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TRAFFIC INDUCED GROUND VIBRATIONS

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1. SUMMARY

Vibration transmitted through the ground from a busy motorway is assumed to be a random and statistically stationary function of time. Random process theory is used to derive an integral equation which expresses the variation with distance from the road of the power spectrum of ground vibration. The power spectrum of road roughness, the mean vehicle spacing, typical vehicle suspension characteristics and the frequency response function of the ground are required as inputs to this expression which is solvable analytically subject to some simplifying assumptions. For solutions under more complex conditions, a computer implementation is required. Theoretical predictions are validated by experimental measurement.

2. NOTATION

A cartesian set of cordinates (x, y, z) is used throughout this paper. The road lies along the x axis and the z axis is an upward normal to the ground surface. u, v & w are displacements in the x, y & z directions respectively. In the frequency domain, angular frequency ω (not to be confused with ground displacement w) and wave number γ are transforms of τ and χ which are the autocorrelation variables corresponding to time t and x respectively. A complex conjugate is denoted *.

3. INTRODUCTION

A long, straight, busy roadway is schematically represented in fig.1b, where a typical force applied to the road by a vehicle moving along it is denoted f_j. Immediately adjacent to the roadway (point 'B'), an observer measuring ground vibration is aware of the passage of each individual vehicle. At distances away from the road significantly greater than the mean vehicle spacing (point 'A'), the ground motion can be said to be random and statistically stationary. The amplitude of vibrations in the ground is small, so the ground can be considered to be a linear system and modelled as an homogeneous, isotropic, viscoelastic halfspace whose frequency response function (vertical displacement output w for force input f) is $H_{wf}(r, \omega)$. Under these conditions, linear random process theory can be used to calculate the power spectrum $S_{ww}(a, \omega)$ of vertical ground motion $w(a, t)$ at a distance 'a' from an infinite, straight road subject to randomly distributed time varying forces along its length $f_j(t)$ defined by their power spectrum $S_{f_j f_j}(\omega)$.

In this paper, we first derive an expression for $S_{ww}(a, \omega)$ in terms of $S_{f_j f_j}(\omega)$ and $H_{wf}(r, \omega)$. The calculation of $H_{wf}(r, \omega)$ and, in particular, $S_{f_j f_j}(\omega)$ is discussed in detail, and subject to simplifications about the choice of vehicle model and mass distribution, an analytical expression for $S_{ww}(a, \omega)$ can be obtained.

4. HALFSPACE RESPONSE TO A RANDOM LINE EXCITATION

On the surface of a linear halfspace (fig.1a), the power spectrum $S_{ww}(\omega)$ of surface displacement $w(t)$ can be expressed [1] in terms of the power spectrum of a vertically applied force

$f(t)$ as follows:

$$S_{ww}(\omega) = |H_{wf}(r,\omega)|^2 \, S_{ff}(\omega) \qquad -(1)$$

where $H_{wf}(r,\omega)$ is the frequency response function for the halfspace and r is the distance from the applied force.

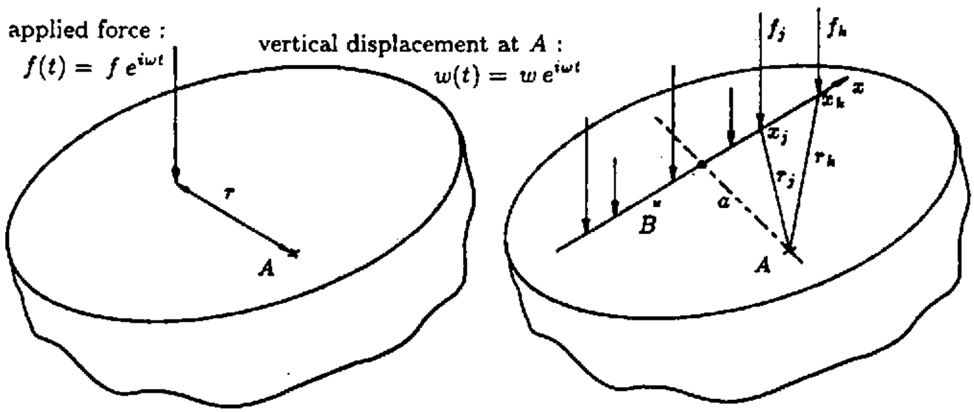

fig.1a - Halfspace frequency response

fig.1b - Random line excitation

For N independent point loads $f_1(t), f_2(t), \ldots, f_N(t)$ acting at points $x_1, x_2, \ldots, x_N$ respectively on the surface of the halfspace (fig.1b), we write the corresponding auto spectral density functions as $S_{f_1f_1}(\omega)$, $S_{f_2f_2}(\omega), \ldots$ and likewise the $N(N-1)$ cross spectral density functions $S_{f_1f_2}(\omega)$, $S_{f_1f_3}(\omega), \ldots$ Newland [1] describes a method for analysing continuous systems subject to multiple discrete random loads where it is shown that the spectrum of surface displacement at a point 'A' (fig.1b) can be expressed as the double summation

$$S_{ww}(\omega) = \sum_{j=1}^{N} \sum_{k=1}^{N} H^*_{wf}(r_j,\omega) H_{wf}(r_k,\omega) S_{f_jf_k}(\omega) \qquad -(2)$$

where r_j and r_k are the distances of the loads $f_j(t)$ & $f_k(t)$ respectively from the point 'A'. For an infinite number of discrete loads, and since $r_j^2 = a^2 + x_j^2$, we can write

$$S_{ww}(a,\omega) = \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} H^*_{wf}(a,x_j,\omega) H_{wf}(a,x_k,\omega) S_{f_jf_k}(\omega). \qquad -(3)$$

Eq. 3 is used to compute the spectrum of vibration at a distance from a road provided that the halfspace frequency response function $H_{wf}(a,x,\omega)$ and the power spectrum of applied force $S_{f_jf_j}(\omega)$ can be obtained.

4.1 Halfspace Frequency Response Function

Disturbances within an homogeneous, isotropic halfspace propagate through the body of the halfspace as compressive and shear waves and along the surface as Rayleigh waves. For the case of an impulsive point force applied to the surface of an elastic halfspace, Lamb [2] demonstrated that Rayleigh waves account for 67% of the radiated energy. Furthermore, the Rayleigh wave decays inversely with the *square root* of the distance from the impulse, whereas the body waves decay inversely with the *square* of distance. At large distances, then, Rayleigh waves account for most of the disturbance felt on the surface of the halfspace. Taking only the Rayleigh wave component, the frequency response function for a lightly damped viscoelastic halfspace can be expressed as [3,8]

$$H_{wf}(r,\omega) = exp(-D\omega^2 r/2c_R)\frac{-i\omega K'}{2\rho c_R^3}H_0^{(2)}\left(\frac{\omega r}{c_R}\right) \qquad -(4)$$

where ρ is the soil density, c_R is the Rayleigh wave speed, K' is a dimensionless material constant (a function only of Poisson's ratio ν, and in the range $0.1 \leq K' \leq 0.22$). D is the damping coefficient derived from the complex shear modulus $G' = G(1 + i\omega D)$, and $H_0^{(2)}(\theta)$ is a Hankel function expressed in standard notation.

4.2 Power Spectrum of Applied Force

It remains to evaluate $S_{f_j f_j}(\omega)$ in terms of the road roughness, the distribution of vehicles along the road and the vehicle dynamics. In order to simplify the discussion that follows, it will be assumed that the vehicles are *not* moving with respect to the halfspace, but only with respect to the rough road surface. The road surface can be thought of as moving with a velocity equal and opposite to that of the vehicles. The final result is not affected by this assumption because we are far away from the road and the vehicles appear to move slowly in and out of the 'field of view'. We only require that the number of vehicles moving within the 'field of view' remains roughly constant.

4.2.1 Power Spectrum of Road Roughness : The road surface profile $Y(x)$ is assumed Gaussian and defined by the power spectrum

$$S_{YY}(\gamma) = S_{Y_0}(\gamma/\gamma_0)^{-n} \qquad -(5)$$

as suggested by the International Standards Organisation [4]. The roadway is moving at speed V beneath the vehicles therefore the power spectrum of the tyre contact point displacement $Y(t)$ can be written [1]

$$S_{YY}(\omega) = \frac{1}{V}S_{YY}(\gamma), \qquad \gamma = \frac{\omega}{V}. \qquad -(6)$$

4.2.2 Vehicle Frequency Response : Each vehicle is modelled as a simple damped two degree-of-freedom oscillator (fig.2). For a range of typical vehicles in Great Britain, the two predominant natural frequencies (whole body 'bounce' and wheel 'hop') are observed to be roughly constant, despite the wide variation in vehicle sizes and geometries. A similar observation can be made for suspension damping. As indicated in fig.2, all vehicles are described by a single oscillator of fixed characteristics, the only difference between vehicles being their total mass m_1+m_2. Subject to this assumption, and remembering that the road profile is moving at speed V beneath the vehicles, we can say for any vehicle at a position x_j, that the accelerations $\ddot{X}_1$ & $\ddot{X}_2$ are functions of $x_j - Vt$, the position of the vehicle relative to the moving road profile. Therefore, the force applied to the road by a vehicle of total mass m_1+m_2 is given by $f = m_1\ddot{X}_1 + m_2\ddot{X}_2$ and is also a function of $x_j - Vt$. It is useful to define the weighted mean acceleration $\ddot{Z} = \ddot{X}_1 + \mu\ddot{X}_2$ so that we can write (dropping the subscript on m_1 so that m_j refers to m_1 of the vehicle at x_j)

$$f_j(t) = m_j\ddot{Z}(x_j - Vt). \qquad -(7)$$

In order to calculate the power spectrum $S_{f_jf_j}(\omega)$ of this applied force distribution $f_j(t)$, an expression for the power spectrum of the mean acceleration $\ddot{Z}$ will be required.

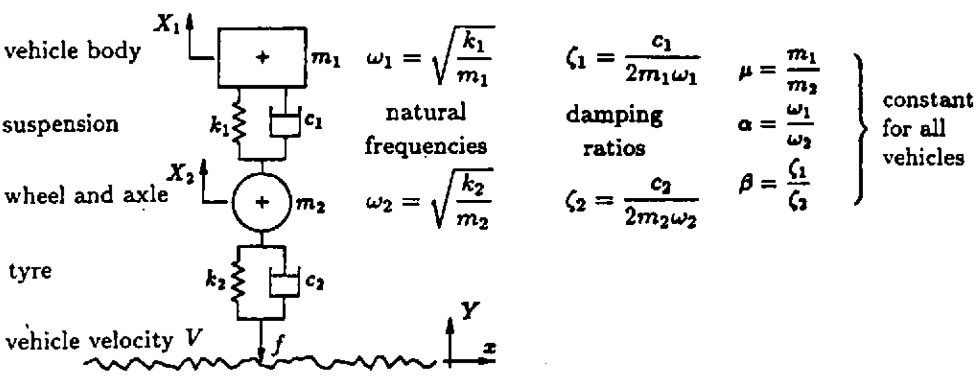

fig.2 - Two degree-of-freedom vehicle model

The vehicle response $\ddot{Z}(t)$ to an harmonic input displacement excitation $Y(t)$ at the tyre contact point is defined in the frequency domain by the expression $S_{\ddot{Z}\ddot{Z}}(\omega) = H_{\ddot{Z}Y}(\omega)S_{YY}(\omega)$. $H_{\ddot{Z}Y}(\omega)$ is the frequency response function of the vehicle model and by considering the equations of motion for the two degree-of-freedom oscillator of fig.2, it can be shown that [8]

$$H_{\ddot{Z}Y}(\omega) = \omega^2\frac{B_2}{\Delta}(A_{12} - \mu A_{11}) \qquad -(8)$$

where $\Delta = A_{11}A_{22} - A_{12}A_{21}$ $\qquad B_2 = 2i\mu\alpha\beta\zeta_1(\frac{\omega}{\omega_1}) + \mu\alpha^2$

$A_{11} = -(\frac{\omega}{\omega_1})^2 + 2i\zeta_1(\frac{\omega}{\omega_1}) + 1$ $\qquad A_{21} = -(2i\zeta_1(\frac{\omega}{\omega_1}) + 1)$

$A_{12} = -(2i\zeta_1(\frac{\omega}{\omega_1}) + 1)$ $\qquad A_{22} = -\mu(\frac{\omega}{\omega_1})^2 + 2i(\mu\alpha\beta + 1)\zeta_1(\frac{\omega}{\omega_1}) + (\mu\alpha^2 + 1).$

The excitation $Y(t)$ is provided by the moving road roughness. It is reasonable to assume that the random input to any one vehicle is uncorrelated with that of any other, because the vehicle spacing is generally large and closely spaced vehicles are not likely to follow the same tracks. So from eq. 6 we can write that the power spectrum of mean acceleration $\ddot{Z}(t)$ for all vehicles

$$S_{\ddot{Z}\ddot{Z}}(\omega) = \frac{1}{V}|H_{\ddot{Z}Y}(\omega)|^2 S_{YY}(\gamma), \qquad \gamma = \frac{\omega}{V}. \qquad -(9)$$

4.2.3 Discrete Mass Distribution : From eq. 7, we have that $f_j(t) = m_j \cdot \ddot{Z}(x_j, t)$. We write $m_j = \bar{m}_j + \hat{m}_j$, where $\bar{m}_j$ and $\hat{m}_j$ are the *mean* and *varying* parts of m_j. The mean and variance of the sprung masses (m_1 in fig.2) of all the vehicles on the road are denoted $\bar{m}$ and σ^2 respectively. We have already assumed that the force applied to the road by any one vehicle is uncorrelated with that applied by any other, so the autocorrelation function for the applied force distribution $R_{f_j f_k}(\tau) = 0$ for $j \neq k$ and for $j = k$ it can be written

$$\begin{aligned}
R_{f_j f_j}(\tau) &= E[f_j(t) \cdot f_j(t+\tau)] \\
&= E[m_j \cdot \ddot{Z}_j(t) \cdot m_j \cdot \ddot{Z}_j(t+\tau)] \\
&= E[(\bar{m}_j + \hat{m}_j) \cdot \ddot{Z}_j(t) \cdot (\bar{m}_j + \hat{m}_j) \cdot \ddot{Z}_j(t+\tau)] \\
&= E[\bar{m}_j \cdot \ddot{Z}_j(t) \cdot \bar{m}_j \cdot \ddot{Z}_j(t+\tau)] + E[\bar{m}_j \cdot \ddot{Z}_j(t) \cdot \hat{m}_j \cdot \ddot{Z}_j(t+\tau)] \\
&\quad + E[\hat{m}_j \cdot \ddot{Z}_j(t) \cdot \bar{m}_j \cdot \ddot{Z}_j(t+\tau)] + E[\hat{m}_j \cdot \ddot{Z}_j(t) \cdot \hat{m}_j \cdot \ddot{Z}_j(t+\tau)]. \qquad -(10)
\end{aligned}$$

The expectation of a multiple product can be expanded in pairs [1,5] provided that all the terms have zero mean and follow a Gaussian distribution. Noting also that $E[\hat{m}_j] = 0$, we can write

$$\begin{aligned}
R_{f_j f_j}(\tau) &= \bar{m}^2 E[\ddot{Z}_j(t) \cdot \ddot{Z}_j(t+\tau)] + 2\bar{m}E[\hat{m}_j \cdot \ddot{Z}_j(t) \cdot \ddot{Z}_j(t+\tau)] \\
&\qquad + E[\hat{m}_j \cdot \hat{m}_j \cdot \ddot{Z}_j(t) \cdot \ddot{Z}_j(t+\tau)] \\
&= \bar{m}^2 R_{\ddot{Z}\ddot{Z}}(\tau) + 0 + E[\hat{m}_j \cdot \hat{m}_j] E[\ddot{Z}_j(t) \cdot \ddot{Z}_j(t+\tau)] \\
&\qquad + E[\hat{m}_j \cdot \ddot{Z}_j(t)] E[\hat{m}_j \cdot \ddot{Z}_j(t+\tau)] \\
&\qquad + E[\hat{m}_j \cdot \ddot{Z}_j(t+\tau)] E[\hat{m}_j \cdot \ddot{Z}_j(t)] \\
&= \bar{m}^2 R_{\ddot{Z}\ddot{Z}}(\tau) + \sigma^2 R_{\ddot{Z}\ddot{Z}}(\tau) \\
&= (\bar{m}^2 + \sigma^2) R_{\ddot{Z}\ddot{Z}}(\tau). \qquad -(11)
\end{aligned}$$

Taking the fourier transform of this autocorrelation function we obtain the power spectrum

$$S_{f_j f_j}(\omega) = \begin{cases} (\bar{m}^2 + \sigma^2) S_{\ddot{Z}\ddot{Z}}(\omega) & \text{for } j = k \\ 0 & \text{for } j \neq k. \end{cases} \qquad -(12)$$

4.3 Power Spectrum of Ground Vibration

Substituting for $S_{f_j f_j}(\omega)$ (eq. 12) into eq. 3, the double summation is reduced to a single summation

$$S_{ww}(a,\omega) = \sum_{j=-\infty}^{\infty} H^*_{wf}(a,x_j,\omega)H_{wf}(a,x_j,\omega)S_{f_j f_j}(\omega)$$

$$= (\bar{m}^2+\sigma^2)S_{\ddot{Z}\ddot{Z}}(\omega)\sum_{j=-\infty}^{\infty}|H_{wf}(a,x_j,\omega)|^2.$$

Provided that the vehicles are close enough so that $H_{wf}(a,x_j,\omega)$ does not change significantly between adjacent vehicles, the summation $\sum_{j=-\infty}^{\infty}$ approximates the integral $\frac{1}{u_0}\int_{-\infty}^{\infty}dx$ so that the power spectrum can be written

$$S_{ww}(a,\omega) = (\bar{m}^2+\sigma^2)S_{\ddot{Z}\ddot{Z}}(\omega)\frac{1}{u_0}\int_{-\infty}^{\infty}|H_{wf}(a,x,\omega)|^2dx.$$

and substituting for the power spectrum of vehicle acceleration $S_{\ddot{Z}\ddot{Z}}(\omega)$ we then obtain

$$S_{ww}(a,\omega) = \frac{(\sigma^2+\bar{m}^2)}{Vu_0}|H_{\ddot{Z}Y}(\omega)|^2S_{YY}(\gamma)\int_{-\infty}^{\infty}|H_{wf}(a,x,\omega)|^2dx, \qquad \gamma=\frac{\omega}{V}. \qquad -(13)$$

A useful observation is that eq. 13 is the product of four *independent* terms:
mass distribution * vehicle frequency response * road roughness * ground response.

For a viscoelastic halfspace model of the ground, the integration of the frequency response function (eq. 4) can be performed analytically so that we obtain a completely analytical expression for the spectrum of vibration at a distance 'a' from a long, straight roadway

$$S_{ww}(a,\omega) = \frac{(\sigma^2+\bar{m}^2)}{Vu_0}|H_{\ddot{Z}Y}(\omega)|^2S_{YY}(\gamma)\frac{\omega K'^2}{\pi\rho^2c_R^5}K_0\left(\frac{D\omega^2a}{c_R}\right), \qquad \gamma=\frac{\omega}{V}. \qquad -(14)$$

$K_0(\theta)$ is a modified Bessel function of the second kind.

5. EXPERIMENTAL RESULTS - VALIDATION OF THE THEORY

An experimental study of ground vibration transmission from a busy road was carried out adjacent to the A604 trunk road near Huntingdon (U.K.). The road is straight and well situated on level ground above a 60m layer of homogeneous Oxford clay.

5.1 Impulse Tests

At the test site, the parameters for the halfspace model of eq. 4 were determined by performing impulse tests. Ground vibration was measured in response to an impulse delivered by a 15kg weight falling from a height of 2m. The compressive and surface wave fronts could be easily distinguished from the measurements and the values of c_R and K' were obtained. The damping coefficient D was obtained by taking the Disrete Fourier Transform (DFT) of the impulse response and comparing the resulting frequency response function with that expected from eq. 4. Accordingly, quite accurate values of these parameters were obtained:

$c_R = 200\ m/s$ $\qquad \rho = 2000\ kg/m^3$ $\qquad D = 0.00015\ s$

$\nu = 0.48$ $\qquad \Rightarrow \qquad K' = 0.103$

5.2 Traffic Vibration Measurements

Ground vibration levels were measured at a distance of 300 m from the road. For the duration of the tests, a video camera was set up to record the traffic flow. Assuming an average of three axles per vehicle (the vibration due to passenger cars is negligible), the parameters for eq. 14 were deduced by considering each axle load independently. Thus taking an average vehicle mass of 24000kg and the observed average three-axle vehicle spacing of 150m:

$\bar{m} = 8000\ kg$ $\sigma = 2000\ kg$ $u_0 = 50\ m$

$V = 30\ m/s$

The parameters for vehicle dynamics (eq. 8) are based on data from several sources [6,7] (Note that the values of ω_1 and ω_2 given here give rise to a 'bounce' frequency of 2Hz and a 'wheel hop' frequency of 10Hz.):

$\mu = 0.15$ $\omega_1 = 15.7\ rad/s$ (2.5Hz) $\omega_2 = 3\omega_1$ (7.5Hz)

$\zeta_1 = 0.082$ $\zeta_2 = 0.022$

For road roughness, the parameters for eq. 5 are those corresponding to a 'good' road profile [4,6]:

$S_{Y_0} = 0.318 \times 10^{-6}\ m^3/rad$ $\gamma_0 = 1.0\ rad/m$ $n = 2$

By substituting the above parameters into eqs. 4.5 & 8, and by using eq. 14, the predicted spectrum of surface ground vibration (acceleration) is calculated and shown in fig.3. Also shown is the spectrum of vibration actually measured under the above conditions.

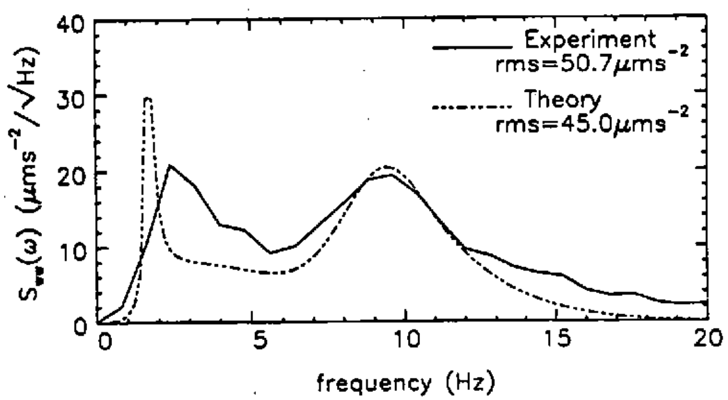

fig.3. Spectrum of ground vibration at 300m - Theory and Experiment

5.3 Discussion of Results

The predicted ground vibration spectrum is substantially borne out by experiment. The sharp spectral peak predicted at 2Hz reflects the assumption that all vehicles have the same 'bounce' frequency. In practice, there appears to be sufficient variation in 'bounce' frequency between vehicles to broaden the peak as observed, without affecting the mean square of the spectrum. Substantial pitching modes in the range 2-5Hz for typical U.K. vehicles [6] may also contribute to this effect. A similar peak broadening is observed at the 'wheel hop' frequency, but this is less prominent since the predicted peak at 10Hz is itself very broad. Vibration levels measured at frequencies above 15Hz are not associated with the large scale vehicle dynamics, but reflect the level of ambient background vibration. A more detailed description of the theory and experimental results will be published shortly [8].

6. CONCLUSION

Random process theory can be used to model accurately the transmission of ground borne vibration from busy roads. Sufficient accuracy is obtained by the use of simple vehicle and halfspace dynamic models and a two parameter form of the road surface profile spectrum.

7. REFERENCES

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