure with that predicted from theory, and this has been done with some success for the response of piles. Measurements of building vibration during the passage of a train are used to investigate the validity of the single-column model and the uncorrelated-input assumption of section 2 by estimating transmissibilities and comparing these with theory. These measurements are also compared with ground vibration measurements made before construction work commenced.

Difficulties are often encountered when making measurements of traffic-induced vibration in an occupied building or on a construction site. Access is often restricted to periods when little or no work is taking place and this greatly restricts the amount of time available to set up equipment and take measurements, particularly since rail- and traffic-induced vibration is usually greatest at times when the site is busy. Even then, noise and vibration on site can swamp any measurements of vibration from passing trains and road vehicles. Distances involved on site are large so that long cable runs are required, and obstacles (such as walls and stairways) give rise to difficulties in communication. For this reason, a two-way radio is very useful.

3.3. Analysis

In a very short space of time, large amounts of data are collected on site. It is necessary to condense this data for ease of handling and presentation, and the methods of data processing used in Cambridge are described briefly here.

In order to analyse impulse data, time traces of both the impulsive force and measured acceleration responses are transformed into the frequency domain using an FFT algorithm. Frequency response functions are then calculated by division. Extraneous noise (from ambient site vibration) is always present in impulse test data, but by calculating the average of several identical impulse responses the level of noise is reduced.

The analysis of vibration data from passing trains not straight-forward. Firstly, the vibration generated by each train pulling into, and out of, a station is a non-stationary process because the amplitude and frequency content of vibration varies considerably with train speed and position. Secondly, each successive train provides a different input, depending upon its size and weight, and also upon which track and in which direction it is travelling at the time - there may even be two trains travelling at once. It is not possible to measure the force input to the track, so these variations cannot be quantified. Thirdly, there is always extraneous noise, and this cannot easily be distinguished from train vibration as both are unknown random processes.

Random-process theory is used to analyse train-vibration data and it is necessary to assume stationarity. Two stationary processes are identified, those of ambient noise and of train vibration. These are analysed separately and designated 'ambient' and 'train' respectively. Auto- and cross-power spectral densities are calculated for both train and ambient vibration, and by plotting these spectra together, those features of the spectrum attributable to ambient vibration and those caused by train vibration can be identified.

Transmissibility functions between two measuring points X and Y are calculated from the measured auto-spectra $S_{xx}(\omega)$ and $S_{yy}(\omega)$ and the cross-spectrum $S_{xy}(\omega)$ and compared with theoretical models. 'Total' and 'direct' transmissibility may be calculated as follows:

$$T_{total}(\omega) = \sqrt{\frac{S_{yy}(\omega)}{S_{xx}(\omega)}}, \qquad T_{direct}(\omega) = \left|\frac{S_{xy}(\omega)}{S_{xx}(\omega)}\right|, \qquad \eta^2(\omega) = \left(\frac{T_{direct}(\omega)}{T_{total}(\omega)}\right)^2 \qquad (2)$$

The 'total' transmissibility takes no account of the correlation between vibration at X and Y, and since a building occupant does not distinguish between correlated and uncorrelated vibration, this is a useful 'user index' of vibration transmission. The 'direct' transmissibility takes account only of vibration which is transmitted linearly to X and Y from common sources of vibration and is more useful for comparison with theoretical models. The 'coherence' $\eta^2(\omega)$ is the square of the ratio of the two transmissibilities and is a measure of linearity and correlation.

For a perfectly noise free linear system, as shown in figure 1, excited by forces $(f_1, f_2, ...)$, the frequency-response functions for the displacement at X and Y due to the j^{th} force alone are $H_{jx}(\omega)$ and $H_{jy}(\omega)$ respectively. If it is assumed that all input forces have the same auto-spectrum but that they are uncorrelated, the two transmissibilities and the coherence may be estimated from theory as

$$T_{total} = \sqrt{\frac{\sum_j H_{jy}^* H_{jy}}{\sum_j H_{jx}^* H_{jx}}}, \qquad T_{direct} = \frac{\left|\sum_j H_{jx}^* H_{jy}\right|}{\sum_j H_{jx}^* H_{jx}}, \qquad \eta^2 = \frac{\sum_j H_{jx}^* H_{jy} \sum_j H_{jx} H_{jy}^*}{\sum_j H_{jx}^* H_{jx} \sum_j H_{jy}^* H_{jy}} \qquad (3)$$

If it is assumed that vibration transmission directly up the column is dominant, then the coherence function becomes close to unity. If a unity coherence function is observed in practice, then there is strong support for the column model for a building. In practice, the observed coherence is not found to be unity and the interpretation of non-unity coherence is currently under investigation.

4. MEASUREMENTS AT GLOUCESTER PARK

A residential, retail and office complex is being constructed above the London Underground's Gloucester Road railway station. The railway lines run diagonally across the site, with the District and Circle lines running close to the surface and the Piccadilly line some 20 metres below ground level. The retail section is built directly above the District and Circle Line platforms on a concrete raft structure. The apartment building is to one side of the train lines although its piles pass within 4 metres of the Piccadilly line's tunnel. Train vibration levels in the residential and retail areas have been monitored at various stages of construction, and impulse tests have also been carried out.

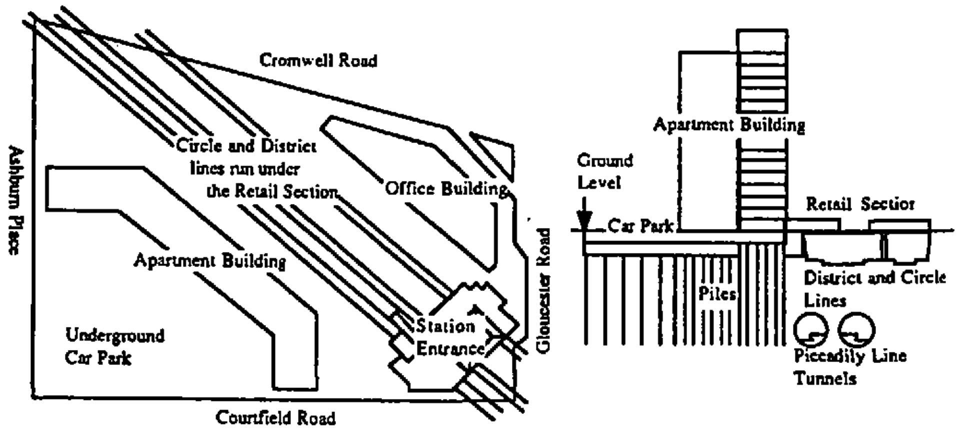

Figure 5 Plan and sectional elevation of the Gloucester Park site

4.1. Impulse on pile cap

At an early stage of construction, a series of impulse tests were performed on the pile caps in order to deduce the dynamic properties of the foundations. Figures 6a and 6b show superimposed the results of a set of six impulses on the top of one of the piles, demonstrating good repeatability of both force and the measured acceleration response of the pile cap. The applied impulsive force was estimated here by measuring the acceleration of the hammer head.

The experimental frequency response function is shown in figures 6c and 6d alongside the theoretical frequency response of a pile based on Novak [7] as given in equation 1. Soil properties for the pile model have been determined by fitting the best theoretical impulse response to the measured data and very good agreement has been obtained between measurement and theory.

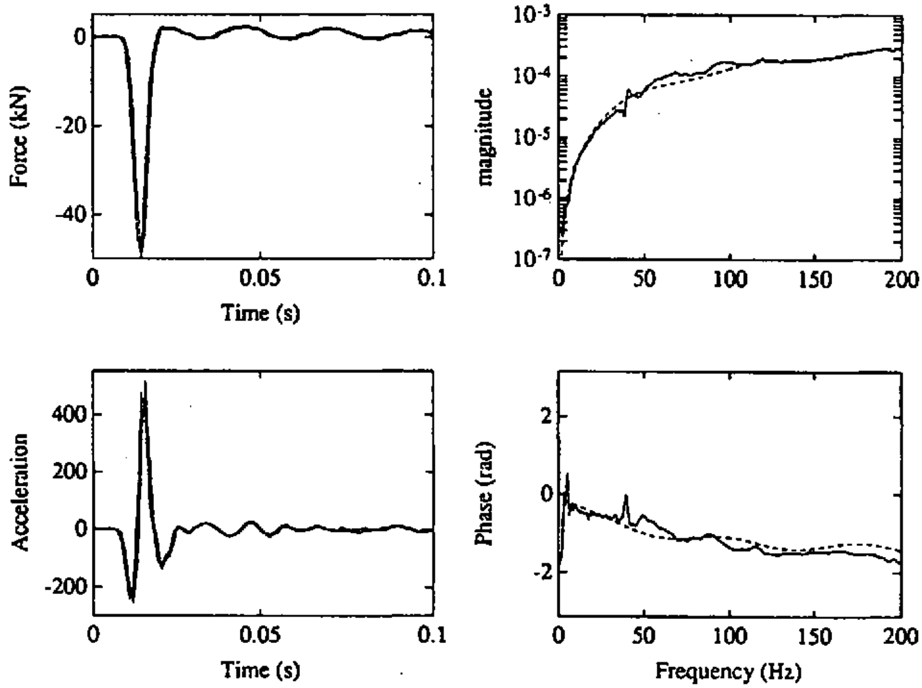

Figure 6 Impulse response of a pile measured at Gloucester Park: (a) impulsive force history (six impulses); (b) acceleration response of the pile cap; (c) and (d) magnitude and phase of the frequency-response function calculated from measured data (solid) and theory (dashed).

4.2. Vibration transmission through isolation pads

Vibration measured above and below an isolation pad is shown in figures 7a and 7b during the passage of a train. It can be seen in the time traces that after the train had departed, there is still a significant amount of ambient noise. The power spectrum of the vibration measured above and below the pad has been calculated both from train-generated and ambient vibration. The line shown between the two phases 'train' and 'ambient' is determined approximately by eye from figure 7a measured below the pad; the same dividing line is used in 7b for the signal above the pad where the two phases are less distinct. In each case, note in figures 7c and 7d that ambient noise is responsible for some of the spectral peaks in the vibration spectrum during the passage of a train.

The transmissibility functions $T_{direct}(\omega)$ and $T_{total}(\omega)$ are shown in figure 8a and the difference between them indicates that there is a large amount of vibration reaching the floors above and below the pad which is uncorrelated. $T_{direct}(\omega)$ shows greater peakiness which is more characteristic of the theoretical column response than the smoother $T_{total}(\omega)$. The coherence is shown in figure 8b which shows more clearly that there is much uncorrelated input to the system.

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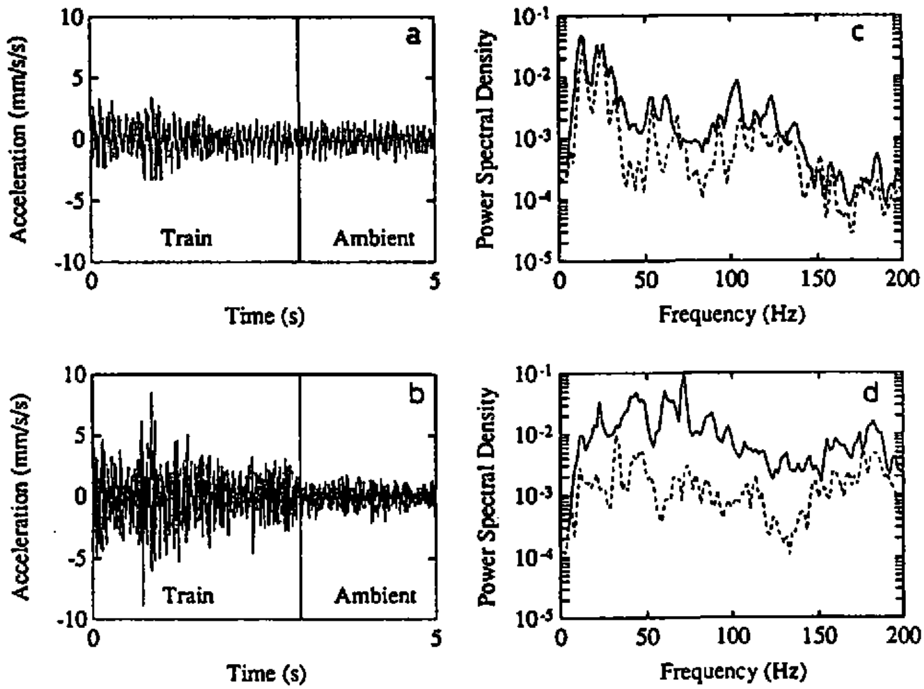

Figure 7 Measurement of transmissibility across an isolation pad at Gloucester park: (a) and (b) acceleration measured below and above the pad, divided into two parts marked 'train' and 'ambient'; (c) and (d) power spectra calculated separately for 'train' (solid) and 'ambient' (dashed) vibration.

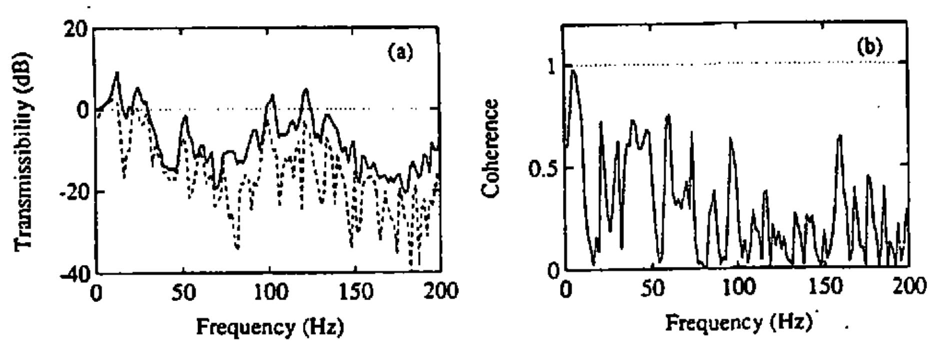

Figure 8 (a) Transmissibility and (b) coherence for the vibration measurements of figure 7. Transmissibility is shown $T_{total}(\omega)$ (solid) and $T_{direct}(\omega)$ (dashed) for train vibration only.

EXPERIMENTS ON VIBRATION TRANSMISSION

Vibration data from trains passing through the railway lines beneath the site have been measured at several stages during construction and vibration spectra at ground level near a pile cap are shown at two stages of construction in figures 9a and 9b. It is important to show many measured spectra for each case to emphasise the variability of train excitation, and to show that it is not sensible to average these spectra together. It appears that vibration levels at pile-cap level are reducing as construction progresses. At the time of publication, the Gloucester Park buildings are still under construction. There is yet much more data to be collected and analysed as the building nears completion which will shed new light on the way in which buildings respond to ground vibration.

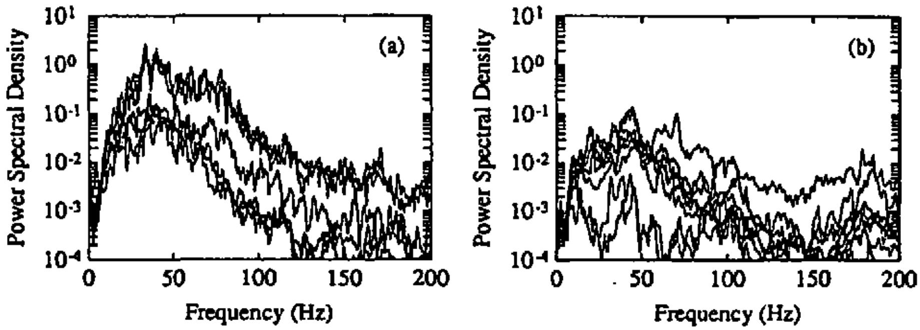

Figure 9 Power spectral densities in $(mms^{-2}/Hz)^2$ of train vibration measured at pile-cap level with construction at (a) pile-cap level and (b) sixth floor level (half way built).

5. LABORATORY TESTS ON CONCRETE COLUMNS

Laboratory tests on the vibration of concrete columns have been performed by Bhaskar [2]. Four circular columns, each 4 metres in length and 0.3 metres in diameter, have been tested as beams and as columns for their free vibration characteristics. The columns, numbered 1 to 4, were cast (grade 50) according to three different reinforcement configurations as follows: (1) and (2) reinforced (8×ϕ16mm) and prestressed; (3) prestressed only and (4) reinforced only. The central prestressing tendon (ϕ25mm) in columns 1 to 3 can be used to simulate building loads.

Each column was tested in bending and in compression. All tests were carried out when the columns were lying horizontally, supported at the nodal points of the first bending mode. The columns were struck with a small impulse hammer at mid-span to excite bending modes and at the column end for the compressive modes. The frequency of the first mode of bending vibration was about 65Hz and for axial vibration about 500Hz.

Modal damping Q factors ($Q=1/2\zeta$) for each mode were estimated by following a method of Woodhouse [8]. Measured column vibration is observed to decay with time after an impulse, and for ten short non-overlapping time intervals during this decay, the Fourier transform is calculated. For each mode, the height of the spectral peak in each successive transform is observed to reduce roughly exponentially with time, and the rate of this decay is proportional to the modal damping.

EXPERIMENTS ON VIBRATION TRANSMISSION

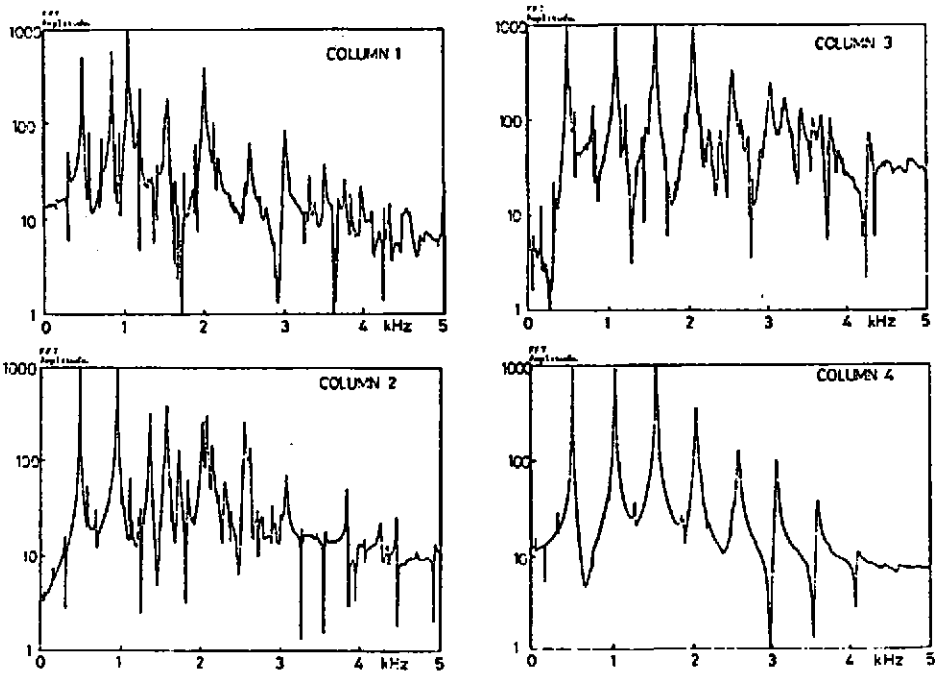

Figure 10 Frequency response to an axial impulse applied to four concrete columns with different reinforcement configurations as described in the text.

By fitting an exponential curve to the peak-height decay with time, an accurate estimate of damping can be obtained. This method is very reliable excepting when there are two modal peaks at very nearly the same frequency, in which case the method often fails because of the difficulty in distinguishing the peaks to determine their rate of decay.

Figure 10 shows a set of typical acceleration responses (DFT's) for axial impulses applied to each of the four columns. The clear and uniformly spaced peaks for column 4 (reinforced only) are typical of the axial characteristics of a column, while the responses shown for columns 1 to 3 are much less clear. It is believed that transverse vibration of the ungrouted prestressing tendon in these first three columns is responsible for many of these additional frequencies. For columns 2, 3 and 4, Q factors for the fundamental mode (500Hz) were calculated in the range 129 to 143. Similar impulse tests were carried out for the bending modes of the same four columns and Q factors at the 65Hz fundamental mode were found to be in the range of 106 to 144. These Q factors are independent of frequency which suggests that a hysteretic model for damping in reinforced concrete should be used. In a real building, however, where the levels of damping are an order of magnitude higher, the additional damping afforded by floors and fittings must be accounted for. A hysteretic damping law for entire buildings does not necessarily follow from these results. Work is in progress to determine an overall damping law for buildings.

6. CONCLUSIONS

Some progress has been made towards an understanding of the response of buildings mounted on resilient pads. A simple column model for an isolated building mounted on piles has been used to interpret building vibration measurements. While the limitations of such a simple model have been described, salient features of building response are found in the response of a single column mounted on a resilient foundation and a pile. Isolation from ground vibration is an important feature of many modern buildings and a design strategy based on a simple model is of use to the designer. More advanced models, of a finite-element type to represent column floor vibration, are under development which will aid the interpretation of site measurements and which will enable a better understanding of damping in buildings.

7. REFERENCES

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