VIBROELASTIC MODELS FOR FLOW NOISE ESTIMATION

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A self-consistent analytical framework is proposed for estimation of relative flow noise intensity for a turbulent flow over elastic boundaries (such as a turbulent boundary layer). The turbulent flow is assumed to be significantly subsonic. The framework employs the well-known analogy between motion of viscous flow and 'soft' elastic media (materials with low shear moduli). The turbulent flow is modelled as an ensemble of viscous (shear or vortex) waves and their conversion into longitudinal waves (sound) is treated as elastic wave transformation in layered elastic media. The proposed approach enables rapid evaluation of numerous 'what-if' scenarios such as changes of flow media, speed of the flow, and elastic properties of coating materials and their effect on flow noise. This framework can be useful as a tool for optimal coating design as well as for planning flow noise experiments. A number of illustrative cases are considered in detail.
Keywords: flow noise, turbulent boundary layer, vibroelastic materials

1. Introduction

Generation of acoustic waves by turbulent flow near elastic boundaries has been a traditional topic of aero- and hydroacoustics [1, 2], environmental acoustics [3, 4] and geophysics [5]. There is a vast amount of literature devoted to this subject [3, 4, 5].

Although the main models for flow noise are nowadays discussed in textbooks it is still a challenging task and an area of active scientific research and engineering effort. The numerical framework for flow noise estimation involves advanced models of a turbulent flow coupled with the equations of an elastic boundary and the model of noise generation (the so-called acoustic analogy [2, 3]). These problems are usually intractable analytically and require intensive computer simulations. There are many software tools available that capture this phenomenology with different levels of fidelity. Unfortunately, the application of these tools very often requires an expert-level knowledge of Computational Fluid Dynamics (CFD) and Finite Element (FE) modelling and advanced computing facilities in order to produce even very basic 'what-if' estimates. This was the motivation for the development of a simplified analytical framework which is easier to implement, in comparison to the full-scale CFD and FE simulations, while still capturing the complex phenomenology of the underlying process and producing relative quantitative results of acceptable accuracy.

Our analysis is restricted to that of a slightly compressible fluid , i.e. flows for which Mach number $M = U/c$ is much less than unity, where c is the speed of sound and U is the velocity of unperturbed flow (far from the the underlying surface).

The proposed framework is used to provide quantitative evaluation of the difference between different surface materials and cannot be used for absolute estimates of flow noise. We also demonstrate its application for the case of flow noise generated by a body with a vibroelastic coating moving in a fluid.

2. Theoretical Framework

It is well known that the velocity field of an arbitrary motion of any slightly compressible medium can be represented as a sum of two components, $\mathbf{v} = \mathbf{v}_{\parallel} + \mathbf{v}_{\perp}$, where $\mathbf{v}_{\parallel}$ is a potential component with $\mathbf{v}_{\parallel} = \nabla\varphi$ (φ is a scalar potential, and ∇ is the gradient operator), and $\mathbf{v}_{\perp}$ is a rotational component with $\mathbf{v}_{\perp} = \nabla \times \mathcal{A}$ ($\mathcal{A}$ is a vector potential, and $\nabla\times$ is the curl operator). For the case of elastic isotropic materials $\mathbf{v}_{\parallel}$ and $\mathbf{v}_{\perp}$ (and potentials φ and $\mathcal{A}$) satisfy the standard wave equations for the longitudinal and shear (transverse) waves

$$\frac{1}{c_l^2}\frac{\partial^2}{\partial t^2}\varphi + \nabla^2\varphi = 0, \tag{1}$$

$$\frac{1}{c_s^2}\frac{\partial^2}{\partial t^2}\mathcal{A} + \nabla^2\mathcal{A} = 0, \tag{2}$$

where

$$c_l = \sqrt{(\lambda + 2\mu)/\rho} \tag{3}$$

and

$$c_s = \sqrt{\mu/\rho} \tag{4}$$

are the speeds of longitudinal and shear waves, respectively. The attenuation of these waves can be taken into account by assuming complex Lamé elastic moduli $\lambda = \lambda_r + i\lambda_i$ and $\mu = \mu_r + i\mu_i$. For a fluid with no mean flow these equations can still be applied but $|c_l| \gg |c_s|$ ($|\lambda| \gg |\mu|$) and the attenuation of transverse waves is high. Note that the longitudinal waves are those for which the motion of the medium is parallel to the wave direction, and shear waves are those for which the motion is transverse (at right angles) to the wave direction.

Our aim is to model turbulent flow as a system of random shear (vortex) waves in a 'soft' (rubber-like) material, governed by Eq. (2). Conventional methods of the elastic wave transformation can then be used to study the process of flow noise generation near the elastic boundary by considering the transformation of vorticity perturbations (shear waves) into sound waves (longitudinal waves) at the boundary. The first step is to find effective parameters of an elastic medium in Eqs. (1) and (2) that correspond to a turbulent flow in a slightly compressible fluid medium. According to the results of Chu and Kovasznay [6] for mode decomposition of compressible flows, in a slightly compressible fluid the dynamics of vorticity perturbations and sound waves can be decoupled. The vorticity perturbations are described by the equation

$$\frac{\partial}{\partial t}\Omega + (\mathbf{v}_{\perp}\nabla)\Omega = \nu\nabla^2\Omega, \tag{5}$$

while for sound waves the wave equation (1) applies. In the above equation $\Omega = \nabla \times \mathbf{v}_{\perp} = \nabla^2\mathcal{A}$ is the vorticity field and ν is the kinematic viscosity of the fluid. For a flow with constant velocity $\mathbf{U}$, the linear approximation leads to $(\mathbf{v}_{\perp}\nabla)\Omega \approx (\mathbf{U}\nabla)\Omega$.

Assuming the vortex perturbations move only in one direction, determined by $U = |\mathbf{U}|$ (which corresponds to boundary layer flow), and by substituting a harmonic component of vorticity $\Omega \propto \exp[i(\mathbf{k_s} \cdot \mathbf{x} - \omega t)]$ into Eq. (5), a dispersion relation for the vorticity perturbations is obtained. For an effective transverse velocity $c_s = \omega/k_s$, where wavenumber $k_s = |\mathbf{k_s}|$, the matching of the dispersion relations leads to the following equivalence [7, 8]

$$c_s = U - i\nu k_s, \tag{6}$$

which is to be matched with c_s from Eq. (4)

$$c_s = \sqrt{(\mu_r + i\mu_i)/\rho}, \tag{7}$$

Matching of the real and imaginary parts of these equations leads to the effective shear modulus for the fluid in the presence of the flow. When $|c_s| \ll |c_l|$ and $U \ll c$ (slightly compressible limit, $M \ll 1$), the components are $\mu_r \approx \rho U^2$ and $\mu_i \approx 2\rho\omega\nu$, providing $\nu k_s \ll U$ and $\nu\omega \ll 1$. The effective longitudinal wave speed of the fluid in the presence of the flow is simply

$$c_l \approx c, \tag{8}$$

with

$$k_l = \omega / c_l. \tag{9}$$

The ‘correspondence’ conditions given by Eqs. (6) and (8) can be employed in the modelling framework to estimate the relative effect of different surface coatings on the noise generated by a turbulent boundary layer [8] adjacent to those surfaces. Transformation of elastic waves in layered structures has been well studied [9, 10, 11, 12]. The coefficients of reflection, transmission and absorption are derived by requiring continuity of pressure, stress and displacement across the interfaces between the layers. The processes are modelled numerically by implementing known theory for plane-wave reflection [9, 10, 11]. Results for arbitrary waves can be modelled as linear combinations of plane waves but the important conclusions do not require this to be done. The boundary layer turbulence is modelled as an ensemble of transverse waves of arbitrary frequency distribution and propagation direction, with vector potential of one wave component

$$\mathcal{A} = \mathcal{A}_0 \exp(-i\omega t + i\xi x - i\eta_s z), \tag{10}$$

incident on the boundary adjacent to the flow from the halfspace $z > 0$, and a reflected longitudinal component with scalar potential

$$\varphi = \varphi_0 \exp(-i\omega t + i\xi x + i\eta_l z). \tag{11}$$

The direction z is normal to the layers and into the fluid, and x is parallel to the layers. The wavenumber components ξ, η_s and η_l are related by

$$\eta_s^2 = k_s^2 - \xi^2, \tag{12}$$

$$\eta_l^2 = k_l^2 - \xi^2, \tag{13}$$

with equivalent expressions for the lower layers. The incidence angle θ is related to the horizontal wavenumber component through

$$\xi = k_s \sin(\theta). \tag{14}$$

The transformation coefficients can be estimated as angle averaged values assuming uniform angle distribution.

As a demonstration of analytical capability of the method the generation of acoustic waves is considered for a single vortex wave (‘harmonic’) of frequency ω of Eq. (10). See also the results of Danilov and Mironov [13] on the same subject. Assume the ratio of the scalar potential of the reflected longitudinal wave to the magnitude of the vector potential of the incident transverse wave is V. Then V represents the coefficient of conversion of transverse waves into longitudinal waves at a plane surface. For a given input medium, the reflected energy is proportional to $|V|^2$ (where $|\ |$ denotes taking the amplitude of a complex quantity). For a simple interface between a fluid and a fluid-like medium (such as a rubber) Ref. [13] gives the approximation

$$\frac{V}{V_*} = \frac{1 - \rho^{(1)}/\rho - 2(\eta_l^{(1)}/k_s)(-1 + \sqrt{\rho\mu^{(1)}/\rho^{(1)}\mu})}{(\rho^{(1)}/\rho + \eta_l^{(1)}/\eta_l)(1 + \sqrt{\rho\mu/\rho^{(1)}\mu^{(1)}})} \tag{15}$$

where $\rho^{(1)}$ and $\mu^{(1)}$ are the density and shear modulus in the reflecting fluid-like half-space, and ρ is the density of the fluid medium. Wave components η_l, $\eta_l^{(1)}$ and k_s are connected to the incidence angle,

Table 1: Water Properties

Parameter	Value	Units
ρ	1000	$\mathrm{kg/m^3}$
c_l	1500	$\mathrm{m/s}$
ν	1×10^{-6}	$\mathrm{m^2/s}$
U	3	$\mathrm{m/s}$

complex moduli and wave speeds through Eqs. (3), (4), (12), (13), and (14). V has been normalised here by V_*, which is the transformation coefficient at a rigid boundary (obtained by setting $\rho^{(1)} = \infty$ in the expression for V).

Eq. (15) provides insightful criteria for material selection for flow noise reduction that would be very difficult to deduce by other means. When $c_s^{(1)} \gg c_s$, as with a water–rubber boundary, Eq. (15) at angles close to normal incidence can be simplified to

$$V/V_* \simeq 1 - \rho^{(1)}/\rho \,, \tag{16}$$

implying that the intensity of turbulent boundary layer noise can be significantly decreased provided the material underlying the turbulent boundary layer has fluid-like properties (such as with rubber) and its density is close to the density of the fluid. Note that Eq. (16) is a complex amplitude ratio and can be negative. This is strikingly different from an intuitive assumption of impedance match, $\rho^{(1)} c_s \simeq \rho c$. The equation is, however, only valid for an infinite half-space boundary. More complex multi-layer-material calculations, which avoid this assumption, can be made using the theory of Lévesque and Piché [9]. Some numerical results are shown in the next section.

More realistic predictions for noise intensity from turbulent flow over elastic interface can be derived by summation of relative contributions from all vorticity harmonics v_ω from a spectrum of a wall-bounded turbulent flow, $E_\omega \propto |v_\omega|^2$ (i.e. by evaluation of convolution integral of (15) and E_ω). For a particular flow the spectrum E_ω can be calculated numerically or deduced from some analytical models (see [14] and references therein).

3. Numerical Results

In this section the formalism proposed above is applied to two test cases to estimate the effect of elastic materials at the flow boundary on turbulent boundary layer noise.

3.1 Noise from turbulent flow over an elastic interface

The first modelling case is for flow noise generated by turbulent boundary layer over an elastic interface moving at 3 $\mathrm{m/s}$ relative to water, as discussed above. The relative efficiencies of two rubber-like materials that can be used as a coating for flow noise reduction are presented in Fig. 1. The rubbers, of centimetre-scale thickness, cover steel of 20 mm thickness which is backed by air. The water parameters are given in Table 1, and the rubber-like materials have full complex frequency dependence (not specified here). The figure shows the field-averaged reflection of longitudinal waves V, with random incidence, relative to the reflection from the steel alone, V_{steel}. The calculation again uses the theory of [9]. The blue curve is for an actual rubber material, with known frequency-dependent elastic moduli, and the red curve is for a nominal rubber material designed as a good absorber of longitudinal pressure waves. Note that there is very little difference between the absorbing coating and a simple uniform layer of rubber. This comparison demonstrates the predictive capability of the proposed method.

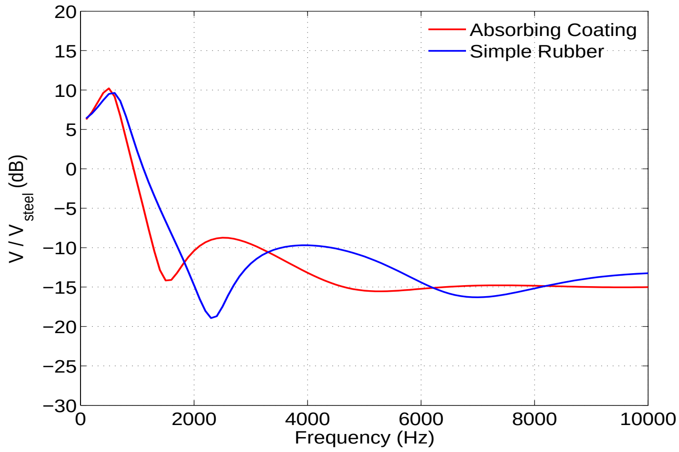

Figure 1: Relative efficiency of two rubber coating layers as a function of flow noise frequency.

Table 2: Rubber properties for section 3.2

Parameter	Value	Units
ρ	1200	$\mathrm{kg/m^3}$
$\lambda + 2\mu$	$3000 \times (1 + 0.01i)$	MPa
μ	$70 \times (1 + 0.8i)$	MPa

3.2 Noise from a body in flow

As the second modelling case we consider the acoustic noise generated by a turbulent flow around body in water with properties given by Table 1. This setting is typical for experimental investigation of flow noise [15, 16, 17]. We assume that the body has a slender shape with an internal air-filled reverberant volume $\mathcal{V} = 1\,\mathrm{m}^3$ and surface area $S = 7\,\mathrm{m}^2$, has a steel shell of 6 mm thickness, and may be externally coated by a 40 mm layer of elastomeric (rubber) material as defined by the properties in Table 2 and Eqs. (3) and (4). Our primary interest is the effect of elastic coating on flow noise and in inferring the relationship between acoustic noise intensity measured inside the body and in the far field. We assume the body moves in the water with velocity $3\,\mathrm{m/s}$, with parameters as defined in Table 1.

This problem was numerically treated with the framework of transformation of elastic waves in layered structures described above [8]. We modelled transformation of waves of each type (shear and sound) at the boundary travelling in both directions through the body surface (inward and outward from the volume $\mathcal{V}$). In order to properly account for conditions of low reverberation and the effect of internal volume losses, both direct and reverberant components of the internal acoustic field need to be considered [18]. A relation between the outward acoustic intensity and acoustic intensity inside the body can be found from the balance of energy flow of the form

$$\frac{dE}{dt} = J_{\mathrm{in}} - J_{\mathrm{out}} - L_{\mathrm{v}}\,, \tag{17}$$

where J_{in} is the inward rate of flow of acoustic energy from turbulence boundary layer excitation, J_{out} is the outward flow of acoustic energy through the shell of the body, and L_{v} is the rate of volume loss (due to air absorption, for example). In the steady state the net energy flow is zero so $dE/dt = 0$.

Both the total acoustic power radiated from the body and the internal pressure can be derived in terms of vortex wave input. The resulting equations can be considerably simplified on the assumption that we can ignore the volume losses inside the body and that the interior is highly reverberent. By

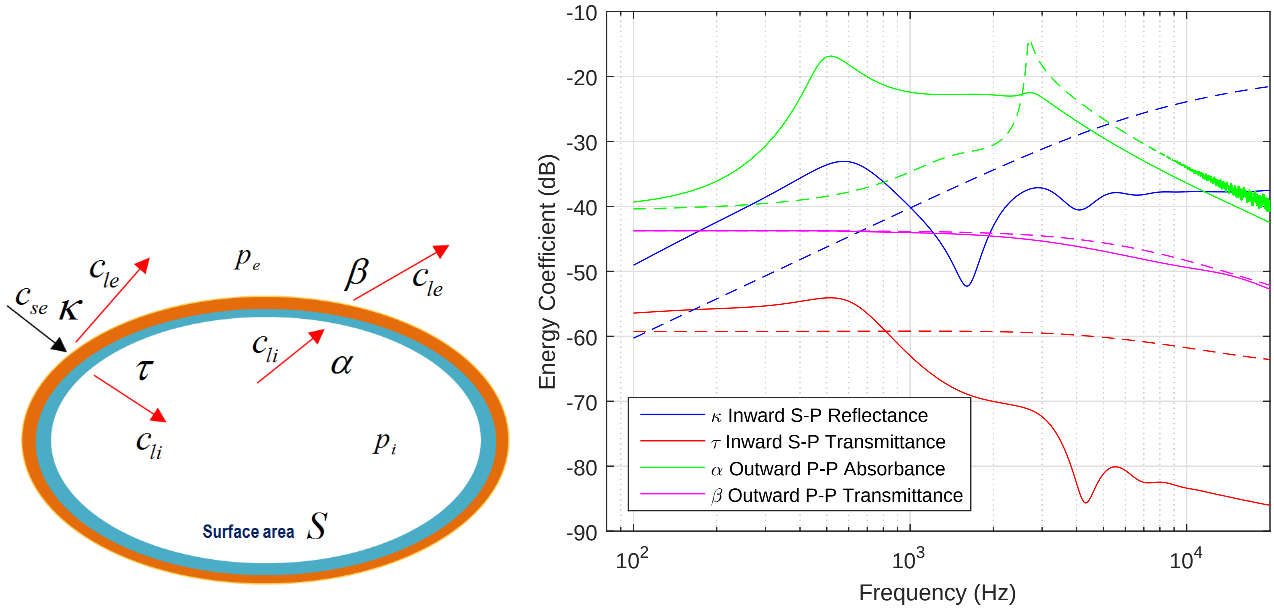

Figure 2: Conceptual diagram of the transmittance and reflectance parameters of the proposed formalism (left) and those parameters plotted (right) for a steel layer (dashed lines) and for steel with rubber coating (solid lines) as a function of flow noise frequency. Parameters α, β, κ and τ appear in Eq. (18).

further assuming a spherical distribution of acoustic power outside the body we obtain the following formula for the ratio

$$\frac{p_e}{p_i} = \frac{1}{4}\left(\frac{\Lambda}{\pi}\frac{Z_e}{Z_i}\frac{S}{R^2}\right)^{1/2}, \quad \Lambda = \left(\beta + \frac{\kappa\alpha}{\tau}\right), \tag{18}$$

where p_e is acoustic pressure at the range R from the body and p_{i} is average acoustic pressure inside the body. Here, $Z_{\mathrm{e}} = \rho_{\mathrm{e}} c_{\mathrm{e}}$ and $Z_{\mathrm{i}} = \rho_{\mathrm{i}} c_{\mathrm{i}}$ are the acoustic impedance of the external and internal media, respectively, and Λ is an aggregated parameter involving averaged energy transmission and reflection parameters of the wall of the body. The parameter κ is the coefficient of transformation of vortex waves (turbulence) into outward sound waves (noise) at the external surface of the body, τ is the coefficient of transformation of vortex waves into inward sound waves, β is the coefficient of transmission of outward acoustic waves at the internal surface, and α is the coefficient of absorption of outward acoustic waves through the surface of the body. All of these parameters must be appropriately spherically averaged (the equivalent of field-incidence values). Note that $\alpha \ll \beta$ and the average reflectance of the outward waves at the internal surface is equivalent to $1 - \alpha$. If there is no loss mechanism in the wall surface then $\alpha = \beta$. These parameters are schematically shown in Fig. 2.

Plots of parameters α, β, κ and τ as a function of frequency are presented in Fig. 2 for the wall of the body with and without the rubber coating, computed using the theory of [9]. These parameters allow estimation of the ratio given by Eq. (18) for any frequency with a targeted flow noise experiment design. This equation could be used to estimated radiated pressure from internal measurements. For instance, from Eq. (18) it is clear that in order to minimise the ratio $p_{\mathrm{e}}/p_{\mathrm{i}}$ (thereby maximising the interior pressure) one needs either to minimise 'material' parameter Λ (i.e. either β or ratio $\kappa\alpha/\tau$) and this can be translated to a selection coating material and its thickness. Note that a more comprehensive form of Eq. (18) includes the effect of bulk losses and includes a volume $\mathcal{V}$ dependence. More analysis of Eq. (18) and associated insights into targeted design of flow noise experiments will be presented elsewhere.

Figure 3 shows the ratio of external pressure to internal pressure of Eq. (18) for the body with both a bare and a rubber coated shell wall. More complex calculations accounting for volume absorption

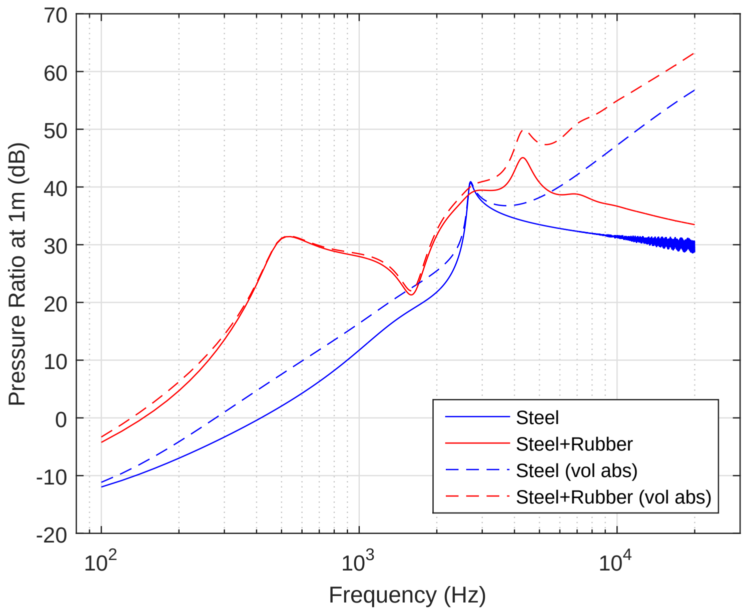

Figure 3: Ratio of external radiated pressure (normalised to 1 m) to internal pressure for the body with and without a rubber coating, ignoring internal volume absorption, using Eq. (18) (solid curves), and also by accounting for volume absorption (dashed curves).

in the internal air space (at 25°C and 50% relative humidity) are also shown as the dashed curves. Note that as the figure shows external pressure relative to internal pressure it should not be assumed from this plot that the case of steel coated with rubber radiates more than the case of pure steel.

The analysis that leads to Eq. (18) can also be used to evaluate the relative effect of a coating on the interior sound. For example, the relative effect of sonar window coating type on sonar self-noise can be estimated using this method.

4. Concluding remarks

We propose an analytical framework that explores the intrinsic analogy between sound generation by wall-bounded turbulent flows and transformation of elastic waves in layered elastic media. Several points must be made about the assumptions behind the analytical framework used in this paper. First, the flow is always assumed significantly subsonic. Second, it is also assumed that the compliance of the surface has little effect on the vorticity sources that are the starting point for this analysis (i.e. they are the same as for rigid surface). For situations where there is strong fluid–structure coupling this would not be the case. Third, plane wave reflection coefficients have been used for flow sources that are clearly not planar. However, arbitrary sources of vorticity in a turbulent boundary flow can often be decomposed into linear combinations of plane waves so if a consistent trend in reduced reflection is observed this is not likely to be an issue.

We believe that the proposed approach will be useful for rapid evaluation of numerous ’what-if’ scenarios in flow noise mitigation including the effects of flow speed, body size, fluid density and viscosity and elastic properties of coating material.

5. Acknowledgment

The authors thank Dr Chris Norwood for useful discussions and support of this work.

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