Proceedings of The Institute of Acoustics

THE APPLICATION OF SIMPLE SOURCE THEORY TO ACTIVE NOISE CONTROL

I. Brown

Dept.of Electrical Engineering Science,University of Essex,Colchester

INTRODUCTION

Active noise cancellation can be applied in a number of ways such as:

a) cancelling at source
b) cancelling at the ear
c) reduction throughout a volume
etc.

The choice of where to cancel is usually determined by the practical situation of interest. The aim of this paper is to evaluate the extent of cancellation achieved when cancelling at source.

The cases that are simplest to tackle at source are those where the noise is radiated from a single, acoustically small outlet, such as the exhaust outlet on reciprocating engines.

In order to see why this is true, it is necessary to consider the radiation pattern of the noise emanating from an exhaust pipe. If the radiating surface of a noise source is small compared with the wavelength of sound, then the far field radiation pattern, assuming no reflections, will be spherical. It is thus possible to model the source as though it were a simple source, i.e. a pulsating sphere of radius a, where ka<<1.

The acoustic pressure of such a source is given by

$$P(r,\Theta) = \frac{j\rho ckQs}{4\pi r} \quad e^{j(wt-kr)} \qquad (1)$$

where Qs = source strength
ρ = density of air
k = 2π/wavelength
c = speed of sound
r = radial distance to the point p.

Consider the case below (Figure 1), where a source of equal strength, but of opposite phase has been placed a distance ℓ from the first source.

THE APPLICATION OF SIMPLE SOURCE THEORY TO ACTIVE NOISE CONTROL

Fig.1

Dipole configuration

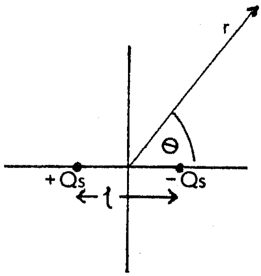
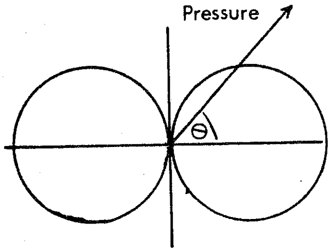


Fig.2

Radiation pattern of dipole



If $\ell<\lambda/2$ then the shape of the radiation pattern is the classic dipole (Figure 2). The ratio of the pressure of a dipole to a monopole has been shown to be [1].

$$\frac{Pd}{Pm} = (k\ell)^2 + (\ell/r)^2 \, \mathrm{Cos}\Theta \qquad (2)$$

As we are normally interested in reducing the far field sound pressure level, i.e. $r>>\ell$

$$\frac{Pd}{Pm} = K\ell\mathrm{Cos}\Theta \qquad (3)$$

Thus it can be seen that the sound pressure goes to zero at $\Theta=\pi/2$ and $3\pi/2$, and that the maximum acoustic pressure, occurring at $\Theta=0$ and π, is a function of the wavelength and separation only. As we are concerned here with noise reduction, it is important to realise that if $k\ell\simeq1$ i.e. $\ell\simeq\lambda/2\pi$ then $Pd/P\simeq1$. That is to say that if the separation is greater than $\lambda/2\pi$, then the maximum sound pressure level of the dipole will be greater than for the noise source alone.

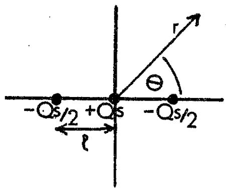
Practically, the limiting criterion for separation is usually the size of the exhaust outlet. It can be easily seen that if the diameters of both the outlet and the port of the cancelling loud-speaker are d, then the minimum distance between their centres is also d. Hence there is a physical limit to the upper frequency at which global cancellation can be achieved.

Another configuration that can be tackled analytically is shown below.

THE APPLICATION OF SIMPLE SOURCE THEORY TO ACTIVE NOISE CONTROL

Fig.3

Dipole with 2 cancelling sources



This consists of two cancelling sources flanking the noise source. If the cancelling sources are each half the amplitude of the noise source, and in antiphase, then the resultant radiation pattern is again a dipole, but this time with a maximum amplitude given by

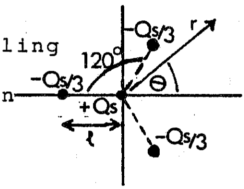
$$\frac{Pd_2}{Pm} = (1 - \mathrm{Cos}\ 2\pi d) \quad \text{where } d = \ell/\lambda = \text{separation in wavelengths} \qquad (4)$$

Comparing this to the cancellation achieved in the first case (Figure 4), we can see that for frequencies below this, there is a significant improvement to the extent of global cancellation. Also, cancellation is now possible up to $\lambda/4$ i.e. 1.5 times greater frequency than for a single dipole.

If the source configuration is changed to that below (Figure 5) with three cancelling sources, then the maximum acoustic pressure is given by (1-Cos2πd) as with the latter case, but now the radiation pattern is circular in the plane of the source.

Fig.5

Three cancelling source configuration



In three dimensions it would have a taurus shape, whereas the dipole is a dumb-bell. Hence this configuration is, in fact, slightly worse than the last case, as more energy is radiated to the far field - a result that is not perhaps intuitively obvious. It might also be pointed out that this analysis is true for any symmetrical arrangement of cancelling sources around a central noise source, up to and including the limiting case of the annulus.

THE APPLICATION OF SIMPLE SOURCE THEORY TO ACTIVE NOISE CONTROL

The next interesting case is the quadrupole, whose configuration of noise and cancelling sources is shown below. (Figure 6)

Fig.6

Quadrupole configuration

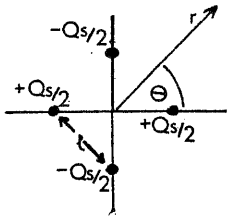
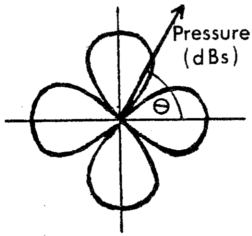


Fig.7

Radiation pattern of quadrupole



This has a radiation pattern like a clover leaf (Figure 7) - in three dimensions this looks like two perpendicular dumb-bells. The maximum amplitude in this case is given by

$$\frac{P_Q}{P} = 2 \sin^2 \frac{K\ell}{4} \quad \text{which, for } K\ell = 1 \text{ can be approximated to} \tag{5}$$

$$\frac{P_Q}{P} = \left(\frac{K\ell}{8}\right)^2 = \left(\frac{\pi d}{2}\right)^2 \tag{6}$$

Putting numbers into this (Figure 4) shows that not only is this a significantly better cancelling configuration, but that cancellation can now be achieved right up to $\lambda/2.8$ i.e. more than twice the frequency of the dipole case. The physical realisation of a quadrupole involves splitting an exhaust outlet pipe into two, hence the source strength of each is $Q/2$, which has been included in the above equation.

When going from a single exhaust outlet to two outlets, the cross sectional area is kept constant in order not to restrict the gas flow. Thus the diameter of each of the two pipes will be less than for a single outlet, which reduces the separation of noise and cancelling sources. This in turn is an additional improvement in the extent of cancellation, which has also been plotted in Figure 4.

As the analytical process of calculating the radiation patterns of simple source configurations is somewhat arduous, a computer program has been written to perform this task. For any given set of noise and cancelling sources, the program calculates the far field radiation pattern, which is then plotted on a polar diagram. In order to mimic the action of the cancellation system, one of the inputs to the program is the residual microphone position. In order to calculate the amplitude and phase of the cancelling sources, the program calculates the amplitude and phase of the sound field at the microphone position, and then calculates the amplitude and phase of the cancelling sources required to produce a null at this point. Finally, the far field radiation pattern is plotted.

This has enabled evaluation of the effect of microphone position on the extent of cancellation. For the case of a single noise source and a single cancelling source, the optimum position is found to be anywhere along a plane perpendicularly bisecting the line joining the sources. If the microphone is moved through 90 degrees so that it is on the same axis as the sources, then the far field radiation pattern takes the form of a cardioid, whose maximum amplitude is some 6dB greater than for the ideal case.

Although the positioning of the microphone in the latter case might be intuitively obvious, for more complex arrangements of noise and cancelling sources, the best microphone position is more difficult, and is often a practical compromise between being far enough from the sources, and yet close enough to get a reasonable signal/noise ratio for the cancelling system to work. In the case of the quadrupolar configuration, the best microphone position is on the line shown in Figure 6, but about 10l from the origin.

It is also possible to incorporate ground reflections into the calculation by putting a set of sources as though a mirror reflection about some plane. The computer simulations have shown the importance of putting the microphone in the ground plane in order to negate further complications.

Conclusion

The application of simple source theory has proved valuable to the investigation of improving active noise control at source, not only by suggesting better cancelling configurations, but also providing a quantitative method of testing microphone positioning and the effect of reflections on cancellation performance.

REFERENCE

[1] L.E. Kinsler and A.R. Frey, 'Fundamentals of Acoustics', Second Edition.

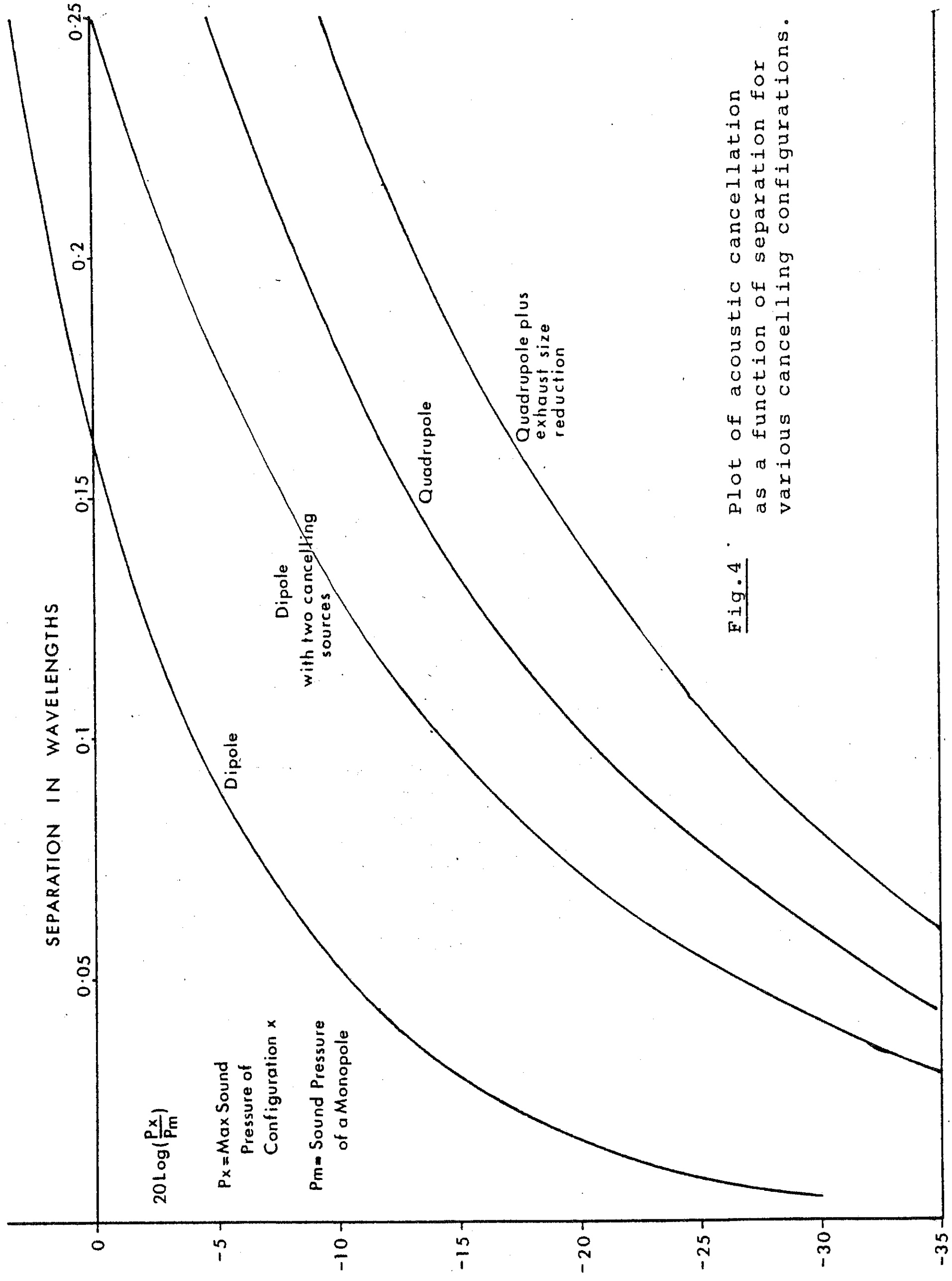


Fig.4 Plot of acoustic cancellation as a function of separation for various cancelling configurations.