Proceedings of The Institute of Acoustics

FREQUENCY RESPONSE TESTING A NON-LINEAR STRUCTURE IN A NOISY ENVIRONMENT WITH A DISTORTING SHAKER

H.G.D. GOYDER

A.E.R.E., HARWELL

1. Introduction

Non-linearities and noise can create serious difficulties when an attempt is made to measure the frequency response function (F.R.F.) of a structure. Unlike a linear structure a non-linear structure does not have a single F.R.F. relating the excitation to the response and can only be characterised by a series of response spectra. In general, measurement of all the spectra in the series is not possible but for random excitation one spectra in the series may be made to correspond to a linear F.R.F. and the remaining term may be treated as uncorrelated noise. The measurement problem created by non-linearities are in this case similar to those problems created by noise. Unfortunately, both the effects of non-linearities and noise are considerably complicated by electro dynamic shakers which are generally used to provide a force to vibrate a structure.

This paper analyses the interaction between the shaker and the structure and proposes a special correlation technique which overcomes the disruptive influence of the shaker, the non-linearities and the noise.

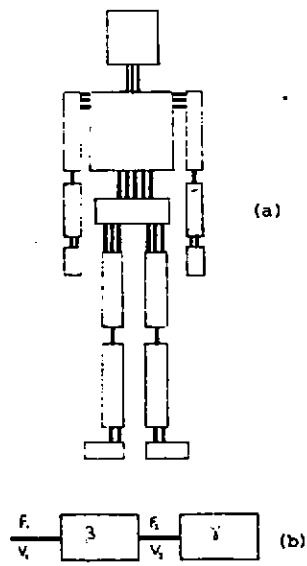

Figure 1. Systems divided into Physical Sub-systems

2. Models of Systems

In general two types of models are used to describe a system, viz: physical models and input-output models. Figure (1) gives two examples of physical models in which a physical system is divided into physical sub-systems. These models are topologically equivalent to engineering drawings, with each box representing a sub-system which is joined to other sub-systems by means of links which couple the co-ordinates of different boxes. In mechanical systems each link has an associated force and velocity. In order to characterise a physical sub-system it is necessary to describe a matrix of F.R.F's which relate all the forces to all the velocities. A distinction between electrical and mechanical systems is not vital and the parameters in the link between electrical systems are voltage and current, thus Figure (1b) could equally well be used to describe the configuration in a frequency response test where sub-system β would be the shaker linked to the test structure by means of a force and velocity, and linked to a signal generator by means of a voltage and current.

FREQUENCY RESPONSE TESTING A NON-LINEAR STRUCTURE IN A NOISY ENVIRONMENT WITH A DISTORTING SHAKER

The second type of model is the input-output model. This model is once again drawn by breaking a system down into linked boxes, but in this case each link is associated with only one parameter (either force or velocity). Each box in an input-output model is characterised by a single F.R.F. with the output spectra being given by the product of the input spectra and F.R.F. Figure (2) is an input-output model of Figure (1b). As can be seen this type of model clearly shows the interaction between forces and velocities, but does not indicate the physical layout of the system. In particular the input-output model of Figure (2) indicates the existence of feedback paths within the physical sub-system of Figure (1b).

3. Shaker Structure Interaction

Figure (3) shows an input-output model for the interaction between a shaker and test structure. The feedback path from the output of the structure to the input is the cause of significant problems when noise and non-linearities are present. The feedback path is due to two effects. First, the motion of the structure causes the coil within the shaker to generate a back e.m.f. which opposes the motion. Second, the mass of the linkage between the shaker and the structure applies a force to the structure which is proportional to the output acceleration. In addition to these two effects Tomlinson [1] has shown that the shaker is itself a non-linear system with considerable amplitude dependence. From this description it is clear that the force arriving at the input to the structure will be a much distorted version of the output of the signal generator.

4. Frequency Response Measurements in a Noisy Environment

A method for measuring the F.R.F. of a linear system in a noisy environment is given below. By using a force transducer and accelerometer electrical signals of the input and output of the linear system are available. As shown in Figure (3) both of these signals are contaminated with noise. The text book method for measuring the F.R.F. is to use a random excitation and to measure the cross spectral density (C.S.D.) between the input and output of the structure and the auto spectral density (A.S.D.) (power spectral density) of the input. The F.R.F. is then determined from the ratio of the C.S.D. to the A.S.D. This method will reject the noise in the output signal which is uncorrelated with the noise in the input signal. Because of the feedback path the noise in the output is correlated with the noise in the input, and thus the textbook method will not be successful.

This problem may be solved by correlating the input and output signals of the structure with the output signal of the random signal generator. The signal supplied by the signal generator is unaffected by the feedback paths of the system and is uncorrelated with the noise. The F.R.F. of the structure under test may be determined by measuring the C.S.D. between $S(t)$ and $V(t)$ and the C.S.D. between $S(t)$ and $F(t)$. The F.R.F. is given by the ratio of the two cross spectral density functions, thus

$$H(i\omega) = \frac{G_{SV}(i\omega)}{G_{SF}(i\omega)}$$

where $H(i\omega)$ is the F.R.F. and G_{xy} is the C.S.D. between x and y

The F.R.F. determined by this means is uncorrupted by the effects of noise or the feedback paths. This measurement procedure has also been used by Wellstead [2] for determining transfer functions in control systems with feedback paths.

5. The Frequency Response of a Non-Linear System

The relationship between the input and output of a non-linear system may generally be written in the form a Volterra series [3].

$$V(t) = \int_{-\infty}^{\infty} h_1(\tau_1) F(t-\tau_1) d\tau_1 + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h_2(\tau_1,\tau_2) F(t-\tau_1) F(t-\tau_2) d\tau_1 d\tau_2$$

$$+ \iiint_{-\infty}^{\infty} h_3(\tau_1,\tau_2,\tau_3) F(t-\tau_1) F(t-\tau_2) F(t-\tau_3) d\tau_1 d\tau_2 d\tau_3 \quad \cdots\cdots\cdots$$

This series is a multi-dimensional convolution integral with the linear convolution integral appearing as the first term. The complete characterisation of a non-linear system would require measurement of all the kernals h_n of this series. This would not, however, be a practical approach since it is difficult to isolate one term of the series. Instead an alternative more limited measurement approach may be adopted. A Gaussian signal can be shaped and applied to the structure in order to simulate the environmental forces to which the structure will be exposed (this is not always easy if it is necessary to conpensate for the distortions of the shaker).

Because the force is derived from a Gaussian source it may be shown [3] that the Volterra series can be simplified and reorganised as a new series known as a Wiener series. The terms of the Wiener series are similar to the Volterra series in taking the form of a multi-dimensional convolution integral, but each term can be measured independently by means of a multi-dimensional correlation technique. This is possible because the Wiener series is organised so that each term produces a contribution to the total output that is orthogonal and thus uncorrelated with the contribution from every other term. Therefore when measuring one term by correlating, the remaining terms appear as noise. The first term of a Wiener series gives the optimum linear relationship between the output and the input of the structure. The second term gives the optimum by bi-linear relationship between the output and input, and similarly the third and higher terms each give an optimum n-linear relationship between the output and input.

The optimum linear term is of particular importance because it enables a linear F.R.F. to be created. This linear F.R.F. when given the same input as the non-linear system under test produces an output which is very close to the output of the non-linear system. The difference between the output of the linear and non-linear system is a minimum and in this sense the linear term is an optimum linear equivalent to the non-linear system. Although the linear F.R.F. is an optimum model of the non-linear structure it is only an optimum model for the input for which it is measured. For other inputs it is possible to derive other optimum models and thus it is important to use a test force which is similar to the force encountered in a working environment.

The correlation technique for measuring the equivalent linear F.R.F. is identical to the correlation technique described in the last section for rejecting noise. The correlation techniques for measuring more terms of the Wiener series are a straightforward generalisation of the technique for the measurement of the linear term.

Thus by treating non-linear effects as being similar to noise and by using a double cross correlation technique to reject noise, a non-linear system may be easily measured.

References

1. TOMLINSON, 1979. J. of Sound and Vibration, 63(3), 337-350. Force Distortion in Resonance Testing of Structures with Electro-Dynamic Vibration Exciters.

2. WELLSTEAD. 5th IFAC Symposium, Ed. R. Iserman, September 1979. Non-parametric Methods of System Identification.

3. SCHETZEN, 1978. John Wiley. The Volterra and Wiener Theories of Non-Linear Systems.

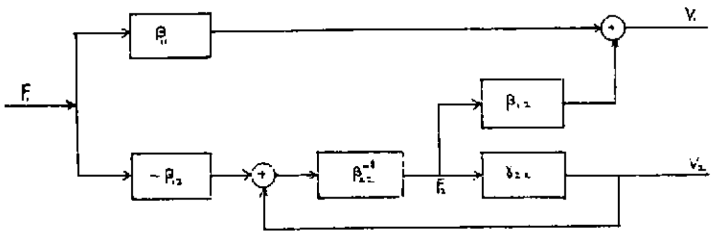

Figure 2. Input-Output Model of Figure 1(b).

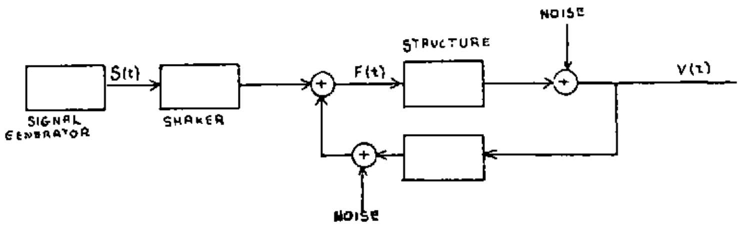

Figure 3. Input-Output Model for Shaker and Test Structure.

HGDG/RMR(3719) - 23.3.81.